INTUITEL and the Hypercube Model - Developing Adaptive Learning Environments

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ABSTRACT

In this paper we introduce an approach for the creation of adaptive learning environments that give human-like recommendations to a learner in the form of a virtual tutor. We use ontologies defining pedagogical, didactic and learner-specific data describing a learner's progress, learning history, capabilities and the learner's current state within the learning environment. Learning recommendations are based on a reasoning process on these ontologies and can be provided in real-time. The ontologies may describe learning content from any domain of knowledge.

Furthermore, we describe an approach to store learning histories as spatio-temporal trajectories and to correlate them with influencing didactic factors. We show how such analysis of spatio-temporal data can be used for learning analytics to improve future adaptive learning environments.

Keywords: computer-assisted learning, adaptive e-learning, spatio-temporal database, learning analytics, learning pathway, instructional design

1. INTRODUCTION

With the raise of ICT, there have been diverse efforts to integrate learning theories into instructional designs of computer-based learning. In this context, fusioning technology with didactic and pedagogical expertise has been a challenge. The costs of design and implementation grow if such systems are supposed to adapt to the user's behavior, progress and personal capabilities.

Whereas early "adaptive" systems were oriented towards behaviorist concepts and were mainly designed as training and testing machines [1]–[5], today's approaches claim to implement more complex mechanisms of adaption. Nonetheless it remains difficult to design adaptive systems that – on the one hand – can adapt to individual learners and – on the other hand – can be used on a general base for various knowledge domains.

In this paper we introduce an approach to create adaptive learning environments that go beyond automated training and testing and that are not limited to specific knowledge domains. We implemented a system that integrates didactic and pedagogical knowledge into learning environments and infers on this knowledge in order to generate human-like learning recommendations in real-time.

The system was developed with the EU-funded INTUITEL project ("Intelligent Tutoring Interface for Technology-Enhanced Learning") [8]–[17]. INTUITEL uses ontologies to map pedagogical and didactic meta data of the learning content as well as data describing the learner's progress and state within the learning environment. The respective instructional design is inferred from these ontologies during runtime. As a consequence, this can be used for any learning material and any knowledge domain.

An innate nature of any instructional design is the transformation of semantically linked learning content into a linear sequence along the time dimension. We give an outlook on how the underlying model of INTUITEL can be expanded such that we can perform learning analytics in an entirely new way that especially focuses on the time dimension. Each learner's history of progress and state within the learning environment is modelled as a high-dimensional trajectory interpolating data over the time dimension. We describe how to use technology of spatio-temporal databases for this and how such data can provide for multi-variate data analysis.

2. THE INTUITEL SYSTEM

Human teachers organize learning content based on its characteristics and the relations between single pieces of that content. They estimate the suitability of the content for their students and seek reasonable forms of didactic progression. With INTUITEL such human knowledge is transformed into machine-processable ontologies that interrelate the learning content to a semantic network. Within the learning environment INTUITEL tracks the behavior of the learners, locating the learning content they have attended to and constantly deduces influencing factors which in the following we call *didactic factors*. Combined with the pedagogical ontology of INTUITEL, these measures form the input for the reasoning and recommendation process of INTUITEL.

Pedagogical Ontology
The pedagogical ontology of INTUITEL is derived from Norbert Meder's Webdidactics [6], [7] and provides a vocabulary to structure and semantically link learning content and form learning pathways. It is the result of an empiric study of a wide range of learning pathways discovered in didactic content in the past. On top, the learning material is subdivided according to the following granularity [21]:

Knowledge Domains (KO) are on the highest level and are equal

TABLE I
DIDACTIC FACTORS

Factor	Description
knowledge actuality	Ranks the time that has passed from the learner's last session until now
course-focused KO learning speed	ranks the average difference in time needed by a learner to proceed a course compared to the time of other course participants.
learning pathway permanence	Ranks the number of KOs a learner has completed along the current learning pathway compared to those of other learners
learning attention	States how much attention a learner pays to the learning content as measured by an eye tracking device
deafness	States if the learner is deaf
gender	Statement about the learner's gender
connectivity level	Ranks the quality of the learner's network connection

to entire courses

Concept Containers (CC) are equivalent to lessons and are contained by Knowledge Domains

Knowledge Objects (KO) are contained by Concept Containers and represent atomic pieces like texts, tests, exercises or media items

On the respective levels of granularity items are semantically linked such that they form learning pathways along which the learner is provided with recommendations [21]:

Macro Learning Pathways are built by linking different Concept Containers. These links may express hierarchical or chronological relations.

