Different Methodologies in Treating Uncertainty

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ABSTRACT

Uncertainty is unavoidable when dealing with data. The errors in measurements, limitations of measuring tools, or imprecise definition of linguistic variables may result in different types of uncertainty. These ambiguities may be due to vagueness in data which results from the imprecise boundaries of data sets; inconsistency that reflects conflict and contradiction between sets; qualitative description of data which sometimes taken by expertise; or some other type. Ignoring dealing with these types of uncertainty affects the reliability of research and the validity of the results.

This article presents three approaches to treat uncertainty using fuzzy logic, intuitionistic logic, and neutrosophic logic and their methodologies in treating these kinds of ambiguity. Fuzzy logic and neutrosophic logic are used in building Rule-based Classification Systems. Different comparisons are presented to illustrate the importance of choosing the suitable logic to tackle the uncertainty in different data sets. These approaches are applied on six real world data sets; Iris, Wine, Wisconsin Diagnostic Breast Cancer, Seeds, Pima, and Statlog (Heart); which are available on UCI Machine Learning Repository web site. The results show that the type of uncertainty in the data set plays a great role in choosing the appropriate logic.

Keywords: Uncertainty, Intuitionistic Fuzzy Logic, Neutrosophic Logic, Classification.

1. INTRODUCTION

In treating data, in any form, one continually comes into possession of information that he/she sums up in general propositions.And it would be a hudge mistake to take these general propositions, as a guarantee, without any dispute.Whether this data is recorded by instruments, or collected by humans, the information obtained has some sort of deficiency. The information might be vague, incomplete, imprecise or contradictory. Which in turn results in different types of uncertainity [9]. In general, these various information deficiencies determine the type of the associated uncertainty. But, In decision-making projects, the problems of uncertainty and hesitancy usually turn out to be unavoidable [11]. Inclustering [8], it is usually the case that some data points are hard to identify their beloningness to exactely one cluster.

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Therefore, considering degrees of belongingness to a data point to more than one cluster would solve the problem. The need for considering uncertainity in application encouraged mathematicians to put theoritical foundation for different theories to treat uncertainity.

The probability theory was the first theory to conceive any uncertainty-based information. And for a long period of time, the probability was the only method to treat uncertainty. However the probability concerns with only the chances of existence of an element to a certain set [6], which is not sufficient in treating several uncertainty types. In addition to the classical probability theory, uncertainty-based information is now very well understood in fuzzy set theory, possibility theory and others [9].

Soft computing methodologies are tolerant for imprecision, partial truth, inconsistency and other uncertainty types [7]. While hard (crisp) computing is useful in applications in which the data is accurately described by mathematical models. Soft computing techniques mimic human mind in forming conclusions from inaccurate or approximated data and in forming propositions using approximate reasoning. These techniques include –not limited to- fuzzy logic, artificial neural networks, genetic algorithms and machine learning. Soft computing technologies have been applied successfully on data with several types of uncertainty. Hybridization between soft computing techniques and these logics have shown great success in ambiguous data.

Each of these techniques may tackle only one certain type of uncertainty. Therefore, hybrid systems of them are used successfully in applications. The advent of very high performance processors makes it possible to applications of soft computing to expand fast.

One of the main advantages of soft computing is that it opened the door for applications utilizing non-bivalent logic. Allowing the truth values to take more than the Boolean two values is a very old idea. For example, the Kleene's three-valued logic uses True, False and Undecided [1]. It allows the fact to be undecided rather than to be just true or false. There have been other versions of the three-valued logic which postulate that some facts may be intermediate between true and false. But the three-valued logic did not seem to have many applications like other logics. Once we have got familiar with the idea of a three-valued logic it seems a natural generalization to have many-valued logic. That allows the facts to have different degrees of truth. The only problem with the many-valued logics is that there are many of them. Each one is designed to overcome a particular problem. We don't have a unified definition for the logic operations. In literature, there has been some other generalizations of fuzzy logic, like interval valued fuzzy, intuitionistic interval valued fuzzy, L-fuzzy and intuitionistic L-fuzzy logics. Each is supported by its logical operations. And each has found its way to applications [1].

The next threesectionsintroduce brief descriptions of the Fuzzy Logic, Intuitionistic Fuzzy Logic and the Neutrosophic Logic. A case study on Rule Based Classification System is discussed in section 4. Results of appling this system using Fuzzy Logic and Neutrosophic Logic are presented in section 5. At the end, section 6 concludes the results obtained and discusses some ideas for future work.

