Optimizing Ship Classification in the Arctic Ocean: A Case Study of Multi-Disciplinary Problem Solving

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ABSTRACT

We describe a multi-disciplinary system model for determining decision making strategies based upon the ability to perform data mining and pattern discovery utilizing open source actionable information to prepare for specific events or situations from multiple information sources. We focus on combining detection theory with game theory for classifying ships in Arctic Ocean to verify ship reporting. More specifically, detection theory is used to determine probability of deciding if a ship or certain ship class is present or not. We use game theory to fuse information for optimal decision making on ship classification. Hierarchy game theory framework enables complex modeling of data in probabilistic modeling. However, applicability to big data is complicated by the difficulties of inference in complex probabilistic models, and by computational constraints. We provide a framework for fusing sensor inputs to help compare if the information of a ship matches its AIS reporting requirements using mixed probabilities from game theory. Our method can be further applied to optimizing other choke point scenarios where a decision is needed for classification of ground assets or signals. We model impact on decision making on accuracy by adding more parameters or sensors to the decision making process as sensitivity analysis.

Keywords: Game Theory; Detection Theory; Decision Making; Fusion

1. INTRODUCTION

The environment is an important part of the Intelligence Community agenda. The Intelligence Community is involved in this work, and involvement is important for citizens of the United States and the world. The Intelligence Community's job is to ensure that senior policymakers and military commanders have objective information that will allow them to make better decisions. Through collection and analytic effort, intelligence reports give our country's leadership insight into how events in all parts of the world will unfold and how these events will affect our national security. Environmental trends, both natural and man-made, are among the underlying forces that affect a nation's economy, its social stability, its behavior in world markets, and its attitude toward neighbors. The environment is one factor. Environmental degradation, encroaching deserts, erosion, and over farming destroy vast tracts of arable land. This forces people from their homes and creates tensions between ethnic and political groups as competition for scarce resources increases. There is an essential connection between environmental degradation, population growth, and poverty that regional analysts must take into account [2].

National reconnaissance systems that track the movement of tanks through the desert, can, at the same time, track the movement of the desert itself and see the sand closing in on formerly productive fields or hillsides laid bare by deforestation and erosion. Satellite systems allow assessment of the magnitude and severity of damage. Adding this environmental dimension to traditional political, economic, and military analysis enhances the ability to alert policymakers to potential instability, conflict, or human disaster and to identify situations which may draw in American involvement. Some events have already dictated that environmental issues are included in our intelligence agenda. When Moscow initially issued misleading information about the accident at the Chernobyl Nuclear Power Plant, U.S. leaders turned to the Intelligence Community to assess the damage and its impact on the former Soviet Union and neighboring countries [2].

The U.S. Coast Guard’s (CG) value to the nation resides in its proven ability to protect those on the sea, protect the United States from threats delivered by sea and protect the sea itself. Its unique authorities, capabilities, competencies and partnerships as a military, law enforcement, regulatory and humanitarian service are central to that value proposition. The CG is recognized worldwide for its ability to execute these diverse maritime missions over vast geographic areas and under the most challenging and demanding conditions [7].

As the CG prepares for the future, the emerging maritime frontier of the Arctic is significantly expanding the operating area. Last September 2013 it was observed that the Arctic had the lowest sea ice extent in recorded history, and there are vast areas of open water where there used to be ice. Activity in the most remote reaches of Alaska continues to evolve and grow, including planned drilling operations in the Chukchi and Beaufort Seas, foreign tankers using the northern sea routes which transit through the Bering Strait and Sea, and small cruise ships pressing even further into the Arctic. As the receding ice invites increased human activity in commercial and private ventures, there is increasing demand for the Coast Guard to ensure the safety, security and stewardship of the nation's Arctic waters [7].