Micro Learning Pathways link different knowledge objects and may express the following relations:

- multi stage learning (behaviorist learning style)
- inquiry based learning (constructivist learning style)
- increasing level of abstraction
- increasing level of concretion

Additionally, the vocabulary allows to specify KOs with respect to their knowledge types and their media types. Amongst others, the set of knowledge types comprises receptive, interactive, cooperative and assignment types and may communicate orientation or explanation knowledge [21]. The set of media types includes text, tables, audio or video.

Didactic Factors

Beyond the definition of learning pathways and semantic relations among Knowledge Objects (KOs), INTUITEL is capable of considering didactic factors. These factors describe facts that are meaningful to rate the suitability of KOs and learning pathways with respect to a learner's personal, environmental and performance parameters. A didactic factor is aggregated by transforming non-nominal data into nominal data denoting an individual that is part of the ontologies on which the INTUITEL system performs its reasoning and recommendation process. Table I explains a few examples of the 47 didactic factors currently specified by the INTUITEL standard [22].

Transformation Rules for Didactic Factors

Didactic factors identify individuals in the scope of the pedagogical ontology. The following two simple examples give an impression of the transformation rules that are used to transform measured non-nominal data into such nominal identifiers.

Knowledge Actuality: The input data of this factor is *ld* = last login date. The according output data is *LastLoginLongAgo* or *LastLoginRecently*

```
IF(ld > 14 days) {
    output = "LastLoginLongAgo"}
ELSE {
    output = "LastLoginRecently"}
```

Learning Pathway Permanence: Input data of this factor is:

- *lAmount* = Number of KOs a learner has completed on the current learning pathway
- *oAmount* = Average number KOs other learners have completed on the same learning pathway
- *sd* denotes the standard deviation σ

Output data: *LpPermanenceHigh*, *LpPermanenceLow*, *LpPermanenceNormal*

```
IF(lAmount > oAmount + sd) {
      output = "LpPermanenceHigh"
} ELSE IF(lAmount < oAmount - sd) {
      output = "LpPermanenceLow"
} ELSE {
      output = "LpPermanenceNormal"
}
```

Recommendation Process

While the pedagogical ontology describes learning pathways and interrelations between KOs, the learning environment is to provide data contributing to learner-specific didactic factors. From this information, INTUITEL creates a learner state ontology (LSO). Combining the LSO and the pedagogical ontology, INTUITEL queries on the resulting custom ontology in order to perform a reduction on the set of available KOs, such that those KOs remain that fit most the learner's state and needs. Based on this data set, the learner is given a respective recommendation, with which KOs she may continue.

LMS Implementation

Currently, INTUITEL support has been implemented in the form of plugins for several open source and commercial Learning Management Systems (LMS): These are Ilias, Moodle, Crayons, Clix and eXact [20]. The plugins are coupled loosely and platform-independent to the INTUITEL system by using REST interfaces over a network. The INTUITEL system imports ids and meta data of KOs that are present in the LMS. These contents are then semantically annotated to form learning pathways as described above providing an editor to a didactic engineer e.g. a teacher. This information is kept entirely inside the INTUITEL system — outside of the LMS. The plugin tracks the ids of the KOs a learner attends to as well as environmental and performance data that contribute to didactic factors. This information is sent to the INTUITEL system which triggers the above described reasoning and recommendation process. The recommendations are finally sent by the INTUITEL system back to the LMS plugin. Within the user interface of the LMS, these recommendations are presented to the learner including textual messages and rankings of further KOs [20].

