CAS – A Journey Has Begun In Aotearoa New Zealand

Derek Smith
Victoria University of Wellington,
Te Whare Wānanga o te Ūpoko o te Ika a Māui
derek.smith@vuw.ac.nz

Abstract

This paper explores a journey through hand-held technology changes in mathematics teaching and learning and raises questions we as mathematics educators should be considering in the shorter and longer term. New Zealand is embarking on a Computer Algebraic Systems (CAS) Pilot Programme in secondary school mathematics. The Ministry of Education and the New Zealand Qualifications Authority have selected secondary schools to be part of a pilot programme in the use of CAS technology in mathematics classes. The aim of the pilot programme is to improve teaching and learning of mathematics through the use of this technology. Six schools in 2005 used CAS technology with Year 9 (13-14 year olds) students and, an additional 16 schools joined the programme in 2006. The pilot is planned to continue with an increasing number of schools in subsequent years. By the time students in the pilot schools reach Years 11, 12 and 13, alternative external assessments using the CAS technology will be available. Professional development support and assistance in obtaining and using the technology will be provided to the pilot schools. The project's emphasis in 2005 was on the Geometry and Algebra strands; the Statistics strand was added in 2006. By 2010 the first cohort of project programme students will have been through their secondary mathematics education via a CAS environment. New Zealand teachers have only a finite time to get into CAS technology and integrate it into their teaching practice. This paper discusses a research project based on a mathematics department professional development that is linked to the pilot.

Theme: Education and Training Systems and Technologies
Teacher Education, CAS Technology

Background

Throughout recorded history people have created and used tools to make our existence easier, in the instance of mathematics to make routine calculations reasonably uneventful. The focus can then be on 'mathematising' the problem rather than the mundane calculations involved in intermediate steps.

The 1992 *Mathematics in the New Zealand Curriculum* document (MiNZC) promotes the use of this technology to make mathematics more accessible, interesting and innovative:

> *Calculators are powerful tools for helping students to discover numerical facts and patterns, and helping them to make generalisations about, for example, repeated operations. Graphics calculators, and computer software such as graphing packages and spreadsheets, are tools which enable students to concentrate on mathematical ideas rather than on routine mechanical manipulation, which often intrudes on the real point of particular learning situations.*
> Ministry of Education, 1992, p.14.

[Note: Presently New Zealand is updating its national mathematics curriculum.]

Although things appear to be changing rapidly around us, in terms of technology development, there is only slow change in pedagogical practice. As technology begins to influence how teachers and learners perceive mathematics in secondary schools, we can wonder whether the slow change is being driven by assessment or because we are clinging on to the mathematical history. Pedegogical practice is not keeping pace with technological development.

Teacher beliefs about the nature of mathematics and mathematics teaching and learning have an impact on teaching practice. With Computer Algebraic Systems (CAS), a question now asked could be, "Will technology in the mathematics classroom provide a richer, mathematical experience for secondary school students?" This question was being investigated as early as 1990 with graphic calculators as the classroom tool, where researchers were advocating technology for classrooms:

> *Computers and graphic calculators should be to mathematics teachers as what laboratory equipment is to science teachers.* Demana & Waits, 1990, p.29.

For technology use in the classroom setting to be meaningful, its integration into classrooms could not be via an ad hoc approach, but a readily available piece of equipment to use in an informed way:

> *Discussions about the use, role and impact of technology in secondary schools have, and will continue to be a topical issue in this present climate of educational reform. Technology, and in particular, a student's own personal technology, needs to be assessable and relevant to the needs of the students and should support and encourage their mathematical learning. Computer and calculator are becoming specialist tools for secondary school mathematics and science students and very soon, developments will see them looking and behaving like a computer.* Smith, 1997, p.1.

Technology is accepted as an important tool in the learning and teaching of mathematics which:

> *…influences the mathematics that is taught and enhances students' learning.* NCTM, 2000, p.24.

The challenge is using the technology to engage the learner in the mathematics being explored to build on learning and understanding concepts rather than procedures, thus building a firm foundation for continued mathematical involvement. Meaningful use of technology in classrooms should be through a planned integrated approach.

