Effects of Interference on Capacity in Multi-Cell CDMA Networks

Robert AKL, Asad PARVEZ, and Son NGUYEN Department of Computer Science and Engineering University of North Texas Denton, TX, 76207

ABSTRACT

An overwhelming number of models in the literature use average interference for calculation of capacity of a CDMA network. In this paper, we calculate the actual per-user interference and analyze the effect of user-distribution on the capacity of a CDMA network. We show that even though the capacity obtained using average interference is a good approximation to the capacity calculated using actual interference for a uniform user distribution, the deviation can be tremendously large for nonuniform user distributions. We also present an analytical model for approximating the user distributions using 2-dimensional Gaussian distributions by determining the means and the standard deviations of the distributions for every cell. This allows us to calculate the inter-cell interference and the reverse-link capacity of the network. We compare our model with simulation results and show that it is fast and accurate enough to be used efficiently in the planning process of large CDMA networks. Keywords: Inter-cell interference, Capacity, CDMA, User dis-

tribution, 2-D Gaussian.

1. INTRODUCTION

The ability to offer greater capacity and multi-rate transmission with backward compatibility, seamless integration, and easier migration path to 3G cellular systems has fueled the widespread deployment of CDMA systems all over the world. But the principal attraction has always been the increased capacity over TDMA and FDMA systems, which explains multitudes of research devoted to study capacity of CDMA systems. And since CDMA capacity is limited by interference [1], [2], it is inevitable to investigate the factors involved in determining interference. One of such variables is the user's distance from its base station.

It has been shown in [3]-[6] that the capacity of a CDMA network is reverse link limited, and hence our study is confined to reverse link capacity. One of the principal characteristics of a CDMA network is that the capacity of the system is a function of total interference experienced by the network, and is upper bounded by the cell experiencing the most interference. Thus, it is imminent to characterize total inter-cell interference seen by a single cell in terms of the user distribution in every other cell for determining capacity in that single cell. Traditionally, the total interference contributed by a cell has been viewed as an approximation, determined by simply multiplying the number of users in that cell by the average interference offered by that cell [1]. In other words, a user placed anywhere within a cell generated the same amount of interference. Clearly, a more realistic approach will use per-user interference as a function of its actual distance to the point of interest. There is a dearth of literature where actual distance was used in the interference model. In [7], even though interference was calculated using actual distance, the capacity calculations were done using mean value of interference. User positions were varied over time, but the number of users was kept constant.

In this paper, we use a model where interference is calculated with actual distance to investigate the effect of user distribution



Fig. 1. Inter-cell interference on cell *i* from users in cell *j*.

over reverse link capacity. Computer simulations of a CDMA network are carried out where interference is calculated in real time as the network is being populated. We assume several user distributions. We investigate the cases of equal capacity in every cell as well as the capacity obtained through optimization techniques discussed in [8]. We also present an analytical model for the approximation of the user distribution using 2-dimensional Gaussian distributions by determining the means and the standard deviations of the distributions for every cell. We verify the numerical analysis results published in [8], and also show that it is possible to have much higher or lower capacity if actual interference is used for specific user distributions.

The remainder of this paper is organized as follows. In section 2, we present the traditional model for calculation of relative average interference. In section 3, we describe our model for calculating capacity using actual relative interference. In section 4, we describe the definition of equal and optimized capacity. In section 5, results from simulation are shown and compared to analytical results. Finally, the conclusions drawn from this paper are summarized in section 6.

2. RELATIVE AVERAGE INTER-CELL INTERFERENCE MODEL

Consider two cells *i* and *j*. The user is power controlled by the base station of cell *j*, and is at distance $r_j(x, y)$ from the base station. The distance of the same user from the base station in cell *i* is $r_i(x, y)$ as shown in Fig. 1. Let n_j be the number of users in cell *j*, denoted by region C_j and area A_j =Area(C_j). The user's transmitter power gain equals the propagation loss in cell *j*. The propagation loss is generally modeled as the product of the *m*th power of distance and a log-normal component representing shadowing losses. The large scale path loss and shadow fading are assumed to be circumvented by the power control mechanism. However, it cannot compensate for the fast fluctuations of the signal power associated with Rayleigh fading [1]. Now let χ_i denote the Rayleigh random variable that

