Statistical traffic modeling of MPEG frame size: Experiments and Analysis

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ABSTRACT

For guaranteed quality of service (QoS) and sufficient bandwidth in a communication network which provides an integrated multimedia service, it is important to obtain an analytical and tractable model of the compressed MPEG data. This paper presents a statistical approach to a group of picture (GOP) MPEG frame size model to increase network traffic performance in a communication network. We extract MPEG frame data from commercial DVD movies and make probability histograms to analyze the statistical characteristics of MPEG frame data. Six candidates of probability distributions are considered here and their parameters are obtained from the empirical data using the maximum likelihood estimation (MLE). This paper shows that the lognormal distribution is the best fitting model of MPEG-2 total frame data.

Keywords: Multimedia traffic, MPEG frame, DVD, quality of service.

1. INTRODUCTION

Recently, telecommunication networks have gradually changed and developed in accordance with the customer's needs for integrated services such as voice, data, multimedia, etc. These demands considerably increased multimedia traffic in entire networks, as well as demands for higher quality of service in the past several years. Thus, multimedia traffic has become one of the major sources of network traffic loads.

Multimedia information is stored and transmitted as compressed data in a device and channel, respectively. Two types of commonly used compression methods are MPEG-2, which is the second version of standards developed by the Moving Pictures Expert Group (MPEG) and H.264, which is also known as MPEG-4 Part 10 or MPEG-4 AVC (Advanced Video Coding). These two encoding methods are widely used commercially. For example, Blu-ray discs and digital HDTV use both MPEG-2 and MPEG-4 AVC as encoding method. An MPEG-2 format is used in current commercial DVD movies.

But if compressed MPEG-2 or MPEG-4 AVC (H.264) data is transmitted through a wired or wireless channel, taking into account the physical limitations of channel characteristics, the transmission must guarantee QoS. To deliver this QoS, the channel must provide sufficient bandwidth. Since the frame size of MPEG data is relatively massive and fluctuates considerably, it is important to achieve an analytical and tractable model of the compressed MPEG data.

Researchers have proposed a series of analytical MPEG source models in the literature. Nomura and colleague suggested the first order AutoRegressive (AR) model, which evaluated the burstiness of video information with measured autocorrelation, coefficient of variation, and probability distribution. This model now no longer fits a single video source [3], [4], [5]. Some researchers proposed a Markov chain model, such as the State Transition (ST) models or Discrete AutoRegressive (DAR) model. The ST model fits well to video data but has high complexity due to its many parameters, whereas the DAR model has just three parameters to be estimated, but is not always a good fit to VBR video data [2], [4], [5], [6]. Maglaris *et al.* presented two types of Markov chain models, Model A using a Gaussian distribution for the source rate and Model B using a discrete binomial distribution. But this model is a poor fit for MPEG data because a mean of the binomial distribution is always greater than its variance, whereas the relationship of the variance and mean of MPEG frame size is the opposite [1], [5]. Heyman, Frey and Lee suggested a gamma distribution-based model. Heyman's Gamma Beta AutoRegressive (GBAR) model consists of a gamma marginal distribution and a geometric autocorrelation. The GBAR model is tractable because it has just three parameters to be estimated. However, this model is not adequate to fit general MPEG data because Heyman built the GBAR model only on the Variable Bit Rate (VBR) videoconferences data and did not take into account the GOP [5], [7]. Lee suggested a sum of two gamma probability density functions (pdf) model, in which he added individual gamma pdfs, normalized them, and obtained parameters using a nonlinear least square algorithm. This model is simple and easily dealt with but, sometimes, each parameter of two gamma pdfs obtained from the nonlinear least square method has the same value [10]. Frey proposed a GOP GBAR model, which is an upgraded version of Heyman's GBAR model.