Hand-held technology

An exploratory study funded by Ministry of Education (MoE) found that technology can be a strong catalyst for change in the learning and teaching of mathematics (Barton, Bullock, Buzeika, Ellis, Regan, Thomas, & Tyrell, 1993). It was not until the restructuring of assessment reforms in New Zealand began in 2000 that mathematics educators saw the relevance of incorporating the graphic calculator (GC) into their teaching practice, but then more as an examination partner than as an investigative, exploratory and problem-solving mathematical tool.

The connecting of the three forms, graphic, algebraic and numeric, can be visualised and utilised with a GC. The 1990s saw the GC move mathematics into a graphical representation mode, establishing the links to the graphical form of mathematics, away from the rigours of formal algebraic reasoning and proofs. Studies such as Demana & Waits (1990), Kutzler (2003), Thomas & Holton (2003), Dick, Dion, & Wright (2003) and Heid (2003) share the view that the use of technology, whether GC or CAS, can support and sustain learning.

CAS technology integrates the numerical nature of the GC brought us in the latter part of the 20th century pushing the boundaries of secondary school mathematics and real life closer, and further and makes the mathematics more tangible in today's classroom. The dilemma emerging now is should the existence of CAS force the issue of rethinking and re-examining curriculum design and pedagogy and assessment? Figure 1 illustrates the moving between the three forms via GC and CAS platforms.

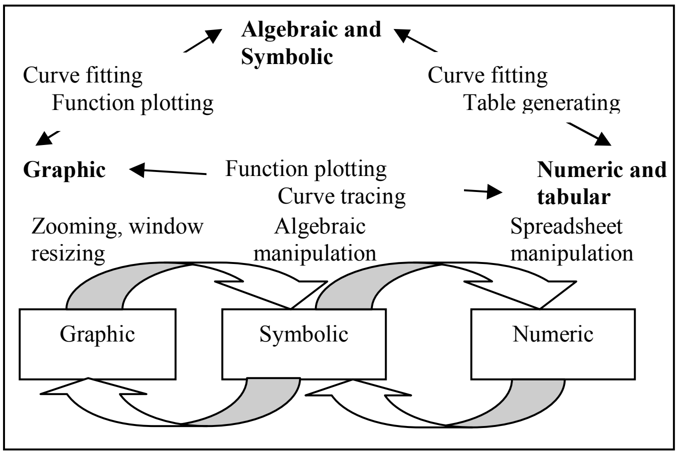

Figure 1: Graphic calculator and CAS representations.

The most talked about feature of hand-held CAS is of the ability to symbolically manipulate, but the CAS calculators available today go beyond that. There is a 'seamless' connection between mathematical strands (e.g. Algebra – Geometry) via CAS, possibly assisting students in making real connections between mathematical concepts, rather than viewing mathematics as isolated topics. Every time a new 'generation' of CAS technology is released we ask, "What's inside this box now?" The cry from the teachers could be, "What will be left for me to teach, if students have access to such technologies, that will factorise, solve, do calculus and more?" and "If students use these tools what will happen to their algebraic skills?"

More questions than answers

Research into CAS use is only at an infancy stage worldwide. There are perhaps more questions being asked than the answering of questions initially posed. Australian researchers Ball and Stacey (2001) focused on algebra, and Pierce and Stacey (2002a, 2002b), algebraic insight, which raised more questions such as: Can CAS have an effect on overall achievement in mathematics? Does CAS technology instruction affect student attitude toward using CAS technology in mathematics? How can we use CAS to promote algebraic thinking? Will the use of CAS place a greater emphasis back on to algebra? What do students record? What about mathematical syntax? Will the CAS syntax notations be accepted into the mathematical community as legitimate mathematical symbolism? How do we get students to think mathematics, not just using the technology? Is it technology for technology's sake or technology for mathematics' sake? Will there be a change in assessment to address the CAS issue? GCs have provided a wonderful experience in mathematics exploring via graphs. Will CAS do the same via algebra? Is there a need to introduce CAS at a level prior to external examination settings? Do 'pencil and paper' methods improve with CAS usage? Studies in Austria, Belgium, Denmark, Scotland and Switzerland have highlighted similar issues. Generally findings endorse the notion that learning can be enhanced by appropriate use of CAS. Heid (2003) noted, after using CAS, "students' mathematical development seemed to proceed more rapidly" (p. 40). Heid and Edwards (2001) also found that weaker students were able to "examine algebraic expressions from a more conceptual point of view ..." (p. 131). Noguera (2001) also commented that "[students'] cognitive development in algebra improved" (p. 263).

