Dynamic Verification of an Object-Rule Knowledge Base Using Colored Petri Nets

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ABSTRACT

In this paper, we propose a formal description for the dynamic verification of an Object-Rule Hybrid Knowledge-based System (HKBS), capitalizing on the work carried out within the verification framework of Frame-Rule Hybrid Expert Systems. The main idea is to model an HKBS by means of a Colored Petri Network (CPN). In this way, method invocations, state class changes, rules and productions will be modeled as components of the CPN. Detection and analysis of the HKBS will be carried out by the construction and analysis of the markings graph, which results from the inference process.

Keywords: Dynamic Verification, Object-Rule Knowledge, Colored Petri Nets.

1. INTRODUCTION

Hybrid Expert Systems (HES) technology is a field in the artificial intelligence domain which has moved successfully from the research laboratory to the commercial or industrial application [1,3]. Generally speaking, HES integrates human expertise in computer programs to enable those programs to execute tasks which normally require the intervention of a human expert. There is a series of problems and difficulties underlying the development of an HES or Hybrid Knowledge-based System (HKBS), ranging from the acquisition of knowledge to the verification of each anomaly in the Knowledge Base. The formal solution using the power of marking graph analysis for modeling definition sets; formal proposition sets for error and anomaly verification in the Object-Rule HKBS. This approach also has:

• the capacity to build the accessibility graph for rules on the basis of the dynamic execution of our rules by the JRules inference engine;
• the capacity to build the occurrence sequences while respecting the formalism defined in [14];
• potential for the analysis of the anomaly situations on the basis of well-defined formal propositions.

The structure of this paper is as follows: In section 2, we present an overview of related work. Sections 3 and 4 present our main contributions. The formal aspects of Object-Rule HKBS modeling by a CPN are defined first, followed by a proposal of a formal solution using the power of marking graph analysis for the verification of each anomaly in the Knowledge Base. The construction of our graph and/or Petri network is performed dynamically. In the final section, we discuss the results obtained during simulation of the proposed examples, and present the conclusions of our work.

2. RELATED WORK

A Knowledge-based System is a category of programs capable of solving problems which do not have a classical algorithmic solution. It is designed to help the user perform a task in an
application area; the system works from knowledge acquired from the area expert. For the last 20 years, the Validation and Verification (V&V) sectors of expert systems have undergone considerable improvement. Indeed, during all the years from 1988 to 1992, the AAAI (American Association for Artificial Intelligence) organized workshops on the verification, validation and testing of intelligent systems. The IJCAI (International Joint Conference on Artificial Intelligence) has organized workshops on V&V since 1989. In addition, the European Conference on Artificial Intelligence (ECAI) has held a number of workshops on V&V.

The need to evaluate the large number of expert systems that have been developed since the mid-1980s have created a special interest in the verification of expert systems. The role and importance of such verification are well documented in [5, 8, 15-18].

The research carried out on the verification of the anomalies in systems based on the Frame-Rule has been considerable in comparison with the research carried out on anomalies in an Object-Rule HKBS, where little work has been done. If the earliest Knowledge-based Systems (KBS) in general, and more particularly the HKBS, were, in the majority of cases, verified using a static verification approach; dynamic verification proved to be efficient in the cases where it was used (INDE, SACCO, etc.). Indeed, in the dynamic verification of a Knowledge Base (KB), we use the deductive power of the Rule Base (RB), which allows us to proceed to a more in-depth verification of the RB.

The research closest to ours is that of Shiu et al. [1, 10-12], which involves the verification of systems which are linked by the production of hierarchical Frame rules. Their approach does not allow the use of method invocations in the condition or action clauses of a rule. They use two types of tokens, the first being the "State Token", which registers the class predicate and information state, and the second being the "Object Instance Token", which represents the instance of a particular object in a particular class in the object hierarchy. That approach is used in a monotonic context, which limits the usability of this approach in the Object-Rule context in a non-monotonic environment.

3. PRINCIPLES OF THE PROPOSED SOLUTION

The HES combines several representation paradigms in a single integrated environment. It is made up of the following features:

- object classes encapsulating the knowledge model – the inheritance relationship describes how these knowledge modules interact;
- rules specifying the functional behavior of objects in the expert system – these functions are presented by method invocations;
- a reasoning strategy controlling and specifying the inference sequence of knowledge in the expert system.