represents the fading on the path from this user to cell *i*. Then, the relative average interference at cell i caused by all the users present in cell j is given by [8],

$$I_{ji} = \mathbb{E}\left[\int_{C_j} \frac{r_j^m(x, y) 10^{\zeta_j/10}}{r_i^m(x, y)/\chi_i^2} \frac{n_j}{A_j} \, dA(x, y)\right], \quad (1)$$

where ζ is the decibel attenuation due to shadowing, with zero mean and standard deviation σ_s . This reduces to [9]

$$I_{ji} = e^{(\gamma \sigma_s)^2} \frac{n_j}{A_j} \int_{C_j} \int_{C_j} \frac{r_j^m(x,y)}{r_i^m(x,y)} \, dA(x,y), \qquad (2)$$

where $\gamma = \ln(10)/10$. Eq. (2) is used to calculate the relative average inter-cell interference for a uniform user distribution. To obtain the per-user inter-cell interference, f_{ii} , I_{ii} is divided by the total number of users in cell j. Note that in this model, f_{ii} equals zero. f_{ji} can be viewed as elements in a two dimensional matrix F with i, j = 1, ..., M, where M is the total number of cells in the network. Each column i of F contains the peruser inter-cell interference exerted by cell j on every other cell *i*. Consequently, the total relative average inter-cell interference experienced by cell i is simply the summation of the product of number of users n_j in cell j and their respective per-user interference factor f_{ji} , which is the column vector i in F,

$$I_{i} = \sum_{j=1}^{M} n_{j} F[j, i].$$
(3)

Once matrix F is computed in advance, the above calculation is adequately fast since it requires only M lookups in the matrix. However, the interference caused by a user is independent of its location within a give cell.

For a general user distribution, the relative average inter-cell interference becomes [10]

г

$$I_{ji} = \mathbb{E}\left[\int_{C_j} \int_{C_j} \frac{r_j^m(x, y) 10^{\zeta_j/10}}{r_i^m(x, y)/\chi_i^2} \ w(x, y) \ dA(x, y)\right], \quad (4)$$

where w(x, y) is the user distribution density at (x, y). We define κ_{ji} to be the per-user relative inter-cell interference factor from cell j to BS i, i.e.,

$$\kappa_{ji} = \frac{e^{(\gamma \sigma_s)^2}}{A_j} \int_{C_j} \int_{C_j} \frac{r_j^m(x,y)}{r_i^m(x,y)} w(x,y) \ dA(x,y).$$
(5)

If the user distribution density can be approximated, then, κ_{ji} needs to be calculated only once. We model the user distribution by a 2-dimensional Gaussian distribution as follows

$$w(x,y) = \frac{\eta}{2\pi\sigma_1\sigma_2} e^{-\frac{1}{2}(\frac{x-\mu_1}{\sigma_1})^2} e^{-\frac{1}{2}(\frac{x-\mu_2}{\sigma_2})^2},$$
 (6)

where η is a user density normalizing parameter. We show that by specifying the means μ_1 and μ_2 and the variances σ_1 and σ_2 of the distribution for every cell, we can approximate a wide range of user distributions ranging from uniform to hot spot clusters. We compare these results with simulations and determine the value of η experimentally.

3. RELATIVE ACTUAL INTER-CELL INTERFERENCE MODEL

If the user's exact location within a cell is taken into account for determining its interference to the network, the matrix F cannot

be calculated in advance. For a user k in cell j, the relative interference offered by this user to cell i is

$$\left(U_{ji}\right)_{k} = e^{\left(\gamma\sigma_{s}\right)^{2}} \left(\frac{r_{j}}{r_{i}}\right)^{m}.$$
(7)

Hence, the total inter-cell interference at cell *i* caused by n_i users in cell j is given by

$$I_{i} = \sum_{j=1}^{M} \sum_{k=1}^{n_{j}} (U_{ji})_{k}, \text{ for } i \neq j.$$
(8)

The calculation of the interference now is much more time consuming since the matrix F has to be constructed every time a user enters the network. However, instead of relative average interference values, it stores the actual interference caused by each user on every cell.