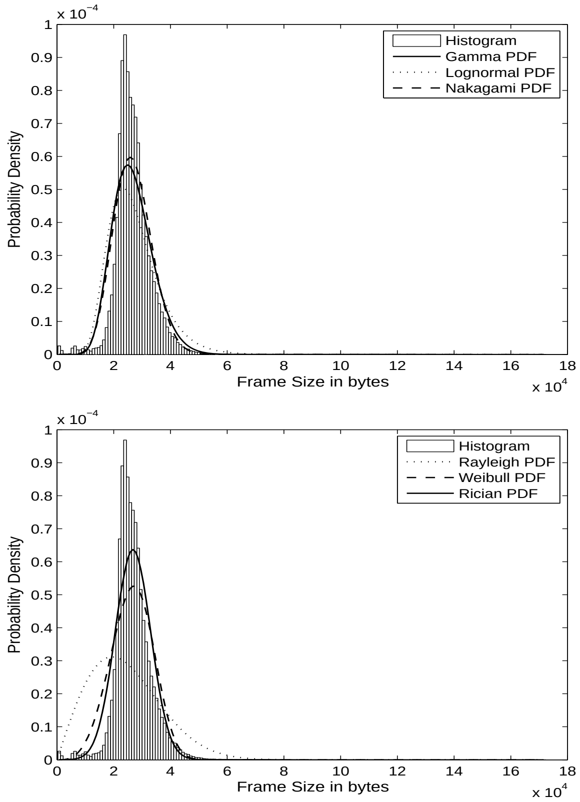

Figure 1 ***Matrix Reloaded*** B-frame pdf

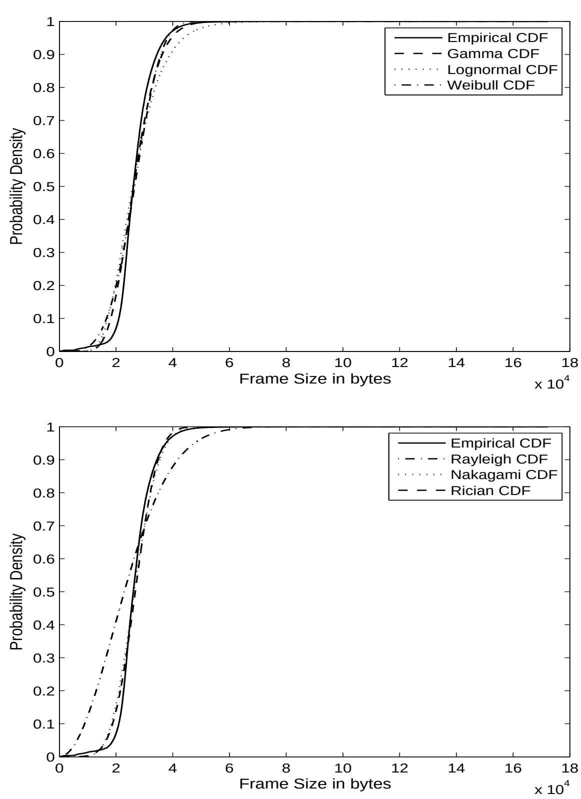

Figure 2 ***Matrix Reloaded*** B-frame cdf

He depicted the MPEG frame size model as a separated intra-(I), predicted-(P), and bidirectional-(B) frame model. B-frame is modeled as one gamma random variable but P- and I-frames are modeled as the sums of two and three gamma random variables, respectively. This model is also simple and analytical but does not mention a total frame size of the MPEG data [5].

In the present paper, a simple MPEG-2 frame size model is proposed to improve bandwidth utilization and to reduce transmission delay. In order to obtain a model that well fits the empirical data, we consider six different probability distributions in which the required parameters are determined by a MLE method, to compare and analyze the statistical characteristics of the MPEG-2 frame size. We show that the best model for the total MPEG-2 frame size is a lognormal distribution by means of a mean squared error (MSE) method of their pdf and cumulative distribution functions (cdf).

The general MPEG-2 format is described in the following section and a statistical model of the MPEG-2 frame data and the results of our experiments are presented in section 3. Section 4 gives a final remark and suggestion for future works.

2. DVD MOVIE CHARACTERISTICS

The DVD frame data used in this paper was extracted from commercial DVDs. This work is done for academic purpose only and there were no edits, collection, or distribution after the research.