The circum-Arctic region and Outer Continental Shelf area ranks second behind the Gulf of Mexico for volume of resources. Sovereign and industrial activities will continue to evolve around access to an abundance of resources. These resources include an estimated 13 percent of the world's undiscovered oil, 30 percent of undiscovered gas, and some one trillion dollars worth of minerals including gold, zinc, palladium, nickel, platinum, lead, rare-earth minerals, and gem-quality diamonds. As Arctic ice recedes and maritime activity increases, the Coast Guard must be prepared to administer and inform national objectives over the long-term. The United States is an Arctic nation, and the Coast Guard supports numerous experienced and capable partners in the region. The aim of this strategy is to ensure safe, secure, and environmentally responsible maritime activity in the Arctic. This strategy establishes objectives to meet this aim and support national policy [7].

There are three strategic objectives in the Arctic for the U.S. Coast Guard. Improving Awareness: Coast Guard operations require precise and ongoing awareness of activities in the maritime domain. Maritime awareness in the Arctic is currently restricted due to limited surveillance, monitoring, and information system capabilities. Modernizing Governance: The concept of governance involves institutions, structures of authority, and capabilities necessary to oversee maritime activities while safeguarding national interests. Limited awareness and oversight challenge maritime sovereignty, including the protection of natural resources and control of maritime borders. Broadening Partnerships: Success in the Arctic requires a collective effort across both the public and private sectors. Such a collective effort must be inclusive of domestic regulatory regimes; international collaborative forums such as the Arctic Council, International Maritime Organization (IMO), and Inuit Circumpolar Council; domestic and international partnerships; and local engagements in Arctic communities focusing on training and volunteer service [7].

An oceanic trade route across the Arctic from the North Atlantic to the North Pacific would represent a transformational shift in maritime trade, akin to the opening of the Panama Canal in the early 20th century. An Arctic marine highway would cut existing oceanic transit between Europe and Asia by an estimated 5,000 nautical miles [7].

Economic factors (e.g., unemployment rates, prices for food, such as bread, or fuel), Political factors (freedoms, type of government), Religious factors (type of religions, religious tensions) combined with trend information such as sentiment analysis on social media, open source data, news, etc. can provide indicators of areas undergoing stress or at risk. An attempt to predict the likelihood of reaction to a future event will be based on correct situation analysis. Efforts to combine the information required for these predictions are time consuming and labor intensive. The availability of open source social media information and implementation of artificial intelligence (AI) methodologies makes this problem tractable. Our GlobalSite system, shown in Figure 1, can be used as a method for decision making and reduce cost of analyses.

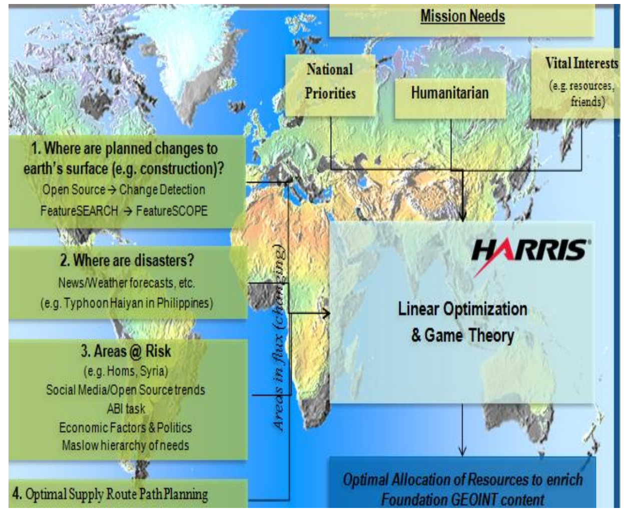

Fig. 1. System Overview

2. ANALYTIC HIERARCHY PROCESS

The Analytic Hierarchy Process (AHP) is a multi-criteria decision-making approach. The AHP is a decision support tool which can be used to solve complex decision problems. The AHP has attracted the interest of many researchers mainly due to the mathematical properties of the method and the fact that the required input data are easy to obtain. It uses a multi-level hierarchical structure of objectives, criteria, sub criteria, and alternatives. The pertinent data are derived by using a set of pair wise comparisons. These comparisons are used to obtain the weights of importance of the decision criteria, and the relative performance measures of the alternatives in terms of each individual decision criterion. If the comparisons are not perfectly consistent, then it provides a mechanism for improving consistency [18].