The use of Petri networks, as shown in Table 1, should allow us to represent classes, rules, conditions, actions and objects, as well as their states. To allow this, we use the properties of CPN to model the HKBS ALCAN [2]. In this section, we will discuss the JRules language that is used in the RB, and then define the modeling of our hybrid system by a CPN and the different types of rule presentation by the CPN.

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Table 1. HKS ALCAN modeling with CPN.

3.1 Formal Description of Modeling

The formal definition of CPN has been published elsewhere [14]. It is reproduced here in order to highlight the extension of our formalization. We define our model as follows:

A simple condition makes it possible to find all the objects of the class that pass the test where a negative condition is true when no object of the class verifies the condition. A "condition that exists" is true when there is at least one object of the class which verifies the condition. The conditions are defined in "continue test". These may include tests on class attributes or class method invocation results. The action "assert" adds an object of a given class with its necessary parameters (Arguments). In the same manner, the action "withdrawal" deletes an object referenced by a variable defined in one of the conditions of the rule, and the action "modify" modifies an object referenced by a variable in one of the conditions of the rule. In this approach, the reasoning is object-oriented. In fact, at the beginning, the work memory is initialized by objects called "initial facts". During a problem-solving session, the inference engine uses the rules, and objects are then added, deleted or modified according to the rules invoked. It should be noted that the conditions and the actions are method invocations on object classes, and that the class attributes are private and thus not accessible by JRules.
DEFINITION (1): The CPN (R) that models the Rule-Object-based HKS is a tuple:

\[ R = \langle \psi, C, \text{Expr}, P, T, A, N, \phi, E, I, \sigma, \text{StateObject} \rangle \]

where:

\[ \psi = \{ w_1, w_2, \ldots, w_i \} \] is a finite set of non-empty types, called color sets.

\[ C = \{ C_1, C_2, \ldots, C_n \} \] is a finite set of objects classes.

**Expr:** A \rightarrow expression, is an arc expression function. It is defined from \( A \) into expression such that:

\[ \forall a \in A : \text{Type}(\text{Expr}(a)) = \phi(p(a)) \land \text{Type}(\text{Var}(\text{Expr}(a))) \subseteq \psi \]

with Type(Expr(a)) is expression type of arc a, and Var(Expr(a)) is a set of expressions of arc a.

\[ P = \{ P_1, P_2, \ldots, P_m \}, n = 1, \ldots, El \] is a finite set of places of object classes.

\[ T = \{ T_1, T_2, \ldots, T_j \} \] is a finite set of transitions (rules).

\[ A = \{ a_1, a_2, \ldots, a_k \} \] is a finite set of arcs.

\[ N: A \rightarrow P \times T \cup T \times P \] is a node function. It is defined from \( A \) into expressions such that: \( P \cap T = P \land A \cap A = \emptyset \).

**StateObject:** \( C \rightarrow E \) is a state function. It is defined from \( C \) into \( E \) and maps for each object class its states, such that:

\[ \{ O : \text{OBJECT}, O \in C : \Rightarrow \text{Type}(\text{Var}(<\text{StateObject}(O))))) \subseteq \psi \} \]

Compared to the classical Petri Net defined in Jensen [14], we have introduced: \( C, \sigma, E, \text{StateObject} \) in order to model the Rule-Object-based HKS. Definitions of CPN found in [14], which have been used here without any change, are as follows:

- **Binding**

DEFINITION (2): A binding of a transition \( t \) is a Boolean function \( b \) defined on Var(t) such that:

\[ \forall v \in \text{Var}(t) : b(v) \in \text{Type}(v) \]

where \( \text{Var}(t) \) is a set of transition variables and \( B(t) \) is the set of all bindings for \( t \).

- **Marking**

DEFINITION (3): A token element is a pair (p,c): where \( p \in P \) and \( c \in \phi(p) \), while a binding element is \( (t,b,e) \) where: \( t \in T, b \in B(T) \) and \( e \in S(t) \). The set of all token elements is denoted by \( TE \) and the set of all binding elements is denoted by \( BE \).

DEFINITION (4): A marking \( M \) is a multi-set over \( TE \), while a step is a non-empty and finite multi-set over \( BE \). The initial marking \( M_0 \) is the marking that is obtained by evaluating the initial expression:

\[ \forall (p,t) \in TE \Rightarrow M_0(p,c) = I(p,c) \]

- **Occurrence Sequence**

DEFINITION (5): A finite occurrences sequence is a sequence of markings and steps:

\[ M_0 \rightarrow M_1 \rightarrow M_2 \rightarrow \ldots \rightarrow M_n \] such that: \( n \in N, M_0 \) is the initial marking and \( M_n \) is the final marking.