MPEG-2 frames consist of three kinds of frames; I-, P-, and B-frames. An I-frame is called an Intra-coded frame, which contains the raw image data. A P-frame is a Predictive-coded frame which takes advantage of the data from the most recent I-frame or P-frame. I-frame and P-frame are called the reference frames. A B-frame is a Bidirectional-coded frame which contains the most compressed picture data because B-frames use both pictures in the following and preceding reference frames. The B-frame and P-frame are encoded based on the preceding and following frame data, meanwhile, the encoding of an I-frame is independent of all other frames [5], [8], [9].

The MPEG-2 video structural hierarchy is as follows: block, macroblock, slice, frame, GOP, frame sequence. Raw image data is divided into 8 by 8 pixel chrominance blocks. The data in each block is a coefficient of a discrete cosine transform. This transform does not change the information in the block. The macroblock is the fundamental unit for motion compensation and is a 16 by 16 array of pixels which consist of 4 blocks [8], [10], [11]. Pictures are divided into slices. A slice consists of an arbitrary number of successive macroblocks, but is typically an entire row of macroblocks [8]. The source picture is an adjacent rectangular array of pixels. A picture has two different types. One is an intra-coded picture that is coded without previous and future frame information. The other is nonintra-coded picture that is coded with other picture information [12]. Each encoded frame in encoded MPEG data is arranged in a GOP layer. It always starts with I-frame and has a length of 12 frames.