Hierarchical game theory can be used to solve for the best strategy for decision making in complex problem solving. Hierarchical game theory can determine cooperating capacity between hierarchies and detect the best united strategy. This can provide a powerful method of resource allocation and asset planning in order to maximize a player's response [8]. Figure 2 shows the hierarchical, game structure for our example. All of these hierarchies are part of the overall player's capability to compete with other players. The four hierarchies are: sensors, ship classes, organizations, and competing nations. In our example we can model uncertainty of sensor detection to determining ship classification and model the probability of a nation verifying ship reporting accuracy. This information is then used to compete with other nations or players for situational awareness of the Arctic region. A feedback loop is used to model sequential time periods as updates are observed [14].

This framework solves for controlling ability in groups and the hierarchical trait in command and control. Thus, to enhance independent decision-making in lower decision-makers and make decision-making between the upper and the lower decision-makers not only have clear hierarchies, but also interact and optimize each other. Sequentially, perfect effects can be obtained with a hierarchy model. [8].

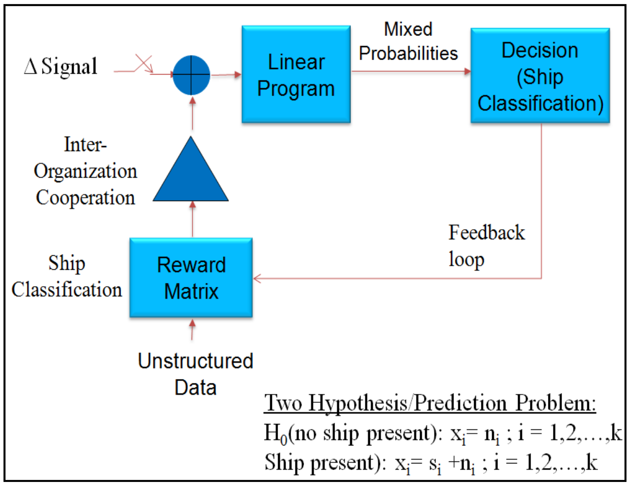

Fig. 2. Model Process Flow

The organizations determine the missions. The missions are to be carried out by the organizations. The sensors are used to carry out the missions. Our example consists of organizations, sensors [5], ship classes, and nation players. Figure 3 shows the elements of each hierarchy.

Hierarchy 1a (Sensor Types)

1. Radar
2. Optical
3. Acoustic
4. Wireless

Hierarchy 1b (Sensors)

1. Visual
2. Spectral
3. Infrared
4. SAR
5. HF Radar
6. LiDAR
7. Acoustic
8. AIS
9. Buoy

Hierarchy 2a (Ship Detection)

1. Ship Present
2. No Ship Present

Hierarchy 2b (Ships of Interest for Reporting)

1. Warship
2. Fishing Vessel
3. Cargo ship
4. Ice Breaker
5. Research Vessel

Hierarchy 3 (Organizations)

1. Coast Guard
2. Private Companies
3. NASA
4. US Navy
5. NATO Partner

Hierarchy 4 (Competing Nations)

1. US
2. China
3. Norway
4. Russia
5. Canada

Fig. 3. Hierarchies and Elements

The goal is to maximize the decision function. The overall performance of the decision is based on several levels of hierarchical decisions. Our example starts with the decision to optimize ship classification decision to verify ship reporting accuracy versus other nation players. Cooperation between organizations is modeled using multi-player Prisoner's Dilemma in our solution [10]. The choice of sensor to use is based on availability. The ship classes are based on real world data. Each level of hierarchy has an impact on the overall ability for a country or blue player to compete on a global basis. Cooperation is less likely to emerge in a large group than a small group. The iterated Prisoner's Dilemma (PD) game has been used extensively in the study of cooperative behaviors in social and biological systems. The N player PD game is realistic for modeling the cooperation strategies [22].