3. THE MULTIDIMENSIONAL COGNITIVE SPACE MODEL

The theoretical foundation of INTUITEL is based on a mathematical model to describe a learner's progress regarding the set of learning content in a learning environment. We introduce the concept of the learner's *cognitive position* in a *cognitive space* the dimensions of which are spanned by the KOs that are contained by a course and that a learner is supposed to attend. We assume that learning progress is measured by the amount of knowledge and skills a learner may have achieved, compared to the learning goal that is expressed by the same measures [22].

Based on this, let N be the number of KOs in a course. Then the learner's cognitive position is an N-dimensional vector $P = \{x_i\}$ with $i = 1, \ldots, N$ and $x_i \in [0,1]$. x_i represents the degree of progress with respect to the i^{th} KO. x_i can be determined the following ways:

1) The learning environment may omit the degree of progress of single KOs as percentage value.
2) The learning environment may omit the result of a test reviewing the learner's progress.
3) In the easiest case the learning environment only provides the information if a learner has attended to a KO or not. This leads to the special case when x_i degenerates to $x_i = \{0;1\}$

The Hypercube Model

The cognitive space model is best understood when visualizing it as a fuzzy hypercube [18], [19]. Each KO forms a dimension and a coordinate axis in a N-dimensional space. The degree of progress is an interval $[0,1]$ along this axis. Taking the interval-borders along each dimension we can build a N-dimensional hypercube. When assigning the current learning progress for each KO we obtain the already explained N-dimensional vector P denoting the cognitive position of the learner as a vector inside the hypercube. At the beginning P will point close to the origin, when a learner has made little progress so far. With increasing progress P will converge to the theoretically maximum position which is given by $P_e = \{x_i | \forall x_i = 1\}$. This would imply that the learner has attended to every single KO in the course with 100% success. Figure 1 shows a 3D-hypercube for $N = 3$. Figure 2 shows a hypercube for $N = 4$ and an exemplary learning pathway through it [22].

Learning Pathways and the Cognitive Distance

Using the hypercube paradigm, we can calculate the distance between the learner's current position and predefined learning pathways. This way INTUITEL determines a recommended learning pathway that is closest to the learner's cognitive position. For this purpose we introduce the *User Learning Pathway (ULP)* that equates to the cognitive position vector P except for the difference that the ordering of the KOs reflects the order in which a learner has processed the KOs so far.

In comparison to the ULP a predefined learning pathway LP is expressed as a permutation of the ULP reflecting the order in which the learner should have processed the KOs if he had followed this LP. As an example, assume a course consisting of 6 KOs. A possible LP may define the following sequence:

$LP \Rightarrow (KO_3, KO_1, KO_5, KO_4, KO_2, KO_6)$

Now suppose, the learner has first processed KO_1 completely, second KO_4 completely and third KO_5 to a degree of 60%. Then the ULP and the respective LP vectors equate to:

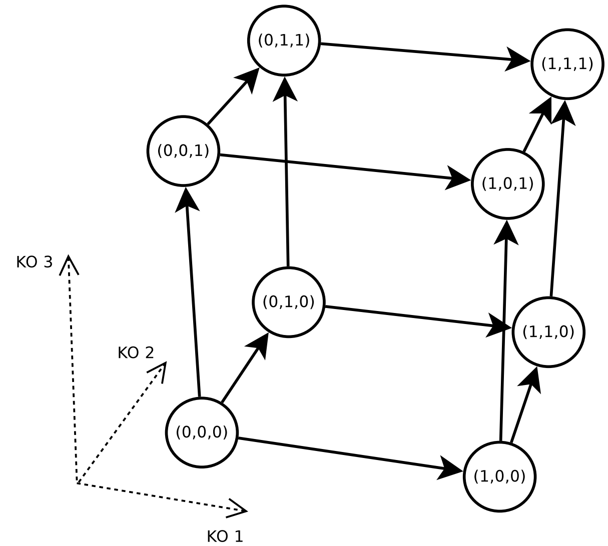

Fig. 1. 3D-hypercube for three KOs named A,B and C

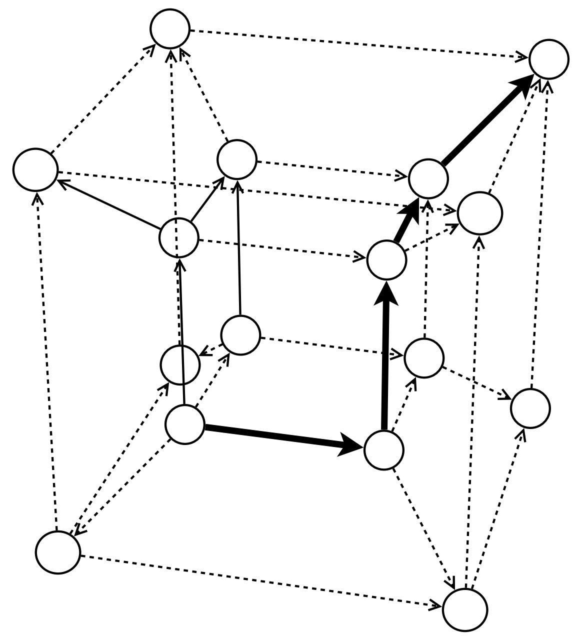

Fig. 2. 4D-hypercube for four KOs with an exemplary learning pathway (bold arrows)

$ULP = \{x_1, x_4, x_5, x_2, x_3, x_6\} = \{1, 1, 0.6, 0, 0, 0\}$
$LP = \{x_3, x_1, x_5, x_4, x_2, x_6\} = \{0, 1, 0.6, 1, 0, 0\}$

INTUITEL calculates a distance measure between these two vectors. This way INTUITEL is able to select the LP that fits most the current cognitive position of the learner [22].

4. FUTURE WORK: LEARNING HISTORIES AS SPATIO-TEMPORAL TRAJECTORIES

The following elaborations are subject of current and future work. Both instructional design and learning pathways have the time dimension as an integral component as the progress of learning always includes the transformation of learning content into a linear sequence along the time dimension. We enhance the hypercube model in a way such it forms the foundation of a new approach to perform learning analytics in experimental as well as in real-life learning situations. The results of these analytics are supposed to discover new didactic factors which may be used to improve future learning environments.

Enhancing the Hypercube Model
First, we enhance the hypercube model by adding indicators that may contribute to didactic factors. The N-dimensional cognitive space is enlarged by K additional dimensions associated with K indicators. Such indicators may be measured by the learning environment or they may be collected separately by the use of surveys accompanying the learning process. In this advanced hypercube model the enhanced vector denoting the learners cognitive position is $P_{(N+K)} = \{1,\ldots,N,1\ldots,K\}$.

Learning Pathways as Spatio-Temporal Trajectories
We will first build a temporal database system to track and store the cognitive position $P_{(N+K)}$ in the form of time-continuous vector data. In a second stage, building upon that temporal database, we transform this time-continuous vector data to $N+K$-dimensional spatio-temporal trajectories, lifting the temporal vector data to a spatio-temporal database. The spatio-temporal space of this database will represent the advanced hypercube model. Trajectories that represent learning histories are time-interpolated cognitive position vectors inside that advanced hypercube. As all dimensions of this hypercube are to be restricted to $x_i \in [0,1]$, the indicators forming the K additional dimensions will have to be normalized to this interval.

Using such a spatio-temporal database approach, data is brought to a highly abstract level where all the information about a learner's progress and corresponding influencing factors are inherent in purely geometric data which is interpolated over the time dimension. Nevertheless, the original vector data will still be available and can be selected by common database operations. For example, regarding projection, one can decide to use only a subset of the K indicators. Moreovover, one can treat the learning pathways seperately from the indicators which becomes important if correlations between indicators and learning pathways are subjected.