There appears to be a definite need for a balance between using technology intelligently, and retaining traditional pencil and paper algorithms and skills. Allowing CAS into courses will obviously affect content, instructional delivery and assessment; in particular how to evaluate student understanding. Judicious use of CAS needs also to be encouraged considering whether to use, or not use, technology. Both teachers and students can appreciate how technology can be used functionally to demonstrate mathematical insight (Ball & Stacey 2001). The teacher needs to be proactive in promoting such efficient decision making about technology usage. A teacher can integrate the technology into curriculum delivery (i.e. teach students how to use it) and will sometimes tactically restrict technology use (forcing the student to use pencil and paper methods) and promote habits in the student of using algebraic insight for overview and self-monitoring (Pierce & Stacey, 2003). With this idea in mind, classroom modelling should be such that the method of attack is similar to what would be reflected in an assessment task that is legitimising both methods of pedagogy, the use of (a) technology and (b) pencil and paper. Kutzler (2003) speaks of the need of sequentially using and not using technology to achieve certain learning goals (p. 53).

Common concerns held by teachers are of CAS as a 'black box' and fear students would carry out the mathematics blindly and without understanding by just pushing calculator keys. Issues that arose from the Dick, Dion and Wright (2003) study can be summarised as:

1. CAS can be a powerful tool for helping people 'do' mathematics. Simply by performing an operation with a CAS is not by itself a mathematical activity.
2. A pencil and paper algorithm can be a powerful tool for helping people 'do' mathematics. Simply performing an operation with a pencil and paper algorithm is not by itself a mathematical activity.

Changes that will be brought about by CAS use will inevitably bring about changes to assessment practice. Many traditionally asked questions become quite trivial, so learning to use new tools is perhaps also about learning to ask new questions. Assessment must be an assessment of student learning that can be valued and provide a valid reflection on the learning that has taken place in the courses provided. Can it also be that students' understanding of course content, even though it has been delivered via a CAS environment, may be better assessed without the technology?

Aim of the CAS pilot in New Zealand

The aim is to improve the teaching and learning of mathematics through the use of CAS technology. Indicators of improvement could include changes in teacher practice, changes in student attitudes, and changes in student learning experiences and assessment outcomes. Other areas to be considered are the professional development provided to schools, and issues of sustainability. Presently the CAS platforms in the trial are the CASIO ClassPad 300, the Voyager 200 from Texas Instruments and in 2006 Hewlett Packard entered the pilot with their HP49G+ model. Focussing on junior secondary school (Year 9 and Year 10) has been considered a strength as this would have minimal effects on national qualifications sought by Year 11, 12 and 13 students (15-19 year olds). The CAS pilot is aiming to give a more structured and supportive approach to the introduction of the CAS technology than that which occurred when graphic calculators entered New Zealand schools.

The Study: Proposed model for classroom delivery

The proposed teaching model offered by the researcher was to focus on how handheld CAS technology could be used in engaging learners in mathematics. The researcher adopted the following teaching approach:

1. problem is posed
2. sketch/solve without the use of technology either in groups or individually
3. teachers share findings with peers
4. discussion is initiated for methods that merit follow-up
5. teachers apply and calculate
6. teacher demonstrations on CAS and/or by pencil and paper
7. teachers discuss the efficiency of both methods.

In modelling the teaching and learning with the mathematics staff the researcher wanted to demonstrate the following:

(i) Inclusive Classroom Climate: Successful teachers create social norms in their classrooms that give students the confidence and ability to take risks, to discuss with others, and to listen actively (Cobb, McClain, & Whitenack, 1995). Valuing student diversity, academically, socially, and culturally, is fundamental to the development of positive relationships between teachers and students (Bishop, Berryman, Richardson, & Tiakiwai, 2003).

(ii) Focused Planning: Target concepts and processes to be learned, by sharing the sessions' learning intentions.

(iii) Problem-centred Activities: Students in high-performing countries spend a large proportion of their class time solving problems (Stigler & Hiebert, 1997). The students do so individually, as well as co-operatively. Also a shared belief between teacher and students is that the responsibility for knowledge creation lies with the students (Clarke & Hoon, 2005).