However, in the real world, individual rational actions are not always taken. In our example, responsible organizations are given incentives to cooperate so that their action can take a better action in the international game so that the blue player can best compete [19]. Results of an open competition are well explained by cognitive hierarchy (CH). In many games it boils down to predicting how deeply other agents in the game will be reasoning. An agent that does not reason enough risks being exploited by its opponents, while an agent that reasons too much, may not be able to interact productively with its opponents [21].

We let the objective function be $F = (F_1, F_2)$ where F_1 could be the blue player. We let x_1 be the decision maker's choice for blue. We let h_{11} to h_{1m} be the lower hierarchical decision maker's response. The objective function for blue's lower decision makers is f_{ij}. We let S_1 be the feasible sets for variable x_1. S_1 depends on x_1 and h_{11} to h_{mj}. The lower decision-maker can modify the upper decision maker's mind according to the actual status [8].

In our example, there are several resource management stages or hierarchies as shown in Figure 4. These stages include information needs, collection objectives, and observables. Our example serves as a method to enhance situational awareness for making best decisions concerning the status of the Arctic region. Situational awareness is used as critical information for activity based intelligence for decisions for allocating resources. Resource management is a component of situational awareness is to translate the decision maker's information needs to real world actions. The orchestration of sensors and identification of sources to produce relevant input for a fusion process is referred to as resource management. Resources are the technical means employed to gather essential data [9]. Operations Research is a branch of mathematics that studies decision making to obtain the best decision. Game theory can help determine the optimal investment strategy [19].

max $F = (F_1(x_1, h_{ij}))$ where j is an element of the i^{th} hierarchy

such as:

max $F = (F_1(x_1, h_{41} \ldots y_{45,} h_{31} \ldots h_{35,} h_{21} \ldots h_{25}\, h_{11} \ldots h_9))$,

s.t.

$x_1 \in S_1$ in which every h_{ij} is the solution as follows:

max f_{4j} (x_1) Competing Nations
Nash/Mutual Information

max f_{3j} (x_1) Organizations
N Player Prisoner's Dilemma

max f_{2j} (x_1) Ship Classifications
Nash
s.t.
$g_{ij}(x_1, h_{ij}) <= 0$

max f_{1j} (x_1) Sensors
Shapley Value Marginal Contributions

Fig. 4. Hierarchical Structure

Generally, players may not possess full information about their opponents. In particular, players may possess private information that others should take into account when forming expectations about how a player would behave. To analyze these interesting situations, a class of games with incomplete information was created as use case scenarios (i.e., games where at least one player is uncertain about another player's payoff function) which are the analogue of the normal form games with complete information similar to Bayesian games or static games of incomplete information [17].

Hierarchy game theory offers important insights and demonstrates superiority of cooperation over competition. Game theory models the heuristics people use in managing their conflicts and helps to explain why rational decisions often miss opportunities for mutual gain [12]. Imperfect information may still be useful to help make decisions. Opponent modeling works by observing the opponent's actions and building a model by combining information from a pre-computed equilibrium strategy with the observations [3]. Cognitive hierarchy is important because it predicts the effect of group size which is not predicted by the Nash equilibrium [1].

3. INFORMATION FUSION

Game theory is the study of strategic decision making. It is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers and is often thought of as an interactive decision theory. It has been applied to economics, political science, psychology, logic, biology and other complex issues. Modern game theory began with the idea regarding the existence of mixed-strategy equilibrium in two-person zero-sum games, applied to economics. Later this evolved to provide a theory of expected utility, which allowed mathematicians and economists to treat decision-making with uncertainty. The notion of probabilistic predictions utilizing game theory is critical in practice to many decision making applications because optimizing user experience requires being able to compute the expected utilities of mutually exclusive pieces of data.

