Mathematical Competitions for University Students

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Introduction

Mathematical competitions among students have two main goals. The first goal is just like any other competition - to discover the strongest competitors. The second is to enhance interest in mathematics. The first aim is quite achievable, however the other goal, is far less attainable. Mathematical competitions in the classical form of the exam is not, in our opinion, the best way to incite the students' interest. Although we invest a lot of time and efforts in choosing suitable competitors, we find that students, who lose in the competition, lose their confidence. As a result, these students are reluctant to participate in future competitions and are left out of our organizing efforts instead of getting additional motivation in studying mathematics, which leads to serious psychological problems. If we want a mass of students to participate in mathematical competitions, we have to choose their forms in a way that the psychological problems of losers will lessen and students will mainly enjoy this competitions. One of possible ways is to make the mathematical competitions more attractive for a mass of students, we have to strengthen their game component.

Games play an important role in child's development: through games, children obtain information on the world at large. In kindergarten and, to some extent, in elementary school, games are used to teach languages and science. However, teaching methods that use games have disappeared from use in high schools and in institutions of higher education, despite the fact that even at these ages, games can help learners learn rapidly and with ease. Our goal is to construct models, which could be used to involve school children and students in educational games. In all games, learners solve mathematical problems in their free time. This is clear to the teacher, but the activities are presented to students as a game.

The Blitz Mathematical Olympiad

One of the advantages of team competition is that nobody takes a full responsibility for the team's loss. This essentially solves negative psychological problems, which appeared in the classical individual competition. The scenario of Blitz Mathematical Olympiad can be described as follows. All teams receive the same problem and have to come up with a solution. In order to score
points, teams must submit the solution to the problem within the allotted time (10-15 minutes). The solutions are immediately checked by the panel of judges, who immediately announce the results to the participants and to the audience. All of this is repeated with the next problems. Dynamics of Blitz Mathematical Olympiad is one of the main principles. We inform participants about their results after solving each problem, rather than after solving all problems, which allows everyone to track the teams' score in real time. The principle of scoring can be, for example, the following. The number of points for solved problem is inversely proportional to the number of teams who solved this problem correct. Consider, for example, the case of 4 teams. Suppose every problem is worth 12 points. In the situation that all of the teams solved correct, each team gets 3 points. If three teams solved correct, then each of them gets 4 points and the fourth team gets 0 points. If two teams solved correct, each of them gets 6 points, and the other two teams - 0 points. In the case of only one right solution, this team gets 12 points and the other three - 0 points. It is important that the order of problems is from simple to more complicated problems. This usually allows us to hold non evidence of the final result till the last problems.

Another option is to propose an alternative scoring method. Every team starts with the same number of participants, for example, five. If a team solved a problem correct, one of its members can go out from the game. If after solving several problems correctly, a team has "disappeared" (all members went out of game), it means that this team won Blitz Mathematical Olympiad.

Modern Internet technologies allow us to organize Blitz Mathematical Olympiads when the teams stay at their Universities in different cities and even different countries.

Our Experience

Blitz Mathematical Olympiad was first organized in Perm Polytechnic Institute, Perm, Soviet Union in the 1980s. Blitz Mathematical Olympiads were regularly organized for high school students in the Department of Youth Activities in the Technion, Haifa, Israel in 1993-1999. In November 2008 the First International Blitz Mathematical Olympiads for University students from Russia, Romania and Israel was successfully organized in Ariel University Center, Israel. In 2011 Super Final of Israeli-Russian Internet Olympiad was organized in this form in Ariel University Center.

The problems from Super Final of September 20, 2011:

Problem 1.

Calculate:

\[ \int_{-1}^{1} \sqrt{4 - x^2} \, dx \]

Problem 2.

An ant, located in a corner of a room of the size 2x 3x 4, wants to get to the opposite corner. What is the shortest distance the ant must travel? (The ant cannot fly.)

Problem 3.
Find the sum of all four-digit numbers whose decimal notation only contains the digits 1, 2 or 3.

**Problem 4.**  
It is given that:  
\[ a + b + c + d + e + f + g + h = 1; \]  
and all summands on the left hand side are non-negative. Find the maximal value of the expression:  
\[ ab + bc + cd + de + ef + fg + gh \]

**Problem 5.**
A toy factory produces carcasses of regular tetrahedrons (triangular pyramids with six edges of equal length), using an unlimited supply of sticks of the same length in six colors. The six edges of each toy are six sticks, all of different colors, glued together at their ends. How many different toys can the factory make?

**Problem 6.**
A unit cube was rotated by 180° around its diagonal. Find the volume of the intersection of the initial cube with the rotated one.

**Problem 7.**
Four sequences of positive integers \(a_n, b_n, c_n\) and \(d_n\) are defined by the relation:  
\[(1 + \sqrt{2} + \sqrt{3})^n = a_n + b_n \sqrt{2} + c_n \sqrt{3} + d_n \sqrt{6}\]

Find the limits of the sequences \( \frac{b_n}{a_n}, \frac{c_n}{a_n} \) and \( \frac{d_n}{a_n} \).

**Problem 8.**
Factor the polynomial \(x^7 + x^5 + 1\) into a product of two polynomials of positive degree with integer coefficients.

We present the problems from Super Final of September 20, 2011 and their solutions in our site [www.i-olymp.net](http://www.i-olymp.net).

Note that the majority of these problems are based on the problems written by Prof. Alexei Kannel-Belov, the coach of the Israeli student team on mathematics, and Dr. Vadim Bugaenko. Both of them are authors of books on nonstandard mathematical problems.

**REFERENCES:**