2. FUZZY LOGIC

Fuzzy logic (FL) deals with imprecise or vague. It facilitates the human common sense reasoning. The propositions in FL can serve as a basis for decision support. It allowsthe truth values to be any number inthe real interval [0, 1] instead of taking just two values 0 and 1 in the Boolean logic. This idea of generalization was not new when introduced by Zadeh. There already has been the many-valued logic, which allows the truth values to be any countable number in [0,1]. However, it was not untill mid sixties that the truth values take any uncountable real value in [0,1] [9].

The truth value of an atomic proposition p is $tv(p) \in [0, 1]$ for any proposition in fuzzy logic.$tv(p) = 1$or $tv(p) = 0$ mean that p is absolutely trueor false, respectively, preserving the Boolean truth meanings while tv(p) = 0.65 just means that the truth of p is 0.65.Since the real world propositions are often only partly true, FL is very representative.

Opertaions on FL

To form sentences in FL, more proposition may be built from atomoic proposition using the negation,defined as,tv(~p) = 1- tv(p) and t-norm and t-conorm functions for the "and" and "or" operations, respectively;where t-norm, t-conorm: $[0,1] \times [0,1] \rightarrow [0,1]$. The definitions of the t-norm and t-conorm functionsare application and person dependent. Researchers define them in many ways, however they have to meet certain conditions. The t-norm has to satisty the following boundary, commutativity, monotonicity, and associativity axioms [9]:

1. *t-norm (a, 1) = a;*

2. *t-norm (a,b) = t-norm (b,a);*

3. *if b ≤c , then t-norm (a, b) ≤t-norm (a, c);*

4. *t-norm(a,t-norm(b, c))=t-norm (t-norm(a, b), c).*

Simillarily any t-conorm has to satisty the following four axioms:

1. *t-conorm (a, 0) = a;*
2. *t-conorm(a, b) = t-conorm(b, a);*
3. *if b ≤ c ,then t-conorm (a,b)≤ t-conorm (a, c);*
4. *t-conorm(a,t-conorm(b,c))=t-conorm(t-conorm(a,b),c).*

t-norm(a,b) = min(a,b) and *t-conorm(a,b) = max(a,b)*are the most used functions for t-norm and t-conorm, respectively.
Systems that are build using FL start with a fuzzification stage to transform the crisp input value into a fuzzy linguistic value. A step that is mandatory done since all existing measurments are in crisp numerical values. Then the inference engine takes these fuzzy inputs and calls the fuzzy rules from the knowledge base to generate fuzzy outputs. The fuzzy rule base systems are in the form of "IF-THEN" rules written using linguisitic values. The last stage is the defuzzification of the fuzzy outputs to crisp output.

3. INTUITIONISTIC LOGIC

The intuitionistic fuzzy Logic was introduced by K. Atanassov in 1986, as one sort of generalization of FL. The fuzzy logic was very successful in handling uncertainties arising from vagueness of a fact. Yet, it cannot model all sorts of uncertainties happening in different real observations specially problems involving imprecise information. In defining *intuitionistic fuzzy set (IFS),* besides the degree of membership $\mu_A(x) \in [0,1]$ of each element $x \in X$ to a set A, Atanassov considered a degree of non-membership $\nu_A(x) \in [0,1]$, such that $\forall\ x \in X\ 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. If $(x) =$ 1- $\mu_A(x)$ the IFS is reduced to a fuzzy set. The Intuitionistic fuzzy sets have the ability to handle imprecise information resulted from incomplete or inconsistent data [2]. Later Atanassov in 1989, introduced the interval-valued intuitionistic fuzzy logic which received little attention from the practical point of view.

4. NEUTROSOPHIC LOGIC

Neutrosophy is one of the new theoriesthat deals with uncertainity. It was Introduced by Smarandache in 1995. The Neutrosophy theory treats uncertainty results from vague, imprecise, incomplete and inconsistent data at the same time [13]. Therefore the Neutrosophic Logic is a very reach logic that generalizes the concept of the classic Boolean Logic, fuzzy logic, intuitionistic fuzzy logic. Table 1 shows this generalization and gives the different types of uncertainity in which each logic is used.

In neutrosophic logic (NL), a propositin has a degree of truth (T), a degree of falsity (F) in addition to a degree of indeterminacy (I). That is any proposition <A>, is to be considered with the negation of the proposition<Anti A> as well as a spectrum of neutralities <Neut A>. The later two forms the term <Non A> which keeps the believe of the proposition balanced and neutralized [3][4][8][13]. NL is very close to human thinking and it hasbeen developed to represent mathematical models which can deal with uncertainty, vagueness, ambiguity, imprecision. That is the knowledge which comes from observations is mostly characterized by imprecise data, as a result of the imprecision of humans or inaccurate measurments. Therefore, Neutrosophic Logic is perfect in treating problems that involve imprecision, partial truth in data. In addition to that, it can treat incompleteness, inconsistency, redundancy, and contradictions in data.