- **Reachable Transition**

DEFINITION (6): A marking \( M_n \) is reachable from a marking \( M_0 \) if and only if (iff) there exists a finite occurrence sequence \( \sigma \) having \( M_n \) as start marking and \( M_0 \) as final marking. We denote by \( \sigma(t_1,t_2,\ldots,t_n) \) the sequence of steps such that:

\[ M_0 \rightarrow M_1 \rightarrow M_2 \rightarrow \ldots \rightarrow M_n \] \( n \in N \) and \( \sigma(t_1,t_2,\ldots,t_n) \) represents the set of markings that is reachable from \( M_0 \) and denoted by: \( [M_0] \).

- **Transition Quasi-Alive**

DEFINITION (7): A transition \( t \) is quasi-alive iff there exists an occurrence sequence \( \sigma \), such that \( t \) is reachable from \( M_0 \).

- **Reachable Property**

DEFINITION (8): The marking \( M_t \) is included in marking \( M_0 \) iff for all place p: \( M_t(p) \leq M_0(p) \).

Formal definition of CPN which has been adapted from [14] in the Rule-Object-based HKS context:

- **Token**

DEFINITION (9): We defined the number of tokens in place \( p \) by:

\[ M(p) = \sum \{ O_i | i > 0 \}, \text{ such that } O_i \text{ is an instance object class in place } p. \]

- **Markings Update**

DEFINITION (10): when a transition \( t \) is enabled, we have the following:

1. If \( A_s \) is produced, the marking \( M_0 \) changes without producing another marking \( M_2 \). We denote that by:

\[ \forall p \in P : M_2(p) = M_0(p) - \sum \{ E(p,t) < b \} \]

2. If \( A_s \) is produced, the marking \( M_0 \) changes to produce another marking \( M_2 \). We denote that by:

\[ \forall p \in P : M_2(p) = (M_0(p) - \sum \{ E(p,t) < b \}) + \sum \{ E(t,p) < b \} \]

3. If \( A_m \) is produced, the value of the instance object (token) in marking \( M_0 \) is updated.

- **Equality of Markings**

DEFINITION (11): Two markings are equal iff their places are equal one to one.

- **Graph of Markings**

DEFINITION (12): The occurrence graph in CPN is a tuple \( \text{OGCPN} = (V,A,E,N) \) satisfying the following requirements:

1. \( V \) is a set of reachable markings from \( M_0 \), denoted by:

\[ V = [M_0] \]

2. \( A \) is a set of finite arcs, denoted by:

\[ A = \{ (M_1,b,e,M_2) \in V \times BE \times SE \times V : M_1(b \in BE \text{ and } e \in SE) \} ; \]
3. "N" is a set of nodes, such as:
   \[ a = (M1,b,e,M2) \in A \text{ and } N(a) = (M1, M2). \]

**Definition (13):** A change of state is a Boolean function \( B \). This function returns TRUE if the instance state object \( O_i \) has changed while the transition \( t \) was fired. We note that:
\[
B(O_i, t) \subseteq \Psi
\]
We also denote by \( S(t) \) the state of binding \( t \).

The formal definition of CPN that has been introduced to complete modeling of the Rule-Object-based HKS:

- **State of Object Class**
  **Definition (14):** To evaluate the state of an object class, we define three functions:
  \[ A_c: T_c \rightarrow \{ \emptyset \} \] is an output class function used when the object instance is removed.
  \[ A_c: T_c \rightarrow P_c \] is an output class function used for modification with a change of state of the object class.
  \[ A_{i_o}: T_c \rightarrow P_c \] is an input class function used for modification without a change of state of the object class.

- **Equalities of Objects**
  **Definition (15):** Two places are equal iff:
  1. They have the same number of tokens (objects);
  2. The tokens are equal one to one; and
  3. They have the same state-of-object class.

  **Definition (16):** Two objects (instances) are equal iff:
  1. They have the same object identification (ID) (each object instance has a unique ID); and
  2. They have the same color.

- **Condition to Firing Rule**
  **Definition (17):** We can fire the same rule iff: after execution of actions in the rule, the attributes of the instance object have changed their values (that is, a JRule reasoning strategy).