We have created a reward matrix of five rows and nine columns. The five rows are the ship classifications as shown in Figure 5. The nine columns are the sensor capabilities values for probability of detection or area under the Receiver Operating Characteristic (ROC) curves (AUC). Other simulations have accounted for games involving more than two players [4]. Determining ship classification which maximizes the a posteriori probability are Nash equilibrium points of the game. The Nash equilibrium points are local maxima have been proven. Relaxation algorithms exist showing efficiency and rapid convergence [8].

	Vis	HS	IR	SAR	HF Radar	LiDAR	Sound	AIS	Buoy
Ship									
No Ship									

	Vis	HS	IR	SAR	HF Radar	LiDAR	Sound	AIS	Buoy
Warship	a11	a12	a13	a14	a15	a16	a17	a18	a19
Fishing Vessel	b21	b22	b23	b24	b25	b26	b27	b28	b29
Cargo Vessel	c31	c32	c33	c34	c35	c36	c37	c38	c39
Ice Breaker	d41	d42	d43	d44	d45	d46	d47	d48	d49
Research Vessel	e51	e52	e53	e54	e55	e56	e57	e58	e59

Fig. 5. Reward Matrix

Maximin equilibrium often is the strategy and is called the Nash theory application of zero or constant sum strategy game [11]. Game theory considers the effect of a player's decision on other decision makers. In many situations, the opponents know the strategy that they are following and what actions are available. The Nash threshold can be used to determine if the player is on the blue or red team. For example, if a reward matrix exists, then the equilibrium point is the one where the reward is the smallest value in its row and the largest number in its column [19].

$$\max_{\text{all rows}} (\text{row min}) = \min_{\text{all columns}} (\text{column max}) \qquad (1)$$

This left half of (1) presents the basic applied theory to decision making of our model under uncertainty. For a possible action, one consideration is to choose the "best" worst outcome. The maximin criterion suggests that the decision-maker should choose the alternative which maximizes the minimum payoff he can get. This pessimistic approach implies that the decision-maker should expect the worst to happen. The maximin criterion is concerned with making worst possible outcome as pleasant as possible [19].

The right half of (1) represents minimax regret criterion which uses the concept of opportunity cost to arrive at a decision. The regret of an outcome is the difference between the value of that outcome and the maximum value of all the possible outcomes. For any action and state, there is opportunity of loss or regret. The decision-maker should choose the alternative that minimizes the maximum regret he/she could suffer [19].

Using different weights allowed for choices is to highlight the ability and need for a tool which can be used to allow the user to dial and modify modeled parameters of the reward matrix to model "what if" scenarios. Additionally saving the weights to a file allows for peer review in order to check and validate decisions. Our approach is modeled, so that the process can be repeated to allow for new or higher quality data/information to be inserted into the process to generate updated results [15]. Equation (2) is the translation of a reward matrix to a linear program which can be solved mathematically.

$$
\begin{aligned}
&\max v \qquad\qquad\qquad\qquad\qquad\qquad (2)\\
&\text{s.t.}\\
&\quad v - a_{11}x1 - b_{21}\,x2 - c_{31}x3 - d_{41}x4 - e_{51}x5 \;\; <= 0\\
&\quad v - a_{12}x1 - b_{22}\,x2 - c_{32}x3 - d_{42}x4 - e_{52}x5 \;\; <= 0\\
&\quad v - a_{13}x1 - b_{23}\,x2 - c_{33}x3 - d_{43}x4 - e_{53}x5 \;\; <= 0\\
&\quad v - a_{14}x1 - b_{24}\,x2 - c_{34}x3 - d_{44}x4 - e_{54}x5 \;\; <= 0\\
&\quad v - a_{15}x1 - b_{25}\,x2 - c_{35}x3 - d_{45}x4 - e_{55}x5 \;\; <= 0\\
&\quad v - a_{16}x1 - b_{26}\,x2 - c_{36}x3 - d_{46}x4 - e_{56}x5 \;\; <= 0\\
&\quad v - a_{17}x1 - b_{27}\,x2 - c_{37}x3 - d_{47}x4 - e_{57}x5 \;\; <= 0\\
&\quad v - a_{18}x1 - b_{28}\,x2 - c_{38}x3 - d_{48}x4 - e_{58}x5 \;\; <= 0\\
&\quad v - a_{19}x1 - b_{29}\,x2 - c_{39}x3 - d_{49}x4 - e_{59}x5 \;\; <= 0\\
&\\
&x1 + x2 + x3 + x4 + x5 = 1\\
&x1, x2, x3, x4, x5 >= 0
\end{aligned}
$$