4. CAPACITY

The capacity of a CDMA network is determined by maintaining a lower bound on the bit energy to interference density ratio which is given by [11]

$$\left(\frac{E_b}{I_0}\right)_i = \frac{E_b}{\alpha(RE_b)\left(n_i - 1 + I_i\right)/W + N_0},$$

for $i = 1, ..., M,$ (9)

where the network has a spread signal bandwidth of W, information rate of R bits/sec, voice activity factor of α , background noise spectral density of N_o , and n_i users in cell *i*. To achieve the required bit error rate $\left(\frac{E_b}{I_0}\right)_i$ must be maintained above a certain threshold. If τ denotes that threshold, then by rewriting the above equations, we get an upper bound on the number of users in every cell *i*:

$$n_i + I_i \le \frac{W/R}{\alpha} \left(\frac{1}{\Gamma} - \frac{1}{E_b/N_0} \right) + 1 \stackrel{\triangle}{=} c_{eff},$$

for $i = 1, ..., M.$ (10)

A feasible user configuration of the network is a set of users in their respective cells that satisfies the above equations. In other words, a new user is admitted to the network if its admittance still maintains the above inequalities for every cell. The right hand side of the equations is a constant that depends on the system parameters, and essentially determines the effective number of channels available to the network.

Equal Capacity

Equal capacity is defined as the requirement that all cells have an equal number of users, i.e. $n_i = n$ for all *i*.

Optimized Capacity

The optimized network capacity is the solution to the following optimization problem [8]

$$\max_{\underline{n}} \qquad \sum_{i=1}^{M} n_i,$$

subject to
$$n_i + I_i \le c_{eff},$$

for $i = 1, ..., M.$ (11)

In [8], the authors describe techniques to solve this optimization problem. The same techniques are used in this paper.



Fig. 2. Equal capacity with uniform user distribution using average interference.

5. NUMERICAL RESULTS

The simulator used for comparison of the two models is an extension of the software tools CCAP (CDMA Capacity Allocation and Planning) [12]. CCAP, written in MATLAB, was developed at Washington University in St. Louis for numerical analysis of optimization techniques developed in [8] to determine and enhance the capacity of CDMA networks. The analysis in CCAP was carried out with relative average interference. Our simulator can verify the numerical analysis results produced by CCAP and also use the actual distance of users for calculation of interference. It has the capability to populate, remove and relocate users within the network.

The testbed is an exact configuration of the network model used in [8]. The COST-231 propagation model with a carrier frequency of 1800 MHz, average base station height of 30 meters and average mobile height of 1.5 meters is used to determine the coverage region. The path loss coefficient, m, is 4. The shadow fading standard deviation, σ_s , is 6 dB. The processing gain, $\frac{W}{R}$, is 21.1 dB. The bit energy to interference ratio threshold, Γ , is 9.2 dB. The interference to background noise ratio, $\frac{I_0}{N_0}$, is 10 dB. The voice activity factor, α , is 0.375. These parameters give c_{eff} of 38.25, which is the implicit upper bound on the relative interference in every cell.

We analyzed a twenty-seven cell CDMA network with uniform user distribution. We compared the capacity calculated with relative average interference as well as relative actual interference. For the equal capacity case, the network capacity was 486 with 18 users per cell if average interference was used. The capacity in each cell is given in parentheses and the total relative average inter-cell interference is given in brackets as shown in Fig. 2. Using the actual distance for interference calculation always resulted in 17 users per cell as shown in Fig. 3.

The optimization in Eq. (10) resulted in a network capacity of 559 users when using average interference as shown in Fig. 4. Using the actual relative interference, a simulation was run for several hundred trials. Figures 5, 6, and 7 show three of those trials, with network capacity of 554, 564, and 568 respectively. The values are found to be adequately close to those in Fig.



Fig. 3. Equal capacity with uniform user distribution using actual interference.



Fig. 4. Optimized network Capacity of 559 using average interference.

4. Fig. 8 shows the cell capacity for those three trials and for the average interference case. Furthermore, fifty simulation trials reveal the average of each cell capacity converging towards the optimized values obtained numerically, as shown in Fig. 9.