### 3.2 Presentation of Certain Rules in the CPN

A color domain is associated with each place, and each token in place is colored by an element of the domain of that place (several tokens may have the same color). The inscription of a place is thus a multi-set of colors, a set in which an element may occur several times.

A transition is enabled if and only if each entry place to the transition contains a sufficient number of tokens for each color of the place domain. Independently of the evaluation of the colored functions, a transition may not be allowed if its attached expression does not satisfy some of the attributes.

In order to differentiate between the instance objects while comparing markings, we use the object ID as a (unique) identification key.

### 4. USING A CPN FOR THE ANALYSIS OF ANOMALIES IN AN OBJECT-RULE HKBS

There are two analysis approaches for researching anomalies in a HKBS. The first is based on marking graph analysis, the second on the analysis of the final places of the Petri network. In this paper, we present the first technique. In this approach, anomaly analysis is based on an analysis of the marking graph or occurrence graphs that represent the accessibility game of the CPN [13, 14].

The idea is to build a graph containing a node for each accessible marking and an arc for each appearance of the marking element. Indeed, we concentrate our analysis by allowing a specific transition (that is, one which corresponds to a few significant initial facts) and verifying next, at a given time, the aggregate of markings.

The problem may then be localized by an analysis of the trace of the transition sequences that may supply the alternative or multiple markings effects.

#### 4.1 Heuristic CPN Marking Graph Construction Algorithm

We propose a heuristic search method to construct the occurrence graph for particular marking. The search for anomalies in HKBS needs an adequate initialization of sequence of transition (Table 2, steps 1, 2, 3, 4) and construction the reachability three (steps 6 to 15).

The strategy used is to start from position “i” and put it initially in the CONFLICT_LIST the list of candidate rules (transitions) to be fired by JRules. Put in CANDIDATE_RULE the rule that JRule fires and general marking \( M_i \) which result from firing CANDIDATE_RULE if this marking \( M_i \) is not an element in MARKING_GRAPH then add Node(\( M_i \)) and arc(\( M_i \), \( R_t \), \( M_{i+1} \)) in MARKING_GRAPH and the new position to be prepared in MARKING_GRAPH is \( i = i+1 \) else add only arc (\( M_{i+1}, M_i \)) in the MARKING_GRAPH. Remove the CANDIDATE_RULE from the LIST_CANDIDATE, if LIST_CANDIDATE is not empty save the context (\( J = J+1 \)), and put first rule in CANDIDATE_LIST. The process is repeated until CANDIDATE_LIST is empty.

```
MarkingConstrGraph MCG {
  1. Construct the initial marking \( M_0 \)
  2. Initialize variables \( I=1 \) and \( J=1 \)
  3. Add (\( M_0 \)) to the occurrence marking graph MARKING_GRAPH
  4. CONFLICT_LIST = list of candidate rules
  5. If CONFLICT_LIST = \{ \emptyset \} | stop by user | number nodes reached Then
    Goto (13)
  6. CANDIDATE_RULE = rule fire by JRule
  7. Fire the rule CANDIDATE_RULE
  8. Construct the actual marking into \( M_{i+1} \)
  9. If Verify(\( M_{i+1}\), Node(MARKING_GRAPH)) Then
    Create ARC (\( M_i, CANDIDATE_RULE, M_i \))
    Else
    Add \( M_{i+1} \) into the occurrence graph MARKING_GRAPH
    Create ARC (\( M_{i+1}, CANDIDATE_RULE, M_{i+1} \))
    I = I + 1
    Endif
  10. Update CONFLICT_LIST
  11. Save context of marking \( M_i \)
  12. IF CONFLICT_LIST \# \{ \emptyset \} Then
    J = J + 1
    CANDIDATE_RULE = the first element in CONFLICT_LIST
    Goto (7)
  Endif
  13. Restore context \( M_i \)
  14. J = J - 1
```

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In an HKS, two rules, $R_i$ and $R_j$, cause redundancy between object classes iff:
1. $R_i$ and $R_j$ are in conflict (i), (ii) and (iii); and
2. For any place $P_s$ in the marking $M_i$, the color of the place $\Phi(P_s)$ in $M_i$ is equal to the color of place $P_s$ in $M_j$. We denote:
$$ [\forall P_s \in \Phi, M_i(P_s, \Phi(P_s)) \neq M_j(P_s, \Phi(P_s)), s > 0] \iff [M_i = M_j] $$

As an example, we consider the following rules written in a simplified notation of JRules.