(iv) Responsive Lessons: Successful teachers constantly monitor their students' thinking and react by continually adjusting the tasks, questions, scaffolding, and feedback.

(v) Connections: Askew, Brown, Rhodes, William, & Johnson, (1997) reported that successful teachers are "connectionist". Teachers use powerful representations of concepts, linking mathematical vocabulary and symbols with actions on materials.

(vi) High Expectations: Ask questions that provoke high-order thinking skills, such as analysing, synthesising, and justifying.

These six points were also taken into account from recent research indicating that quality teaching is critical to the improvement of student outcomes (Alton-Lee, 2003; Hattie, 2002). Classrooms are complex social situations. Acknowledging this complexity, it is possible to use activities with teachers that can make a positive difference to achievement.

Participants

The teachers involved in this study taught in a state co-educational secondary school (Year 9-13, 13-19 year olds) which serves a large rural town and region. All full-time staff in the Mathematics Department hold qualifications in mathematics and/or statistics and their teaching experience ranges from 12-35 years.

Four full days of Professional Development (PD) over the 2006 academic year was provided by the researcher with part of each day focusing on teaching and learning with CAS. The sessions were aimed to contribute to the teacher's personal understanding of CAS and promote positive growth as a teacher in technology use, with the aim to change classroom practices, leading to improved student learning outcomes and teacher beliefs and attitudes. The teachers had ownership of the PD and the researcher provided a framework for teachers to provide an opportunity to challenge their existing classroom practices. The staff had other activities to work on between these PD days to increase their confidence in using CAS.

Case Study 1

The following scenario was posed:

"Take one of the 6 cm by 6 cm pieces of paper provided, and after following the instructions on folding that piece of paper, compare yours with your neighbour's."

Instructions "Create a fold so that AC = AE, then a fold such that BC = BG and finally, fold so that DH = FG.

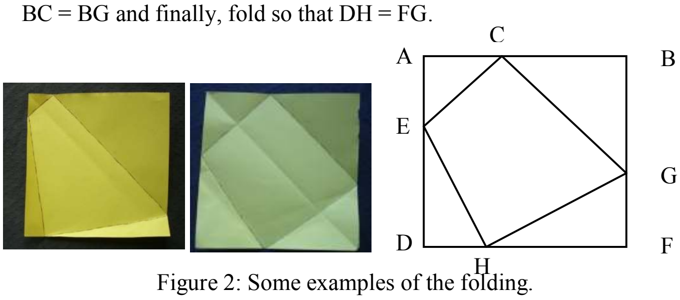

Figure 2: Some examples of the folding.

The question was then asked, "Can you suggest something about the areas of the quadrilaterals that have been formed inside the original squares?"

One of the teachers suggested that they might be equal when she had compared hers with others around her. We called it 'Lee's Conjecture' [teacher's name] and they began to investigate this by using the geometrical animation capability of the ClassPad 300 to show a series of differently folded squares to produce quadrilaterals. Two examples of the paper folding are shown in Figure 2.

The construction sequence was discussed and decisions were made such as where the positions of the quadrilaterals start in the animation sequence. Collectively the group came up with a series of instruction sequences, listed below, beginning with 'draw the line AB' and then place 'C anywhere on the line AB', as the group discovered in their trialling that the animation always begins from the starting place from which a line is constructed.

- Draw the line AB and draw a point C
- Change the tool to a 'pointer'. See Figure 3a
- Tap a point C and the line AB
- Open **Edit** and selecting **Add animation**. See Figure 3a
- Repeat this process for producing the line AD and the point E and add the animation
- Create the line FB and the point G and add the animation
- Construct the line DF and point H and add the animation
- Then form the quadrilateral 'snapping the vertices' to the points EFHG. Add some shading to distinguish the area of the quadrilateral by selecting the four vertices [changing the tool to 'pointer'] and opening **Edit** then selecting **Shade On/Off.** See Figure 3b**.** The Animation is captured Figure 3c, d, e and f.