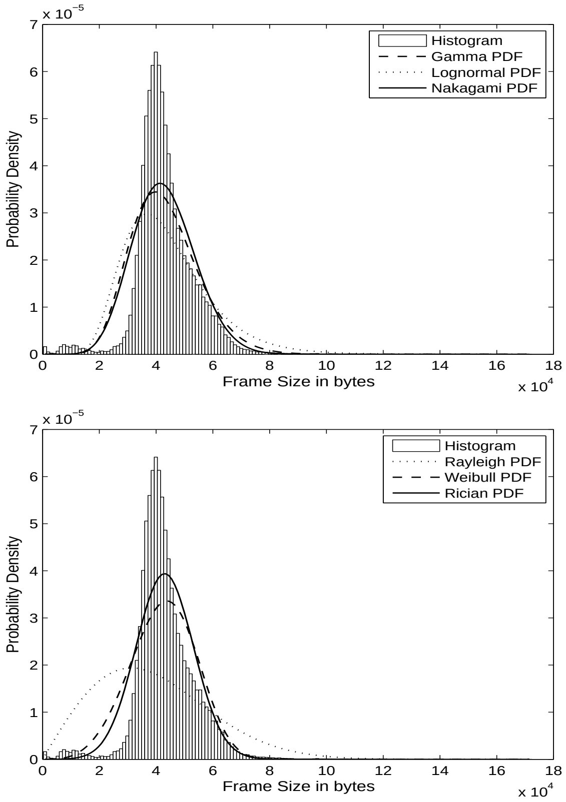

Figure 3 ***Matrix Reloaded*** P-frame pdf

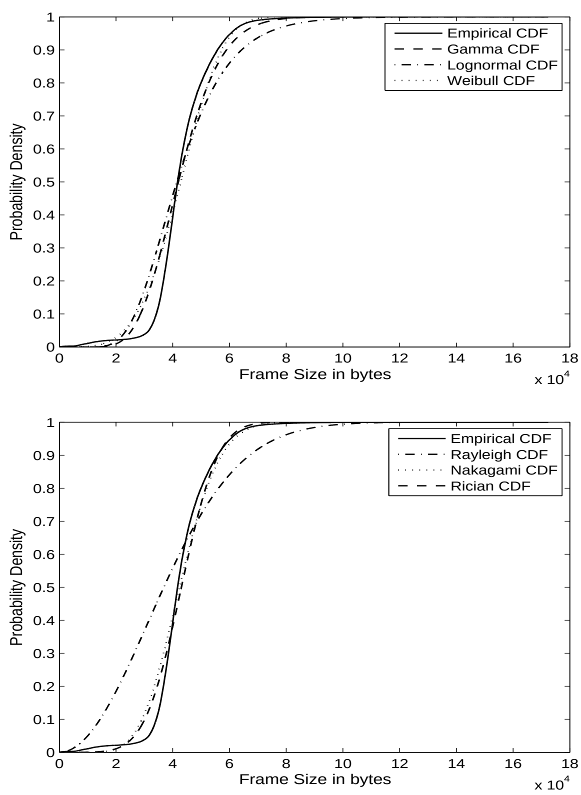

Figure 4 ***Matrix Reloaded*** P-frame cdf

The P- and B-frames are inserted into the sequence at regular intervals. A general structure of GOP is, therefore, a sequence of 12 frames, IBBPBBPBBPBB, but it is not a regular format [10], [11].

3. STATISTICAL MODELING OF MPEG FRAMES

In this section we perform an experimental analysis of the statistical characteristics of the MPEG-encoded commercial DVD movie frame data and show that the application of our results to empirical data is good enough to specify the characteristics of MPEG frame data. The procedure of the MPEG traffic modeling is generally composed of two parts. The first part is to find the statistical characteristics of the MPEG traffic data and build a precise model for a statistical analysis, and the other is to figure out a relationship between the statistical model and its traffic characteristics. This paper is especially focused on the first part, a modeling of the MPEG traffic data which is based on the B-, P-, and I-frames. The frame size model of the MPEG traffic has a similar statistical characteristic (i.e. the shape of pdf) of a gamma distribution, a log-normal distribution, a rayleigh distribution, a weibull distribution, a nakagami distribution, and a rician distribution.

This paper presents six similar shapes of probability distributions and compares these pdfs and cdfs with that of the original data. To figure out a probability histogram from the frame data, we use a Freedman-Diaconis method as a decision rule of a bin size: [13]

Table 1 ***Matrix Reloaded*** B-frame error

Probability Distributions	PDF errors [bytes]	CDF errors [bytes]
Rician	3.2762×10^{-11}	0.0017
Nakagami	3.6301×10^{-11}	0.0022
Gamma	4.1884×10^{-11}	0.0030
Weibull	5.7357×10^{-11}	0.0044
Lognormal	6.9409×10^{-11}	0.0070
Rayleigh	1.6511×10^{-10}	0.0307

$$\text{Bin\_Size} = 2 \times \text{IQR}(x) \times n^{-1/3} \quad (1)$$

where the IQR(x) is the interquartile range of the frame and n is the number of observations in sample x. The parameters which specify the statistical characteristics of each probability distribution are estimated by the MLE method. In addition, we measured the MSE to evaluate the best fitting model of the MPEG frame size. The followings are pdf formulae of each candidate distribution. For a gamma distribution,

$$f_{GAM}(x) = \frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)}, \quad x > 0 \quad (2)$$

where k is a shape parameter and θ is a scale parameter. Both parameters are positive and real. For a lognormal distribution,

$$f_{LOGN}(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}, \quad x > 0 \quad (3)$$

where μ is a location parameter (mean) and σ is a scale parameter (standard deviation).