$$ \text{rule } R_1 \{ \text{when } \{ \text{obj1.Tank: } \text{obj1.getCode() = 'CP'}; \text{obj1.getPercentageFull()} > 90 \}; \text{obj2.Site: } \text{obj2.getCode() = 'CP'} \}; \text{then } \{ \text{modify obj1;} \text{qblzzleDischargeRiskShortTime(100); } \} ; \} ; \} $$

$$ \text{rule } R_2 \{ \text{when } \{ \text{obj1.Tank: } \text{obj1.getPercentageFull()} > 90 ; \text{obj2.Site: } \text{obj2.getCode() = 'CP'} \}; \text{then } \{ \text{modify obj1;} \text{qblzzleDischargeRiskShortTime(100); } \text{modify hist{ obj2 rêve milieR: risk(100) } } \} ; \} ; \} $$

Figure 2 shows the markings after a complete simulation of the firing rules in the above example.

**Table 2.** CPN marking graph construction algorithm.

**4.2 Types of Anomalies**

Our research is a contribution to the study of the accuracy of a Knowledge Base. This includes completeness (the problems of redundancy and subsumption), consistency (the problems of incoherence or contradiction, and useless premises) and perfection (the problem of unreachable rules) of a Knowledge Base.

We describe below a set of the most frequently treated anomalies in verification currently; some of these anomalies, such as contradiction, are deemed serious since they cause incorrect functioning of the KB.

- **a) Proposition of Conflict**

  In an HKS, two rules $R_i$ and $R_j$ are in conflict (Figure 1) iff:

  - there exist two final sequences $\sigma_i$ and $\sigma_j$ resulting from final markings $M_i$ and $M_j$ respectively, such that $M_i$ is produced by firing $R_i$, noted $M_i = \delta(M_0, \sigma_i)$ and $M_j$ is produced by firing $R_j$ and noted $M_j = \delta(M_0, \sigma_j)$;
  - they do not have the same path: $\sigma_i \cap \sigma_j = \emptyset$;
  - there exists a set of places $\Phi \neq \emptyset$, in which a place in $M_i$ and $M_j$ have the same color, and is noted by:
    $$ \Phi = \{P \in P: P, M_i(P, \Phi(P)) = M_j(P, \Phi(P)), s > 0\}.$$  

- **b) Proposition of Redundancy**

  In an HKS, two rules, $R_i$ and $R_j$, cause redundancy between object classes iff:

  1. $R_i$ and $R_j$ are in conflict (i), (ii) and (iii); and
  2. For any place $P_s$ in the marking $M_i$, the color of the place $\Phi(P_s)$ in $M_i$ is equal to the color of place $P_s$ in $M_j$. We denote:
    $$ [\forall P_s \in \Phi, M_i(P_s, \Phi(P_s)) \neq M_j(P_s, \Phi(P_s)), s > 0] \iff [M_i = M_j] $$

- **c) Proposition of subsumption**

  A rule $R_j$ is subsumed by rule $R_i$ iff:

  1. $R_i$ and $R_j$ are in conflict (i), (ii) and (iii); and
  2. there exists a set $E$, such that $E \neq \emptyset$, where...
3. For any place belonging to Event E, we have:
   a) The number of tokens of P, in M, is the same as in M, denoted by:
      \( \forall P \in E : \|M(P, \varphi(P)) \| = \|M(P, \varphi(P)) \| \)
   b) For any place \( P \in M \), its state is the same as in M, denoted by:
      \( \forall P \in E \Rightarrow \text{State}(P)_{M_i} = \text{State}(P)_{M_0} \)

4. There exists a set of transitions \( \Omega M_i \) for which \( R_i \) does not change its state while firing these last transitions, and/or there exists a set of transitions \( \Omega M_i \) for which \( R_i \) does not change state while firing these transitions, and then: \( \Omega M_i \neq \emptyset \) and/or \( \Omega M_i \neq \emptyset \).

5. If \( \{R_i \subseteq \Omega M_i\} \), we say that \( R_i \) is subsumed by \( R_j \), i.e. the set of consequents of \( R_i \) includes the same consequents as \( R_j \), and the set of premises of \( R_i \) is the same as for \( R_j \).

6. If \( \{R_i \subseteq \Omega M_i\} \), we say that \( R_i \) is subsumed by \( R_j \), i.e. the set of consequents of \( R_i \) includes the same consequents as \( R_j \), and the set of premises of \( R_i \) is the same as for \( R_j \).