The neutrosophic values T, I, and F, are real subsets of the non-standard unit interval $]^-0,1^+[$. However, for pratical reasons the non-standard unit interval is replaced by the unit interval [3].

In intuitionistic fuzzy sets, the incorporated uncertainty - represented by the falsty degree- is dependent on the degree of belongingness. But, here, the uncertainty in neurosophic fuzzy sets is presented independentely. A neutrosophic set A in X is defined by T_A, I_A, F_A, the truth, the indeterminacy and falsity membership functions, respectiley. These T_A, I_A and F_A are real standard or non-standard subsets of $]^-0,1^+[$ with no restriction on their sum, i.e.

$$^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+.$$

Table 1: A comparison between the above logic and the different types of uncertainity they measure

	Fuzzy Logic	Intuitionistic Fuzzy Logic	Neutrosophic Logic
Logic founder	Zadeh, 1965	Atanssov, 1983	Smarandach, 1999
Membership	Truth Degree	Truth Degree Falsity Degree	Truth Degree Indeterminacy Degree Falsity Degree
Uncertainity Type it treats	vagueness	vagueness, Imprecision	vagueness, Imprecision, Inconsistency, Incompleteness

Opertaions on NL

Like other non-bivalent logics, the connectives are defined in many ways [13]. Two functions N-norm and N-conorm are used; where *N-norm* and *N-conorm* are from $(\ ]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[\ )^2$ to $]^-0,1^+[$.

Any *N-norm* or *N-conorm*has to satisfy the four axioms for the boundary, commutativity, monotonicity, and associativity. An example of the logical connectives is to define them as [1]:

$$\sim(t_1, i_1, f_1) = (f_1, i_1, t_1)$$

$$N\text{-}norm\big((t_1, i_1, f_1), (t_2, i_2, f_2)\big) = (\min\{t_1, t_2\}, 1 - (\min\{t_1, t_2\} + \max\{f_1, f_2\}), \max\{f_1, f_2\})$$

$$N\text{-}conorm\big((t_1, i_1, f_1), (t_2, i_2, f_2)\big) = (\max\{t_1, t_2\}, 1 - (\max\{t_1, t_2\} + \min\{f_1, f_2\}), \min\{f_1, f_2\})$$

Similar to the FL systems, any NL system strarts with a neutrosofication stage to transform the crisp input value into a neutrosophic value. Then the inference engine runs on neutrosophic based "IF-THEN" rules. And the result has to pass through de-neutrosophication step to transfer the output to crisp output.

5. CASE STUDY

RULE BASED CLASSIFICATION SYSTEM

One of the earliest applications of fuzzy logicis Rule Based Systems. In such systems, the core consists of fuzzy "IF-THEN" rules. Fuzzy sets are used to form the antecedent and the consequent parts of the "IF-THEN" rules and a logical fuzzy implication is used, as well [11]. Reasoning based on fuzzy propositions is referred to as approximate reasoning. The fundamental components of approximate reasoning are these "IF-THEN" fuzzy propositions [9]. And like the classical logic, the most common inference rule is the generalized modus ponens.

These systems drive their rules directly from numerical data using soft computing techniques. One of the earilest systems was by Kosko [10].Then many applications have been introduced for Fuzzy Rule Based Classification Systems [11][5][12].These fuzzy systems have been generalized using different logics. Yet, the most recent ones use Neutrosophic Logic. In [3], generalization of a fuzzy rule based classification system to the corresponding neutrosophic system, is done using the *truth, indeterminacy,*and *falsity* membership funtions. It turns out that the overall classification accuracy has been improved using the neutrosophic logic, especially, in data sets with intereleaved and overlapped classes. An improvement of this system in [4], in which genetic algorithm is used in designing and optimizing the knowledge base of the system.

6. RESULTS AND DISCUSSIONS

Fuzzy Logic and Neutrosophic Logic have been used in rule based classification of six different world wide datasets; Iris, Wine, Wdbc, Seeds, Pima, and Statlog(Heart). They all have vague boundries between their classes. Moreover, some of them have intersected areas with misplaced objects.