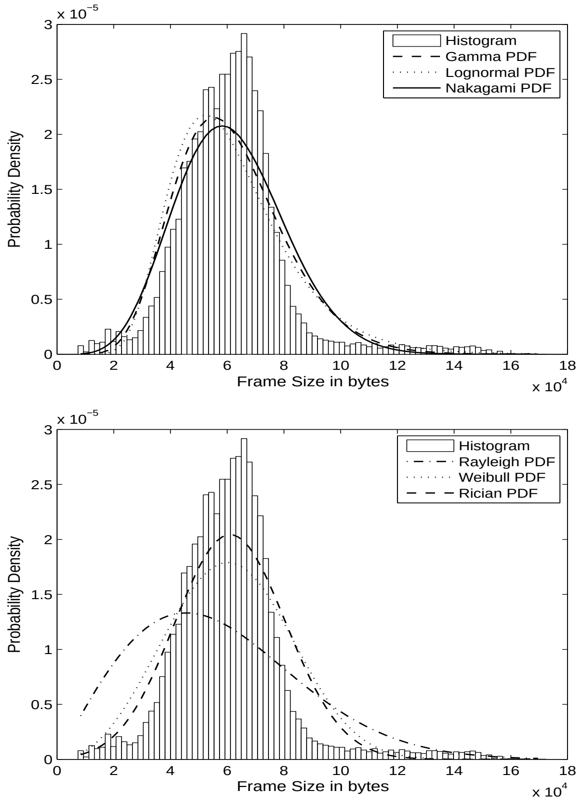

Figure 5 ***Matrix Reloaded*** I-frame pdf

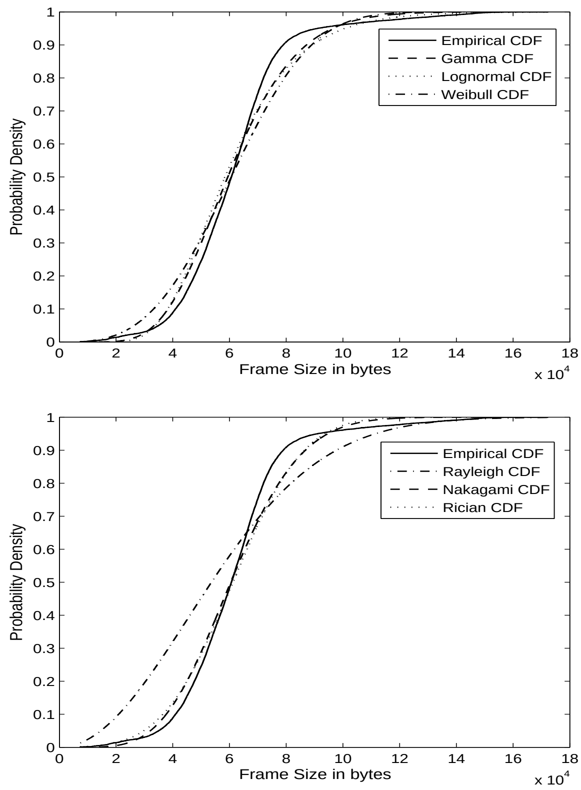

Figure 6 ***Matrix Reloaded*** I-frame cdf

For a nakagami distribution,

$$f_{NAKA}(x) = \frac{2\mu^{\mu}}{\omega^{\mu}\Gamma(\mu)} x^{2\mu-1} e^{-\frac{\mu}{\omega}x^2}, \quad x > 0 \tag{4}$$

where μ is a shape parameter and ω is a spread parameter. Both parameters are positive and real ($\mu \geq 0.5$). For a weibull distribution,

$$f_{WEIB}(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}, \quad x > 0 \tag{5}$$

where λ is a scale parameter and k is a shape parameter. Both parameters are positive and real. For a rayleigh distribution,

$$f_{RAYL}(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, \quad x > 0 \tag{6}$$

where σ is a scale parameter and is positive and real. For a rician distribution,

$$f_{RICI}(x) = \frac{x}{\sigma^2} e^{-\frac{x^2+\nu^2}{2\sigma^2}} I_0\left(x\nu/\sigma^2\right), \quad x > 0 \tag{7}$$

where $I_0\left(x\nu/\sigma^2\right)$ is the modified Bessel function of the first kind with order zero. σ is a scale parameter ($\sigma \geq 0, \nu \geq 0$).

Heyman and Frey already proposed a gamma-based frame size model. Especially, Frey and his colleague suggested that the size of an MPEG B-, P-, and I-frame could be modeled as one gamma random variable, the sum of two gamma random variables, and the sum of three gamma random variables, respectively [5], [7].