A problem of subsumption between two rules is encountered if, starting from the same initial marking \( M_0 \), we construct two different occurrence sequences, leading to two markings \( M_i \) and \( M_j \), one of which is included in the other (if \( M_i = M_j \), we find the redundancy case). Note that a subsumption anomaly is a special case of a redundancy.

As an example, we consider the following rules written in a simplified notation of JRules.

**Rule S1**: When \( \text{Simuulation}() \), object Tank (object CF get Code() = "RCP"); object CF get PercentageFull() = 95; object CF get DateLastFull() = "2013-01-01"; object Site (object CF get Code() = "CP"); Then modify object (object get MiddleRisk(100)); modify object discharge short Time(4));

**Rule S2**: When \( \text{Simulation}() \), object Tank (object get Code() = "CP"); object Site (object get Code() = "CP"); Then modify object \( \beta \) (object get MiddleRisk(100)); modify object discharge short Time(4));

Figure 3 shows the markings after a complete simulation of firing rules of the above example.

We note that:
1. \( S_1 \) and \( S_2 \) are in conflict:
   a) If there exist two final markings \( M_1 \) and \( M_2 \), such that \( M_1 \) is reachable from \( M_0 \), \( M_1 \) and \( M_2 \) are reachable from \( M_0 \), \( M_2 \neq M_1 \) and denoted by \( \sigma_1 = \delta(M_0, \sigma_1) \), and \( M_2 \) is reachable from \( M_0 \), \( M_2 \neq M_1 \) and denoted by \( \sigma_2 = \delta(M_0, \sigma_2) \).
   b) If \( \sigma_1 \cap \sigma_2 = \emptyset \).
   c) If there exists \( \Phi = \{P_2, P_3\} \) such that \( M_1(P_2, \varphi(P_2)) = M_2(P_2, \varphi(P_2)) \).
2. The color in place \( P_3 \) is the same in \( M_1 \) and \( M_2 \) and noted by \( \Phi = \{P_3\} \) such that \( M_1(P_3, \varphi(P_3)) = M_2(P_3, \varphi(P_3)) \).
3. There exists a non-empty set \( E = \{P : P \in \Phi \} \) such that we have:
   a) \( M_1(P_1, \varphi(P_1)) = M_2(P_1, \varphi(P_1)) \)
   b) \( \text{StateObject}(P, M_i) = \text{StateObject}(P, M_j) = \text{TankFull} \)

4. There exists \( \Omega M_i \) for which \( R_i \) is included in \( M_i \) and denoted by \( \Omega M_i \) work on the same variables (methods) and they lead to the state of \( S_1 \) (Tank ‘RCP’ Full) and same states \( S_2 \) (Site ‘CP’ InProd) in \( M_i \) and \( M_j \).

**d) Proposition of contradiction**

An HES has contradictory rules \( R_i \) and \( R_j \), if:
1. \( R_i \) and \( R_j \) are in conflict (i), (ii) and (iii).
2. There exists a set \( E \neq \emptyset \) containing all the places that do not belong to \( \Phi \), i.e.:
   \( E = \{P : P \in \Phi \} \Rightarrow \{\Phi \cap E = \emptyset \} \)

3. For any place \( P \), \( \forall P \in E \), all objects instances in place \( P \), \( M_i \), are identical to the object instances of \( P \), \( M_j \), and denoted by \( \Omega P_1(P, M_i) = \Omega P_2(P, M_j) \). Moreover, the state of place \( P \) of object \( O \) in \( M_i \) is different from the state of this object in \( M_j \), We denote this by: \( \text{State}(O, (P, M_i)) \neq \text{State}(O, (P, M_j)) \).

As an example, we consider the following rules written in a simplified notation of JRules.