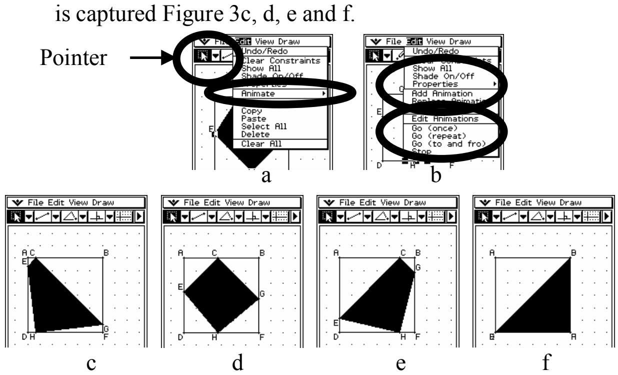

Figure 3: Construction of the square, points and quadrilateral completed, the animation captured.

Collecting table values from the changing quadrilateral

Running the animation creates a 'log' of data values that can then be displayed. These include angles, lengths, gradients, and areas. Time was given to explore what was happening to the four triangular areas that have been formed inside the original square by selecting the triangles in turn and creating a table of values for these respective areas (Figure 4). Some copy and pasted the data into the spreadsheet and used cell formulae to verify that the areas of the four triangles added to 18.

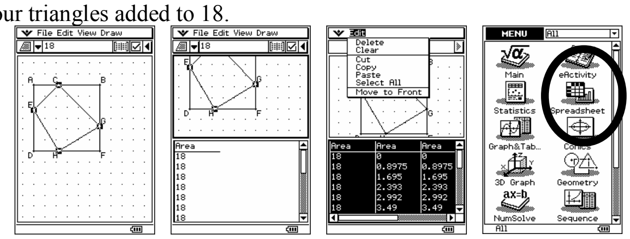

Figure 4: Collecting the data from the animations of the areas.

The algebra

The question, "What is another way to write 18?" was asked. This was used as a lead-in to the algebra that is embedded in this geometrical problem.

Area of quadrilateral

$$= 36 - \tfrac{1}{2}x^2 - \tfrac{1}{2}(6 - x)(6 - x) - \tfrac{1}{2}(6 - x)\,x - \tfrac{1}{2}(6 - x)x$$
$$= 36 - \tfrac{1}{2}x^2 - \tfrac{1}{2}(36 - 12x + x^2) - \tfrac{1}{2}(6x - x^2) - \tfrac{1}{2}(6x - x^2)$$
$$= 36 - \tfrac{1}{2}x^2 - 18 + 6x - \tfrac{1}{2}x^2 - 3x + \tfrac{1}{2}x^2 - 3x + \tfrac{1}{2}x^2$$
$$= 36 - 18 - \tfrac{1}{2}x^2 - \tfrac{1}{2}x^2 + \tfrac{1}{2}x^2 + \tfrac{1}{2}x^2 + 6x - 3x - 3x$$
$$= 18$$

The use of the **Main** icon, confirmed the algebra illustrated above. Using the commands 'expand' and 'simplify', as illustrated in Figure 5, the expression for the relationship found between the original square, the quadrilateral and the four triangles, was confirmed. Some of the teachers realised that the 'expand' command was not necessary using CAS. But the power of 'expand' was not lost, the "white box–black box" metaphor introduced by Buchberger (1990, cited in Cedillo and Kieran, 2003, p. 221), distinguishes between using the technology blindly to perform routine mathematical tasks (black box), and using it to help students construct meaning for mathematical concepts and procedures (white box). Kutzler (2003), in his exploration of pedagogical approaches to CAS usage, stated:

> *The reason that so many students are at odds with mathematics may be related to their lack of experimentation.* Kutzler, 2003 , p.61.

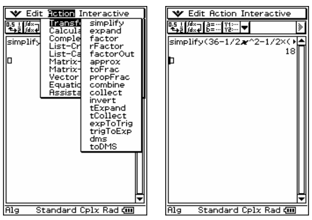

Figure 5: Commands 'expand' and 'simplify'.

This activity linked the concepts of geometrical construction, animating the geometrical construction along with logic and reasoning within the construction sequence. The animation demonstrated a number of possible constructions based on a varying length of the original AE = AF fold. Calculations for area, and algebra use, illustrated the area of the quadrilateral created was half of the original area of the square.

Feedback from the session was:

> "You certainly can have fun with this and are learning."
> "CAS is about pedagogy, and what learning we want to occur."
> "The ideas that you have demonstrated with this activity are redesigning the way we do mathematics now."

This indicates recognition by the teachers of the importance of pedagogical practice. A number said that this style of teaching would influence their future teaching practice.