As in any real world problem, the data here contains imbalanced data sets, where one class (or more) represents large number of the examples (majority class) while the other classes contain just few examples (minority classes) [12].This drives the classifier to be skewed towards the majority class. Therefore, it is important -when dealing with real world data sets- to select an appropriate measure of performance. The most common method is analysis based on the confusion matrix [12][8]. Table 2 shows a confusion matrix for classification of two classes *A, B*, where:

True Positive(*TP)* is the percentage of correctly classified examples in class *A*,

False Negative(*FN)* is the percentage of examples classified in *A* while it should be in *B*, False Positive(*FP)* is the percentage of examples classified in *B* while it should be in *A*, and True Negative(*FN)* is the percentage of correctly classified examples in class *B*.

Table 2: Confusion matrix for Classes *A,B*

	Actual Class A	Actual Class B
Class A Prediction	True Positive(TP)	False Negative(FN)
Class B Prediction	False Positive(FP)	True Negative(TN)

Accuracy = $\frac{TP+TN}{TP+TN+FP+FN}$ is the most commonly used metric for empirical evaluations but if one of these classes were a majority class, the minority class would have a little impact [12]. Therefore, three other measures have been used [8]:

The Precision = $\frac{TP}{TP+FP}$, the Sensitivity =$\frac{TP}{TP+FN}$, and the Specifcity =$\frac{TN}{TN+FP}$

Fig. 1 shows the total accuracy of the two rule based classification systems one is using fuzzy logic and the other is using neutrosophic logic. It is clear that NL gives a more accurate classification than FL for the six data sets. Which explains the importance of using the indetermincy term for these data sets. The other measures for each data set are explained later.

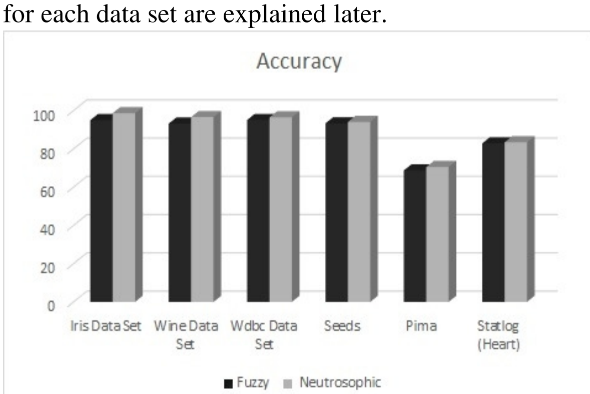

Figure 1: Accuracy of Classifing the data sets in Fuzzy and Neutrosophic Logics

The Iris dataset contains three classes Iris-Setosa, Iris-Versicolour, and Iris-Virginica. The Iris-Setosa is completely separated from the other two classes. And therefore, the classification using FL has the same result as the classification using NL for the three measures; Precision, Sensitivity, and Specificity. However, The second and third classes do have vague boundries and intersected areas. As a result, the Classification using NL gives better results than the classification using FL.

The Wine dataset contains three classes. Most objects of the first are separated from the other two classes. But some objects in second and the third class are misplaced near the center of the wrong class. Here the indeterminacy term, in NL, plays a good role. The difference between the two classification systems is shown in Fig. 3.

The Wdbc dataset has two classes class M, and class B. There were objects belong to both classes and also nearby the center of each class, which reflects inconsistency of the data. Fig. 4 shows that the NL system has reached better results than the FL one.

The Seeds dataset has three classes with very vague boundries. i.e. the misplased objects lie only on the edges. As a result, we can see the precision and specifity of the first class in the FL is better than the ones using NL, Fig 5. However, the overall measures were better using NL.

Pima dataset has two classes, where the two classes are interleaved and may have overlapped centers. It is difficult to identify them separately. Similar to the Wdbs, the indeterminacy term, here, gives better results. The results of the classification using NL, with respect to the three measures, is always better than the classification using FL, Fig. 6.

Statlog(Heart) dataset contains two almost overlapped classes. It is difficult to identify one from the other. It is an example of the imprecision and inconsistency that arises in data. Therefore, there is a big difference between the results of the NL and the FL, Fig. 7.

7. CONCLUSION AND FUTURE WORK

Studying well the data set of any project is important. And treating the uncertainty in the data is essential and affects the reliability of the results. This article compares between fuzzy, intuitionistic, and neutrosophic logics and introduces the uncertainty types each logic can handle. A case study of Rule Based Classification System -applied to six world wide data sets-, is presented to show the importance of choosing the appropriate logic according to the data set. The results showed that using neutrosophic logic is in general better. Fuzzy logic suits systems that has only vague data, i.e. the boundaries between the classes are unclear. However, the neutrosophic logic -in most measures- gives better results for intersected data sets.

In the future work, building hybrid systems with other soft computing techniques seems promissing in extracting rules for the rule based classification systems.