**Rule C1**: When \( \text{Obji Tank}( \text{Obji} get Code() = \text{"RCP"}) \); \( \text{Obji get PercentageAll}() >= 90 \); \( \text{Obj2 Site} \) (object get Code() = \text{"CP"}); Then modify object \( \text{Obj2 middleRisk}(100) \);

**Rule C2**: When \( \text{Obji Tank}( \text{Obji} get Code() = \text{"RCP"}) \); \( \text{Obji get PercentageAll}() >= 90 \); \( \text{Obj2 Site} \) (object get Code() = \text{"CP"}); Then modify object \( \text{Obj2 middleRisk}(100) \);

Figure 4 shows the markings after a complete simulation of firing rules in the above example.
We note that:
1. \( C_1 \) and \( C_2 \) are in conflict:
   a) if there exist two final markings \( M_1 \) and \( M_2 \), such that
      \( M_1 \) is reachable from \( M_0 \) and \( M_2 \) and denoted by \( \sigma_1 = \delta(M_0, \sigma_1) \) and \( M_2 \) is reachable from \( M_0 \) and denoted by \( \sigma_2 = \delta(M_0, \sigma_2) \),
   b) \( \delta_1 \cap \delta_2 = \emptyset \),
   c) if there exists \( \Phi = \{ P_i \} \) such that \( M_1(P_i, \Phi(P_i)) = M_2(P_i, \Phi(P_i)) \);
2. There exists a set \( E = \{ P_i : P / P_2 \notin \Phi \} \) for which:
   for any object instance \( O_i \) in \( P \), \( O_i \) exists in marking \( M_1 \) and \( M_2 \).
   We denote this by:
   \( O_i : \text{Object: } O_i(P_1, M_1) = O_i(P_2, M_2) = (\text{"CP"},100) \) and,
   StateObject\((P_2, M_1)\) = InProd and,
   StateObject\((P_2, M_2)\) = NormalProd.

This results in \( \text{StateObject}(P_2,M_1) \neq \text{StateObject}(P_2,M_2) \).

We can say that firing \( C_1 \) and \( C_2 \) produces places with the same object instances but in different states, which is obviously a contradiction.

e) **Proposition of unnecessary premises**

An HES has unnecessary IF conditions between two rules \( R_i \) and \( R_j \):
1. There exist two final sequences \( \sigma_i \) and \( \sigma_j \), resulting from final markings \( M_i \) and \( M_j \) respectively, such that \( M_i \) is produced by firing \( R_i \), denoted \( \delta(M_0, \sigma_i) \) and \( M_j \) is produced by firing \( R_j \) and denoted \( \delta(M_0, \sigma_j) \);
2. They do not have the same path: \( \sigma_i \cap \sigma_j = \emptyset \);
3. For any place \( P_i \) from the set \( P \) of places, there exist two different colors \( (c, c') \) for this place in \( M_i \) and \( M_j \) respectively, denoted by:
   \[ \forall \, p : P, \exists \, c, c' \in \nu / (p,c) \in M_i \text{ and } (p,c') \in M_j \Rightarrow c \cap c' = \emptyset. \]

f) **Proposition of unreachability**

We know that for a rule to be reachable, it is necessary that one of its triggering states be obtained directly from an initial marking or indirectly from an intermediate marking in the reachability graph. If in the graph a rule does not use any initial or intermediate marking, then this rule is unreachable. In an HES, a rule \( R_i \) is unreachable iff:
1. \( \forall \, \sigma_i \), a marking sequence such that \( M' \) is obtained from \( M_0 \), and thus we have:
   \[ M' = \delta(M_0, \sigma_i) \Rightarrow \sigma_i \cap \{ R_i \} = \emptyset. \]

**4.3 Dynamic Verification Principle**

The users of the system are experts in their field. It is the case, however, that after having built the knowledge base, or any modification or update to it, inconsistencies or anomalies can easily occur, since no tool for checking anomalies is integrated into the automatic rules manager. The following steps show how we construct a graph-marking-based verification model:

1. Construct reachability graph nodes (markings) progressively, while there is an available rule to be fired and according to formal definitions 2,3,4,5,6,7 and 10;
2. Starting with the initial node (initial marking), build all the occurrence sequences as defined in section 3.1, definitions 1 and 4;
3. Analyze these occurrence sequences to detect anomalies according to formal definitions 12,13,14,15,16 and the formal propositions.

**5. EXPERIMENTS AND DISCUSSION**

Implementation of the simulation is composed of a set of Java classes and of an RB in which to present the anomalies (.ilr extension files). Once the Java classes are compiled (Figure 5), the inference engine is fired via JRules API imported into Java. The latter uses a rule file as the entrance, compiles the file to check for errors and initializes the work memory according to the initial facts contained in that rule file.

The reasoning session will start at that moment with a search for the rules that will verify the objects in the work memory. Only one rule is then chosen to be fired. That cycle will continue until there are no more tangible rules to apply or until the inference session is explicitly stopped by the user.