The initial solution for the blue player's mixed strategy in terms of probabilities: $\mathbf{x} = (x1, x2, x3, x4, x5)$.

4. MODELING AND SIMULATION

When you use a mathematical model to describe reality you must make approximations. The world is more complicated than the kinds of optimization problems that we are able to solve.

Linearity assumptions usually are significant approximations. Another important approximation comes because you cannot be sure of the data that you put into the model. Your knowledge of the relevant technology may be imprecise, forcing you to approximate values in A, b, or c in a linear equation. Moreover, information may change. Sensitivity analysis is a systematic study of how sensitive solutions are to changes in data [6].

Figure 6 shows our sensitivity analysis using several different signal to noise ratios (SNRs). The graph shows the accuracy as a function of the number of parameters (sensors). In our example we have added a signal to one column parameter and Gaussian noise to each parameter in the reward matrix. The SNR, d, is the distance between the means on the two hypotheses, ship class present or not, with a variance normalized to one.

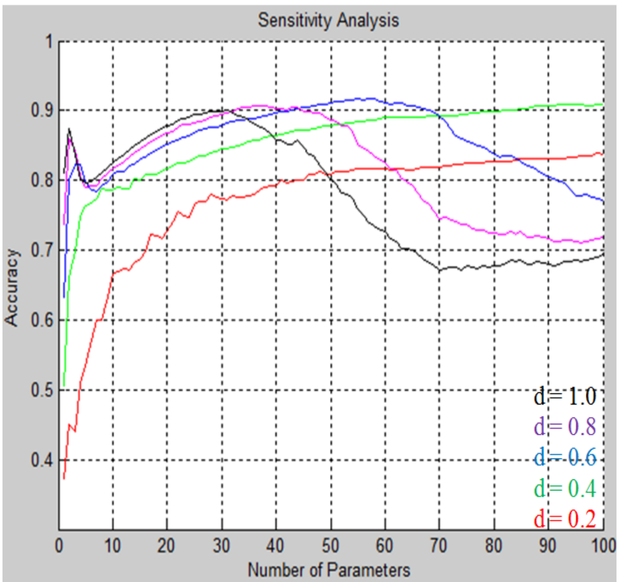

Fig. 6. Linear Programming Sensitivity Analysis

Our sensitivity analysis shows that more parameters are useful when the SNR is low. The analysis also shows that at higher SNR, two or three sensors are enough. The reason for higher accuracy at low SNR is that more information, sensors, helps. The reason for a lower accuracy at higher SNRs is because we have added more constraints as we add more parameters to the linear program. This is similar to principal component analysis where most of the information is contained in the first few variables [13].

If you add a constraint to a problem, two things can happen. Your original solution satisfies the constraint or it doesn't. If it does, then you are finished. If you had a solution before and the solution is still feasible for the new problem, then you must still have a solution. If the original solution does not satisfy the new constraint, then possibly the new problem is infeasible. If not, then there is another solution. The value must go down. (Adding a constraint makes the problem harder to satisfy, so you cannot possibly do better than before). If your original solution satisfies your new constraint, then you can do as well as before. If not, then you will do worse [6].

Figure 7 shows the sensitivity analysis for the Shapley method for calculating accuracy due to marginal contributions based on order [16]. Our Matlab implementation of treating each ship class as a player in a game uses the Shapley value as the probability of choosing a class based on sensor parameters. Our solution currently considers the running average while adding another sensor parameter or column to the reward matrix.