Table 2 ***Matrix Reloaded*** P-frame error

Probability Distributions	PDF errors [bytes]	CDF errors [bytes]
Rician	2.9434×10^{-11}	0.0029
Nakagami	3.3355×10^{-11}	0.0037
Gamma	3.8642×10^{-11}	0.0051
Weibull	4.3932×10^{-11}	0.0054
Lognormal	6.0069×10^{-11}	0.0105
Rayleigh	1.1406×10^{-10}	0.0324

Table 3 ***Matrix Reloaded*** I-frame error

Probability Distributions	PDF errors [bytes]	CDF errors [bytes]
Rician	9.4135×10^{-12}	0.0025
Nakagami	8.9577×10^{-12}	0.0020
Gamma	9.1945×10^{-12}	0.0020
Weibull	1.5444×10^{-11}	0.0045
Lognormal	1.2100×10^{-11}	0.0029
Rayleigh	3.9521×10^{-11}	0.0198

However, this model is not always successful in fitting the general MPEG frame data. Figure 1 and Figure 2 show the *Matrix Reloaded* DVD B-frame histogram, pdf and its empirical cdf, respectively. The six probability distributions overlap on the B-frame data histogram and empirical cdf in the figure 1 and figure 2. As we can see, the *Matrix Reloaded* B-frame histogram is more fitted to a rician distribution rather than a gamma distribution. We can validate the numerical results of the fitting problem. Table 1 contains the pdf and cdf errors of each probability distribution measured by MSE method.