Case Study 2

The following scenario was posed:

"The local council wishes to construct a memorial to the war veterans of the region. They plan to construct a circular water fountain in the local park in a triangular area that has been set aside for this purpose. Use geometrical constructions of any triangle to find the largest possible circle that can fit inside."

After initially working by sketches via pencil and paper the teachers were able to transfer these skills to the CAS and construct the in-circle using the appropriate 'drop down menus' and 'drawing tools.' The animation was done by the teachers in three ways:

1. The vertices of the triangle were manipulated and the captured data gathered.
2. A vertex was animated along a line segment and the data captured in the table format and then manipulated in the spreadsheet icon.
3. All three vertices were animated along a line segment and the data captured in the table format and then manipulated in the spreadsheet.

Figure 6 illustrates these three construction methods and data gathering, moving information between icons and then using animation features and the spreadsheet.

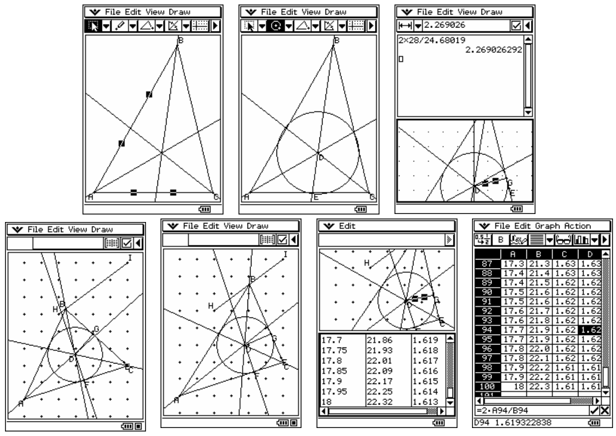

Figure 6: Constructions and calculations.

The question, "Is there a relationship between the triangles area and perimeter and the radius of the in-circle?" was then posed, as a lead into the algebra embedded in this problem.

The algebra

Consider a 5, 12, 13 triangle as shown in Figure 7. Let r be the radius of the in-circle. Then the area of the triangle is $\tfrac{1}{2}\times5\times12 = 30$ square units. Equating each of the three smaller triangles yields: $\tfrac{1}{2}.5.r + \tfrac{1}{2}.12.r + \tfrac{1}{2}.13.r = 30$ [See dotted lines.]

$$r = {}^{15r}/_{15} = {}^{30}/_{15} = 2$$

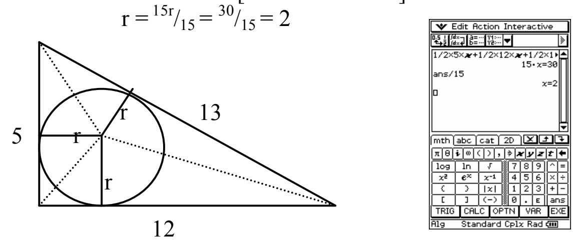

Figure 7: An example using a 5,12,13 triangle.

In general:
½ar + ½br + ½cr = Area [Note: A represents Area, this includes]
½r(a + b + c) = A
½r×P = A[Note: P represents Perimeter]
$r = {}^{2A}/_{P}$
Feedback from this session was constructive and caused the teacher to see the technology used as a tool for generating higher order thinking:

> "We actually have to think about what we are doing and what was happening."
> "I did not know that there was a relationship between the area and perimeter of a triangle with the incircle. This has opened up my eyes to the ways that CAS can be used to explore mathematical ideas."

Findings
Interacting with CAS geometrically, symbolically, and numerically can promote positive engagement and interactions. Teachers need to be judicious users of the technology, selecting when, and when not to, use CAS, which requires a good understanding of the concepts with and without using CAS. But as a teaching and learning tool, CAS offers great potential as shown in these selected teacher comments:

> "We have been challenged mathematically."
> "This activity has challenged me and has made me make sound links between the math strands."
> "These sessions have opened my eyes to the richness in what CAS has to offer my students and my teaching."
> "We explored the geometry and used the numbers to discover a formula."
> "We have been challenged and it has been provided with support that has been non judgemental in our learning. I am beginning to see how the students can feel in my class when the understanding is missing."
> "Mathematics is my subject specialty but there were processes that I had not thought of until now."
> "Staff really enjoyed learning a new skill – clearly they will need continued exposure to this new technology in order to feel confident in adapting lessons to accommodate it. It will be an ongoing learning curve. [Advisor] shared some draft exemplars with us which gave us an indication of the changes to come."
> "Somehow [Advisor] always had a unique challenge to present to us which got us thinking outside the square."
> "I like the way that we drew from our knowledge and used pencil and paper to get the ideas and concepts on paper and then used the CAS."