In what follows, we show that by using 2-D Gaussian distribution, we can model many different scenarios including users uniformly distributed, users clustered at the center of the cells, and users at the cells' boundaries. We analyzed the network with different values of σ_1 and σ_2 , while keeping μ_1 and μ_2 equal to zero in (6). Table I shows the maximum number of users in every cell for the 27 cell WCDMA network as the values of σ_1 and σ_2 are increased from 5000 to 15000 while μ_1 =0 and μ_2 =0. This results in users spread out (almost uniformly) in the cells. Fig. 10 shows the 2-D Gaussian approximation of users uniformly distributed in the cells with $\sigma_1=\sigma_2=12000$. The total number of users is 548. This compares well with simulation results presented in Fig. 5, which yields a total number of users



Fig. 5. Simulated network capacity of 554 using actual interference.



Fig. 6. Simulated network capacity of 564 using actual interference.

equal to 554 when they are placed uniformly in the cells.

The user distribution was uniform for all the above cases. However, for non-uniform user distribution in the cells, our simulation results showed the extreme diversity in the capacity of the network. Changing the placement of the users in such a way that they cause minimum interference to the whole network, instead of placing them randomly in their cells, yielded a much higher capacity of 1026 users with 38 users in each cell. Fig. 11 shows the pattern in which users were placed. The high concentration of users near their respective base station is justified, since intuitively, the obvious way to have minimum interference on the network is to remain closer to one's base station, reducing the power gain required to maintain a desired signal to noise ratio.

Table II shows the maximum number of users in every cell for the 27 cell WCDMA network as the values of σ_1 and σ_2 are increased from 100 to 400 while $\mu_1=0$ and $\mu_2=0$. This results



Fig. 7. Simulated network capacity of 568 using actual interference.



Fig. 8. Comparison of cell capacity for 3 simulation trials, with optimized capacity obtained numerically.



Fig. 9. Comparison of average cell capacity for 50 simulation trials, with optimized capacity obtained numerically.

TABLE I

The maximum number of users in every cell for the 27 cell WCDMA network as the values of σ_1 and σ_2 are increased from 5000 to 15000 while μ_1 =0 and μ_2 =0. This results in users spread out (almost uniformly) in the cells.

$\sigma = \sigma_1, \sigma_2$	5000	7000	10000	12000	15000	Uni Dist
$Cell_1$	18	18	18	18	18	18
$Cell_2$	18	18	18	18	18	18
$Cell_3$	18	18	18	17	17	17
$Cell_4$	18	18	18	17	17	17
$Cell_5$	18	18	18	18	18	18
$Cell_6$	18	18	18	17	17	17
Cell ₇	18	18	18	17	17	17
$Cell_8$	18	18	18	18	18	18
$Cell_9$	18	17	17	17	17	17
$Cell_{10}$	22	21	21	21	21	21
$Cell_{11}$	22	22	22	21	21	21
$Cell_{12}$	22	21	21	21	21	21
$Cell_{13}$	17	17	17	17	17	17
$Cell_{14}$	18	18	18	18	18	18
$Cell_{15}$	18	17	17	17	17	17
$Cell_{16}$	22	21	21	21	21	21
$Cell_{17}$	22	22	21	21	21	21
$Cell_{18}$	22	21	21	21	21	21
$Cell_{19}$	18	17	17	17	17	17
$Cell_{20}$	25	25	25	25	25	25
$Cell_{21}$	25	25	24	24	24	24
$Cell_{22}$	25	25	24	24	24	24
$Cell_{23}$	25	25	25	25	25	25
$Cell_{24}$	25	25	25	25	25	25
$Cell_{25}$	25	25	24	24	24	24
$Cell_{26}$	25	25	24	24	24	24
$Cell_{27}$	25	25	25	25	25	25
Total Users	565	558	553	548	548	548



Fig. 10. 2-D Gaussian approximation of users uniformly distributed in the cells. $\sigma_1=\sigma_2=12000$, $\mu_1=\mu_2=0$. The maximum number of users is 548.



Fig. 11. Maximum network capacity of 1026 using actual interference with best case non-uniform user distribution.