We carried out two types of simulations. The first uses the examples presented in section 4 and extracted from HKS ALCAN [2]. The second uses the well-known Game of Life for several types of patterns [22].

**Figure 4.** Marking graph – contradiction rule.

**Figure 5.** JRules - Java interaction.
the knowledge base in a coherent state, thus guaranteeing reasoning devoid of inconsistencies and anomalies.

5.2 Simulations with the Game of Life
The Game Of Life is an animated representation of a population of cells developed first by the John Horton Conway [22]. In fact, it is a lattice of cells with a set of rules describing how successive generations of cells develop. The main interest of the game is that it enables complex phenomena to emerge from simple rules. The cell automaton in two dimensions is considered as a reference by researchers who are interested in the field of artificial life, because it shows that very simple rules may make it possible to bring to light non-trivial operations, and because it may simulate the abundance and diversity of life. Here is an example of an RB written in JRules which we simulated to create the situation of a redundancy.

```jrules
import GameOfLife;
rule setLife {
  property context = GameOfLife.ConstructGraph();
  rule fillCell { when (?o:Cell(isDead?:#isDead; neighbour():#neighbour)==3) then setAlive(?o); } // R3
  rule fillRandomCell { when (?o:Cell(neighbour():#neighbour)==3) then setAlive(?o); } // R7
  rule leaveEmpty { when (?o:Cell(isDead?:#isDead; neighbour():#neighbour)==5) then setDead(?o); } // R2
  rule loneliness { when (?o:Cell(isAlive?:#isAlive; neighbour():#neighbour)==1) then setDead(?o); } // R5
  rule happiness { when (?o:Cell(isAlive?:#isAlive; neighbour():#neighbour)==2) then setAlive(?o); } // R4
  rule overcrowding { when (?o:Cell(isAlive?:#isAlive; neighbour():#neighbour)==3) then setDead(?o); } // R5
}
```

Globally speaking, simulations have revealed that:

1. The results manually predicted in section 4 were confirmed by the computerized application; we can confirm that, the modelisation of HKBS and checking anomalies proposed in this paper can be automated by a program software;
2. In order to verify the correctness of the heuristic CPN marking graph construction algorithm, we have conducted simulations for several patterns ("Glider", "Small Exploder", "Exploder", "J0 Cell Row", "Fish", "Pump", "Shooter", "Slow", "Fast", "Hyper") in the Game of Life application and all these simulations gave us satisfactory results. However, only pattern Exploder is presented here for brevity.
3. The Game of Life case enabled us to confirm that the construction of the marking graph and the analysis of occurrence sequences would definitely make it possible to predict an anomaly. Actually, the results of the inferences of the RB without anomalies (Figure 6.a) and those with redundancy (Figure 6.b) are the same. Only the analysis of occurrence sequences allows us to predict the differences (Figure 7).
4. To the author's best knowledge, this work is the first research for dynamic verification of rule-object KBSH; it can be used as a reference approach for all new research in this area.

6. CONCLUSION
Even though an non-monotonic Object-Rule system is not a simple task, few researchers have dealt with that set of problems (section 3). In this paper, we have proposed an approach for the dynamic verification of anomalies in an Object-Rule HKBS which may be implemented by a computer program. We successfully tested this technique on some examples. However, because of the complexity of the problem and the lack of similar work, certain aspects should be improved. Our main contributions are:

a) to propose a complete modeling of the Object-Rule Hybrid Knowledge-Based System by a CPN; in fact, we propose five redefinitions and four new formal definitions of the CPN for constructing the model of our system, a model in which all the parts of an HKBS (classes, rules, conditions, actions and objects) are presented by the modeling elements of the Petri networks (places, transitions, expression of arcs and tokens);
b) to propose a formal solution based on the analysis of the sequences of the markings graph to analyze the anomalies in Object-Rule HKBS.

This paper illustrates the capacity of the proposed technique to identify situations of incompleteness anomalies in an Object-Rule HKBS in an environment of interactive execution. That verification is based on the analysis of the traces of the sequences and occurrence sequences and on marks.

Extensions of this research work are currently being studied:

- to complete the formal analysis proposals for the case of concentricity;
- to propose a new approach, based strictly on an analysis of the final places of the actual Petri network;
- to carry out simulations on large Knowledge Bases.

![Figure 6. Game of Life](image)

(a) Result without anomaly. (b) Result with redundancy.

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8. REFERENCES


Figure 7. Game of Life - occurrence sequences.