Building a community of learners to promote mathematical interactions between teachers and learners is a model that can assist in teacher professional development. Promotion of this learning community is an aspect that can be transferred to the teachers' classroom, particularly when the learning is to be collaborative and constructivist in nature.
The department intended to adopt the proposed model to deliver parts of their courses to junior and senior classes. The teachers described the following ways that CAS could influence their teaching practice:

- scaffolding the development with a balance of CAS and 'pencil and paper' skills
- enabling exploration and investigation of the geometry and algebra
- focussing on formulation of algebraic expressions, e.g. sameness in expressions such as $(x + y)^2 = x^2 + 2xy + y^2$ or $y = 4x + 17$ and $y = 4(x + 3) + 5$ or 32 and $\frac{1}{2}x^2 + \frac{1}{2}(8 - x)^2 + x(8-x)$
- thinking about the problem posed and logically working through it - from 'muddling through' → sequencing systematically → logical steps → minimum steps required. This 'packing' and 'unpacking' is like layers of an onion? Working from the inside to the outside
- making the learning more student centred
- making links to geometry and algebra more interactive
- more mathematical discussion and questioning being evident when CAS was included in the activity.

Conclusion
The case studies primarily focused on the effects that technology could have on teacher practice. Technology use can transform learning both in terms of what is learnt and how it is learnt. Inquiry-based learning is consistent with a constructivist view of knowledge acquisition, stimulating mathematical thinking and creating opportunities for reflection on the learning. Group discussions where teachers (and students) share roles and engagement in the tasks provided an illustration of how the environment promotes learning.
The extent of impact that CAS technology has in classrooms depends to a large extent on the teachers' attitudes, views, or philosophies concerning its use. The following summarises the teachers' beliefs towards technology in the classroom after this PD:

- CAS technology is a mathematical tool to enhance learning and understanding. It allows explorations and discoveries
- It allows for open-ended questions and exploration of real life investigations
- It can scaffold learning. The focus is on mathematical concepts rather than on the manipulative details. Moving between the multiple representations, algebraic, graphical, or tabular, gave alternative ways of expressing a problem and leading on to a possible solution.

Increasingly, research suggests effective teaching support is required for teachers to move from traditional methods of teaching mathematics to incorporate CAS into classroom practices. These aspects are crucial in the successful integration of technology into the teaching and learning programmes in secondary schools. The impact that this technology will have in shaping assessments of the future is still to be addressed. Mathematics for all and not just the privileged is paramount if the integration of CAS is to be a successful development in secondary school education. As educators we cannot ignore hand-held CAS technology. There are plenty of positives regarding the integration of CAS into learning programmes, whether it is to scaffold weaker mathematics students or assist others in pushing the boundaries of mathematical excellence. CAS is a new mathematical voyage, as we begin yet another paradigm shift in mathematics education.

References.

Alton-Lee, A. (2003). *Quality teaching for diverse students in schooling: Best evidence synthesis.* Wellington: Ministry of Education.

Askew, M., Brown, M., Rhodes, V., William, D., & Johnson, D. (1997). *Effective teachers of numeracy.* London: King's College, University of London.

Ball, L., & Stacey, K. (2001). *What should students record when solving problems with CAS? Reasons, information, the plan and some answers.* In J.T. Fey, A. Cuoco, C. Kieran, L. Mullin & R.M. Zbiek (Eds.), *Computer algebra systems in secondary* *school mathematics education* (pp.289-303). Reston, VA: The National Council of Teachers of Mathematics.

Barton, B., Bullock, J., Buzeika, A., Ellis, J., Regan, M., Thomas, M.O.J., & Tyrell, J. (1993). *Technology in mathematics education: An exploratory study.* Auckland: Mathematics Education Unit, University of Auckland.