Fig. 12. 2-D Gaussian approximation of users densely clustered around the BSs. $\sigma_1=\sigma_2=100$, $\mu_1=\mu_2=0$. The maximum number of users is 1026.

in users densely clustered around the BSs. Fig. 12 shows the 2-D Gaussian approximation with $\sigma_1=\sigma_2=100$. The maximum number of users is 1026. This compares exactly with simulation results presented in Fig. 11, which yields a total number of users equal also to 1026. In this configuration, the users cause the least amount of interference to the network, by reducing the power gain required to maintain a desired signal-to-noise ratio.

On the other hand, the maximum network capacity was very low by having the simulator place the users such that they cause maximum interference to the network. The simulation yielded a total capacity of 108 users, with only 4 users in each cell. The pattern seen in Fig. 13 shows that the simulator placed the users at the extreme corners of their respective cells. The placement at extremities would require users to increase their power gain causing a lot more interference to other users. It is interesting to see that cells 9, 13, 15, and 19 where interference peaked are not in the center of the network. This is justified since the users in the cells at the boundaries of the network are placed

TABLE II

The maximum number of users in every cell for the 27 cell WCDMA network as the values of σ_1 and σ_2 are increased from 100 to 400 while μ_1 =0 and μ_2 =0. This results in users densely clusters around the BSs.

$\sigma = \sigma_1, \sigma_2$	$\sigma = 100$	$\sigma = 200$	$\sigma = 300$	$\sigma = 400$
$Cell_1$	38	38	37	34
$Cell_2$	38	38	37	34
$Cell_3$	38	38	37	35
$Cell_4$	38	38	37	35
$Cell_5$	38	38	37	34
$Cell_6$	38	38	37	35
Cell ₇	38	38	37	35
$Cell_8$	38	38	37	35
$Cell_9$	38	38	37	35
$Cell_{10}$	38	38	37	36
$Cell_{11}$	38	38	37	36
$Cell_{12}$	38	38	37	36
$Cell_{13}$	38	38	37	35
$Cell_{14}$	38	38	37	35
$Cell_{15}$	38	38	37	35
$Cell_{16}$	38	38	37	35
$Cell_{17}$	38	38	37	35
$Cell_{18}$	38	38	37	35
$Cell_{19}$	38	38	37	35
$Cell_{20}$	38	38	37	36
$Cell_{21}$	38	38	38	36
$Cell_{22}$	38	38	38	37
$Cell_{23}$	38	38	38	36
$Cell_{24}$	38	38	38	36
$Cell_{25}$	38	38	37	36
$Cell_{26}$	38	38	37	36
$Cell_{27}$	38	38	37	36
Total Users	1026	1026	1003	954

near the periphery of these cells, and towards the center of the network, resulting in much higher concentration of users along the boundary of these 4 cells.

Fig. 14 shows the 2-D Gaussian approximation of users clustered at the boundaries of the cells. The values of σ_1 , σ_2 , μ_1 , and μ_2 may be different in the different cells and are given in Table III. The maximum number of users is 133. This result is close to what was attained through simulation. The maximum network capacity was made low by having the simulator place the users such that they cause maximum interference to the network. The simulation yielded a total capacity of 108 users, with only 4 users in each cell. The pattern seen in Fig. 13 shows that the simulator placed the users at the extreme corners of their respective cells. The placement at extremities would require users to increase their power gain causing a lot more interference to other users.

6. CONCLUSIONS

In this paper we investigated the effect of using the actual distance of users for calculation of inter-cell interference when calculating the capacity of a CDMA network. The simulations were carried out for a twenty-seven cell CDMA network. For comparison, we first simulated and verified the results that were obtained analytically using the relative average interference. Our simulation results show that for a uniform user distribution, the difference in capacity determined using relative actual interference and relative average interference is too small to warrant the incursion of heavy computational load involved in the former case. However, our simulations also showed the extreme variation in total capacity under certain user placements, which



Fig. 13. Maximum network capacity of 108 using actual interference with worst case non-uniform user distribution.