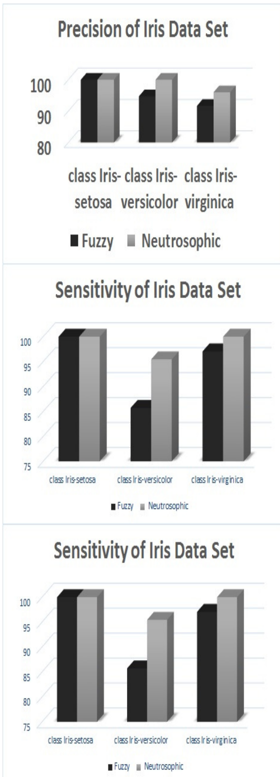

Figure 2: The measures for the Iris dataset

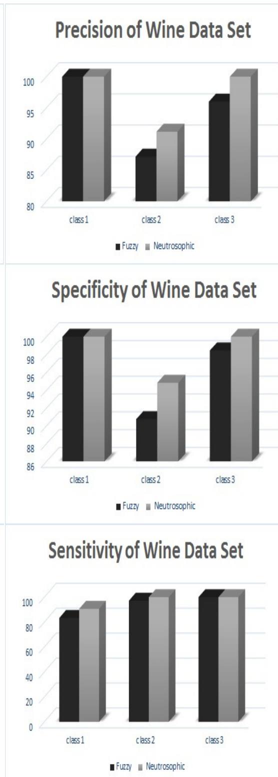

Figure 3: The measures for the Wine dataset

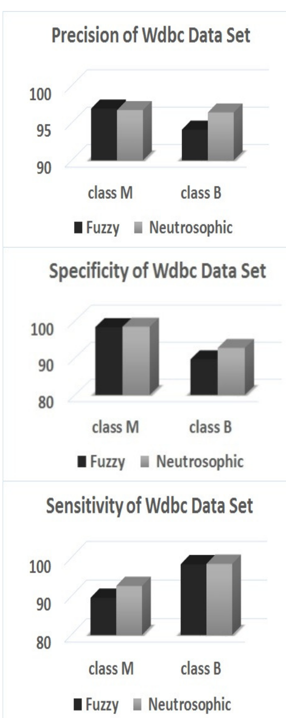

Figure 4: The measures for the Wdbc dataset

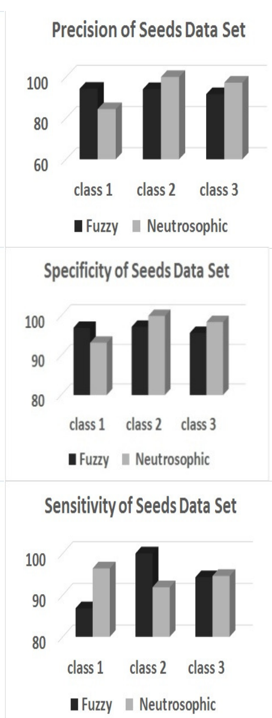

Figure 5: The measures for the Seeds dataset

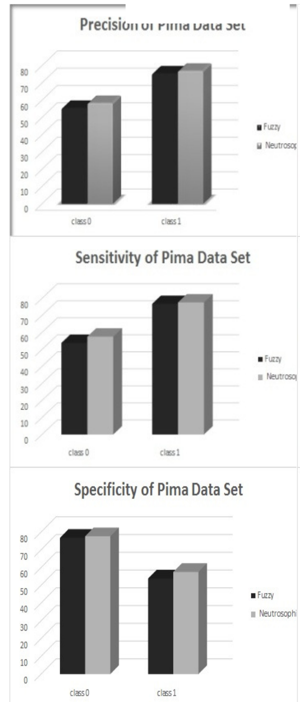

Figure 6: The measures for the Pima dataset

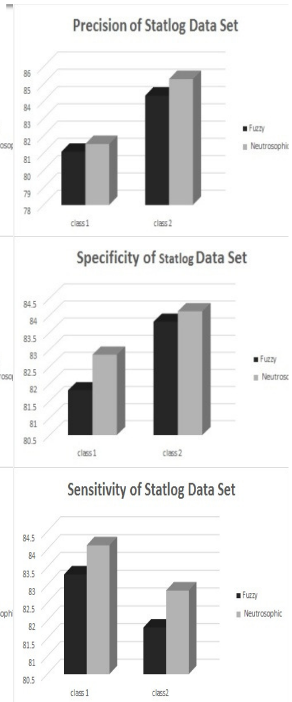

Figure 7: The measures for the Statlog dataset

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