Bishop, R., Tiakiwai, S., Richardson, E., & Berryman, M. (2003). *Te kotahitanga: The experiences of year 9 and 10 Māori students in mainstream classes.* Wellington: Ministry of Education.

Cedillo, T., & Kieran, C. (2003). Initiating students into algebra with symbol-manipulating calculators. In J. Fey, A. Cuoco, C. Kieran, L. McMullin & R. Zbiek (Eds.), *Computer algebra systems in secondary school mathematics education* (pp. 219-239). Reston, VA: The National Council of Teachers of Mathematics.

Clarke, D., & Hoon, S.L. (2005). Studying the responsibility for the generation of knowledge in mathematics classrooms in Hong Kong, Melbourne, San Diego and Shanghai. In H. Chick & J. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education,* July 10-15. Melbourne: PME.

Cobb, P., McClain, K., & Whitenack, J. (1995). Supporting young children's development of mathematical power. In A. Richards (Ed.), *FLAIR: Forging links and integrating resources* (Proceedings of the 15th biennial conference of the Australian Association of Mathematics Teachers, Darwin). Adelaide: AAMT.

Demana, F., & Waits, B. (1990). The role of technology in teaching mathematics. *Mathematics Teacher*, *83*(1), 27-31.

Dick, T., Dion, G., & Wright, C. (2003). Case study: Advanced placement calculus in the age of the computer algebra system. *Mathematics Teacher*, *96* (8), 588-596.

Hattie, J. (2002). *What are the attributes of excellent teachers?* A paper presented at the New Zealand Council for Educational Research Annual Conference. Wellington: NZCER.

Heid, K. (2003). Theories for thinking about the use of CAS in teaching and learning mathematics. In J. Fey, A. Cuoco, C. Kieran, L. McMullin & R. Zbiek (Eds.), *Computer algebra systems in secondary school mathematics education* (pp.33-51). Reston, VA: The National Council of Teachers of Mathematics.

Heid, K., & Edwards, M. (2001). Computer algebra systems: Revolution or retrofit for today's mathematics classrooms? *Theory into Practice*, *40*(2), 128-136.

Kutzler, B. (2003). CAS as pedagogical tools for teaching and learning mathematics. In J. Fey, A. Cuoco, C. Kieran, L. McMullin & R. Zbiek (Eds.), *Computer algebra systems in secondary school mathematics education* (pp.53-71). Reston, VA: The National Council of Teachers of Mathematics.

Ministry of Education. (1992). *Mathematics in the New Zealand curriculum.* Wellington: Learning Media.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Noguera, N. (2001). A description of tenth grade algebra students' attitudes and cognitive development when learning algebra using CAS. *The International Journal of Computer Algebra in Mathematics Education, 8*(4), 257-270.

Pierce, R., & Stacey, K. (2002a). Monitoring effective use of computer algebra systems. *Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia*, University of Auckland.

Pierce, R., & Stacey, K. (2002b). Algebraic insight: The algebra needed to use computer algebra systems. *Mathematics Teacher*, *95*(8), 622-627.

Piez, C., & Voxman, M. (1997). Multiple representations: Using different perspectives to form a clearer picture. *The Mathematics Teacher*, *90*(2), 164-166.

Smith, D. (1997). *Graphical calculators in the classroom: The effects that graphical calculators have in the enhancement of mathematical learning in the classroom.* Unpublished Master's Thesis. Massey University, Palmerston North.

Stigler, J., & Hiebert, J. (1997). Understanding and improving classroom mathematics instruction: An overview of the TIMSS video study. In *Raising Australian standards in mathematics and science: Insights from TIMSS*. Melbourne: ACER.

Thomas, M.O.J., & Holton, D. (2003). Technology as a tool for teaching undergraduate mathematics. In A.J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick & F. K. S. Leung (Eds.), *Second international handbook of mathematics education: Vol 1* (pp.347-390). Dordrecht: Kluwer.

Author

Derek Smith has taught mathematics and statistics at secondary schools in the Wellington region for 16 years. He is now a senior lecturer at VUWCE. He is involved in Mathematics Education in Primary and Secondary pre-service programmes and is a Regional Facilitator for the Numeracy Development Project. He has a long history of delivering workshops countrywide and at NZAMT conferences for teachers and students in the use of hand-held technology in teaching and learning mathematics and statistics.
Email: derek.smith@vuw.ac.nz