The spatio-temporal database is yet to be implemented and will be based on data structures and algorithms for spatio-temporal indexing and querying as they are used in already existing spatio-temporal databases. By example this includes the use of R-trees [25] or X-trees [26] for indexing. Existing systems mainly focus on Geographical Information Systems (GIS), Network and Facility Management, Land Information Systems (LIS) and Image Processing [27]. They mostly provide support for only two or three spatial dimensions [28]–[32]. A particular approach for dealing with higher dimensions was introduced with the DEDALE system using a constraint database technique [33]–[38]. Moreover, the database will provide algorithms for the clustering of the $(N+K)$-dimensional trajectories. Based only on the geometric relations between different trajectories of different learners such trajectory clusters can be used as input for further multivariate data analysis. We sketch two examples to illustrate how such data analysis can contribute to the improvement of adaptive learning environments.

Discovery of Unknown Didactic Factors: In the context of an experimental learning situation, arbitrary indicators are measured. The resulting data is transferred into the above described system and the data is converted into persistent learning histories together with their indicators. Using factor analysis, new didactic factors can be identified together with indicators that are represented by these factors. In a second step, the original set of indicators can be reduced to a smaller one, restricted to indicators that are easy to measure in a non-experimental learning environment.

Real-Time Learning Pathway Prediction: Like in the previous example, learning histories as well as influencing indicators are stored with the advanced hypercube model. In a first stage, the learning histories are subjected to a cluster analysis in order to identify common classes of learning histories. In the second stage, taking these clusters on the one side and the measured indicators on the other side, one can perform for example either a discriminant analysis or a logistical regression. As a result, we can determine, which variation of indicators of a specific learner will probably lead to a specific learning history. Built on this knowledge and measuring these indicators in the learning environment, i.e. in an LMS, we can predict the learner's future learning history and recommend according learning pathways and KOs.

5. CONCLUSION

INTUITEL provides a novel and innovative way of transforming human-based didactic and pedagogical knowledge into machine-processable information. It furthermore introduces a new way to develop adaptive learning environments that can guide their learners similar to a virtual tutor. The didactic and pedagogical expertise is integrated into the system in a plug-and-play style using ontologies. The ontology-based approach ensures that the knowledge, that is needed for the annotation of learning material as well as for the creation of recommended learning pathways, can be formulated in a way that reflects human language and thinking. This way, no specific technical expertise about the system is needed by the didactic engineer. With INTUITEL, we have separated the underlying technology from the didactic and pedagogical domain. The creation of annotated material is therefore intuitive and completely unrestricted with respect to the according domains of knowledge. Also, the technical implementation for already existing learning environments is inexpensive. The use of a REST-based protocol for the communication between the learning environment and the INTUITEL system makes it possible to implement plugins for a wide range of learning environments.

In particular, INTUITEL does not force the learner to follow a certain learning pathway or to take some specific knowledge objects. Instead INTUITEL is capable of adapting itself to the decisions of the learner. The hypercube model and the calculation of cognitive distances between the learner's state and recommended learning pathways allows INTUITEL to constantly generate new recommendations regarding the learner's movement through the cognitive space.

INTUITEL was tested with real learners with respect to learning success, usability and the learners' general satisfaction. The tests were taken at the University of Vienna, University of Reading and the University of Valladolid. The summary and conclusions of these tests can be retrieved from [24]. More detailed information about the overall pedagogical testing plan can be found under [23]. In addition, the Karlsruhe University of Applied Sciences continues to design more INTUITEL courses for real learners in order to gain more experience for the enhancement and improvement of the INTUITEL system.

The utilization of the advanced hypercube model for a spatio-temporal database approach in order to model learning histories as spatio-temporal trajectories will offer a new way to perform learning analytics. Integrating indicators related to learning behaviour into the hypercube model will contribute to finding new didactic factors. It will help to understand how these factors influence learning behaviour. And finally, we can find out how to measure and implement them in a learning environment.

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