TABLE III

The values of σ_1 , σ_2 , μ_1 , and μ_2 for the 2-D Gaussian approximation of users clustered at the boundaries of the cells as shown in Fig. 14. The maximum number of users is 133

	μ_1	σ_1	μ_2	σ_2
$Cell_1$	-1400	300	-900	300
$Cell_2$	-1400	300	800	300
$Cell_3$	-1400	300	800	300
$Cell_4$	0	300	-1700	300
$Cell_5$	0	300	-1600	300
$Cell_6$	1300	300	-800	300
$Cell_7$	-1400	300	900	300
$Cell_8$	-1300	300	900	300
$Cell_9$	0	300	1500	300
$Cell_{10}$	0	300	1600	300
$Cell_{11}$	0	300	1550	300
$Cell_{12}$	-1400	300	900	300
$Cell_{13}$	0	300	1500	300
$Cell_{14}$	1300	300	900	300
$Cell_{15}$	1300	300	-800	300
$Cell_{16}$	-1350	300	-850	300
$Cell_{17}$	-1400	300	-900	300
$Cell_{18}$	0	300	-1600	300
$Cell_{19}$	-1400	300	-800	300
$Cell_{20}$	-1400	300	-800	300
$Cell_{21}$	-1350	300	800	300
$Cell_{22}$	0	300	1600	300
$Cell_{23}$	1350	300	800	300
$Cell_{24}$	1400	300	-800	300
$Cell_{25}$	0	300	-1700	300
$Cell_{26}$	0	300	-1600	300
$Cell_{27}$	-1350	300	-850	300



Fig. 14. 2-D Gaussian approximation of users clustered at the boundaries of the cells. The values of σ_1 , σ_2 , μ_1 , and μ_2 may be different in the different cells and are given in Table III. The maximum number of users is 133.

could not have been predicted using the average interference for non-uniform user distributions.

7. REFERENCES

- K. Gilhousen, I. Jacobs, R. Padovani, A. Viterbi, L. Weaver, and C. Wheatley, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Technol.*, vol. 40, no. 2, pp. 303–312, May 1991.
- [2] J. Yang and W. Lee, "Design aspects and system evaluation of IS-95 based CDMA systems," *IEEE International Conf. on Universal Personal Commun.*, pp. 381–385, October 1997.
- [3] J. Evans and D. Everitt, "On the teletraffic capacity of CDMA cellular networks," *IEEE Trans. Veh. Technol.*, vol. 48, no. 1, pp. 153–165, January 1999.
- [4] R. Padovani, "Reverse link performance of IS-95 based cellular systems," *IEEE Personal Commun. Mag.*, vol. 1, no. 3, pp. 28–34, Third Quarter 1994.
- [5] K. Takeo and S. Sato, "Evaluation of a CDMA cell design algorithm considering non-uniformity of traffic and base station locations," *IEICE Trans. Fundamentals*, vol. E81-A, no. 7, pp. 1367–1377, July 1998.
- [6] A. Viterbi, A. Viterbi, K. Gilhousen, and E. Zehavi, "Soft handoff extends CDMA cell coverage and increases reverse link capacity," *IEEE J. Select. Areas Commun.*, vol. 12, no. 8, pp. 1281–1288, October 1994.
- [7] D. W. Matolak and A. Thakur, "Outside cell interference dynamics in cellular CDMA," *Proceedings of the 35th Southeastern Symposium*, pp. 418–152, March 2003.
- [8] R. Akl, M. Hegde, M. Naraghi-Pour, and P. Min, "Multi-cell CDMA network design," *IEEE Trans. Veh. Technol.*, vol. 50, no. 3, pp. 711–722, May 2001.
- [9] R. Akl and A. Parvez, "Impact of interference model on capacity in CDMA cellular networks," *Proceedings of SCI 04: Communication* and Network Systems, Technologies and Applications, vol. 3, pp. 404–408, July 2004.
- [10] S. Nguyen and R. Akl, "Approximating user distributions in WCDMA networks using 2-D Gaussian," *Proceedings of International Conf. on Comput., Commun., and Control Technol.*, July 2005.
- [11] D. Everitt, "Analytic traffic models of CDMA cellular networks," *Proceedings of the 14th International Teletraffic Congress*, pp. 349– 356, June 1994.
- [12] R. Akl, M. Hegde, A. Chandra, and P. Min, "CCAP: CDMA Capacity Allocation and Planning," Washington University, Tech. Rep., April 1998.