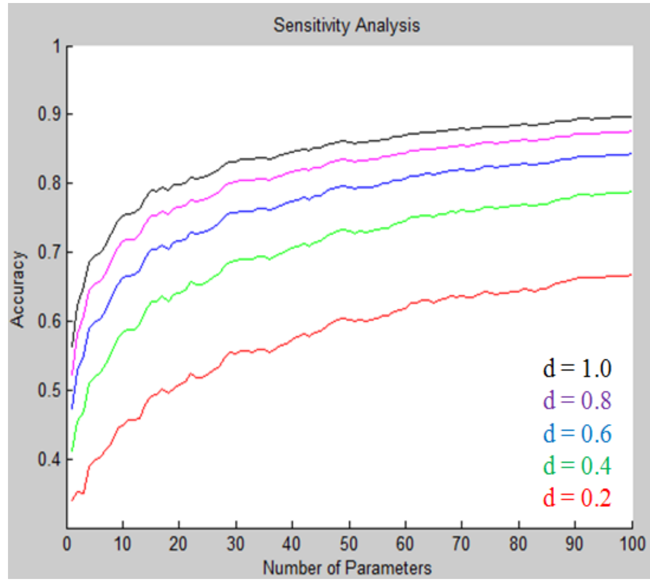

Fig 7. Shapley Sensitivity Analysis

It is interesting to compare the linear programming solution with the Shapley solution. This shows that there is some decision making process for choosing a modeling method. The human brain still needs to be involved in sorting out complex results.

Other work includes an analysis of strategic behavior of countries when there is imperfect verification of an arms control agreement. It provides a framework for determining whether an arms control agreement is desirable, shows which factors are needed for the agreement to be maintained in the absence of third-party enforcers, and develops propositions relating changes in verification capabilities to changes in the likelihood of cheating and the use of verification technology. These propositions yield several paradoxes of information (for example, the better the verification technology, the less often it will be employed). Since the analysis incorporates both simultaneous and sequential moves by the players, it provides new insights into other applied areas as well as game theory [20].

5. CONCLUSION

No decision is ever 100% correct; however, understanding the effects of algorithmic decisions based upon multiple variables, attributes, or factors and strategies with probability assignments can increase the probability for the best decision for a particular situation or event. We discussed a linear programming method for modeling ship verification reporting activities with limited resources. We realize that solution presented is only a guide and is not intended to replace the human brain in decision making. Multi-disciplinary solutions including automated game theory is promising for solving real world strategies and helps an analyst make optimal decisions.

Our contribution in this paper is to combine linear programming, hierarchical game theory with uncertainty modeling in order to plan for activities based on open source intelligence. Our example shows mixed probabilities of ship classification to help a player's situational awareness in order stay knowledgeable about a region of interest. Our solution provides the ability to populate a reward matrix from unstructured big data. We combine a number of technologies for data fusion. Our solution is a multi-use application: course of action planning, resource management, and risk assessment. In the presence of game theory and hierarchical theory, and on the basis of dynamic state attrition-models, our strategy can solve this kind of problem favorably.

Automated processing techniques are needed to augment tactical intelligence-analysis capabilities by identifying and recognizing patterns, weighting them appropriately, providing near real time objective decisions where the user can interact with the information based upon their experiences and knowledge base. GlobalSite is a probabilistic decision solution which allows for users to interact with information in near real time using game theory to provide a reward matrix of best possible outcomes.

Our approach adds computational intelligence to provide the analyst with a decision making capability to reduce time to collect and process data while retaining the information needed to complete the mission analysis. Additionally the probabilities of successfully performing ship reporting verification are filtered by the level of cooperation between participating organizations. Proper execution is critical for attaining the desired impact with respect to other nation players. Our sensitivity analysis models the accuracy as a function of the number of available sensor assets.

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