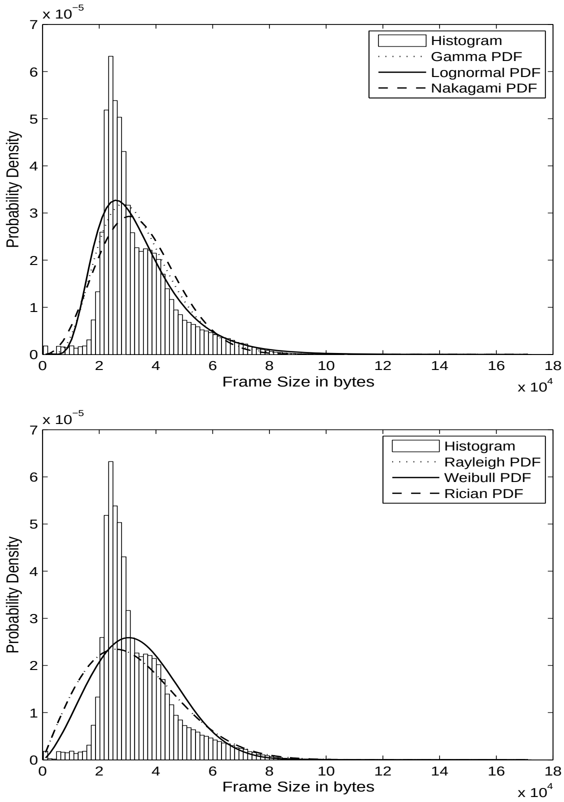

Figure 7 ***Matrix Reloaded*** total frame pdf

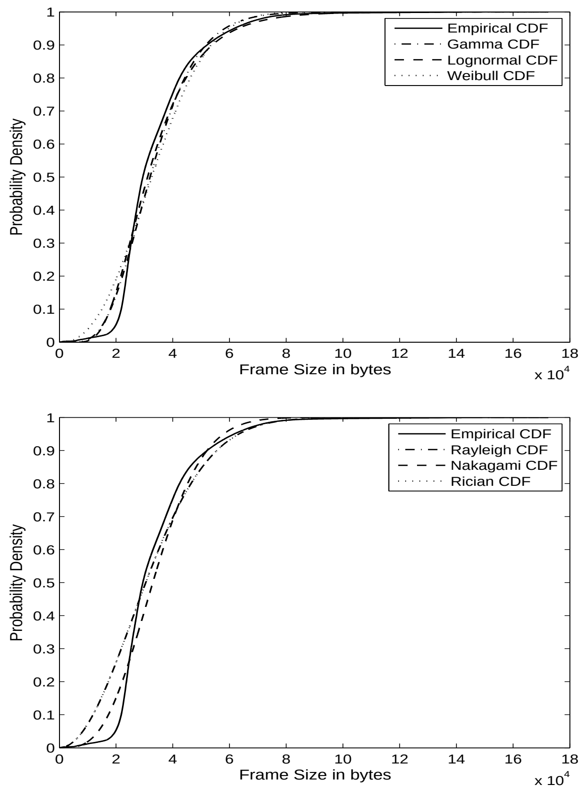

Figure 8 ***Matrix Reloaded*** total frame cdf

Table 4 Estimated parameters for the *Matrix Reloaded* Total frame

Probability Distributions	Parameters	
Gamma	k : 6.3285	θ : 5.33×10^{3}
Lognormal	μ : 10.3457	σ : 0.4296
Nakagami	μ : 1.6879	ω : 1.34×10^{9}
Weibull	λ : 3.79×10^{4}	k : 2.4164
Rician	ν : 64.0363	σ : 2.59×10^{4}
Rayleigh	σ : 2.59×10^{4}	

Table 5 *Matrix Reloaded* Total frame error

Probability Distributions	PDF errors [bytes]	CDF errors [bytes]
Lognormal	2.9391×10^{-11}	0.0017
Gamma	3.2055×10^{-11}	0.0020
Nakagami	4.0755×10^{-11}	0.0035
Weibull	4.8376×10^{-11}	0.0045
Rayleigh	5.7566×10^{-11}	0.0066
Rician	5.7566×10^{-11}	0.0066

As a result of numerical and graphical evaluations, the best fitting model to the *Matrix Reloaded* B-frame data is not a gamma distribution but a rician distribution. From figure 3 to figure 6, they show the statistical characteristics of other frame data (i.e. P- and I- frame). The best fitted models of the P-frame and I-frame are a rician distribution and a nakagami distribution, respectively. Table 2 and 3 also support these results numerically.

The statistical characteristics of the *Matrix Reloaded* total frames that B-, P-, and I- frames are merged together is shown in figure 7. Apparently, a lognormal distribution is well fitted to the total frame histogram. And numerical results in Table 5 support the fitting results. In the second fitted distributions of figure 7, the rician distribution coincides with the rayleigh distribution. The cause of this correspondence is the modified Bessel function in the rician pdf and its parameter. Table 4 shows the estimated parameters of the *Matrix Reloaded* total frame. In general, when $\nu = 0$, the rician distribution is reduced to the rayleigh distribution. We can make sure of this relationship in Eq. (6) and Eq. (7). A general equation of the modified Bessel function of the first kind is

$$I_{\nu}(z) = \left(\frac{z}{2}\right)^{\nu} \sum_{k=0}^{\infty} \frac{\left(z^2/4\right)^k}{k!\Gamma(\nu+k+1)} \tag{8}$$

The order zero of the modified Bessel function of the first kind is

$$I_0\left(\frac{x\nu}{\sigma^2}\right) = \sum_{k=0}^{\infty} \frac{\left(z^2/4\right)^k}{\left(k!\right)^2}\Bigg|_{z=\frac{x\nu}{\sigma^2}} \tag{9}$$

If we consider the Eq. (7) and Eq. (9) together with estimated parameters, we can get the result of a rayleigh distribution. We extended this experiment to several DVD movies. The statistical characteristics of total frame size of other DVD movies also follow the lognormal distribution.

8. CONCLUSIONS

In the present paper, we propose a statistical model of the multimedia traffic, especially the MPEG-2 total frame size. We measure and find the best fitting model of empirical MPEG-2 frame data. The data used in this paper is extracted from the commercial DVD movie *M-atrix Reloaded*. We evaluate a histogram of total frame data by means of the MLE and MSE methods. The results of these experiments are that the best fitting model of an MPEG-2 frame data is a lognormal distribution.
Moreover, the extended analyses applied to other DVD data also show the similar results. Based on this result, future research will investigate the statistical relationship of each individual frame and total frame. This research will help the design of multimedia home network systems.

9. REFERENCES

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