Autodriver algorithm

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ABSTRACT

The autodriver algorithm is an intelligent method to eliminate the need of steering by a driver on a well-defined road. The proposed method performs best on a four-wheel steering (4WS) vehicle, though it is also applicable to two-wheel-steering (TWS) vehicles. The algorithm is based on coinciding the actual vehicle center of rotation and road center of curvature, by adjusting the kinematic center of rotation. The road center of curvature is assumed prior information for a given road, while the dynamic center of rotation is the output of dynamic equations of motion of the vehicle using steering angle and velocity measurements as inputs.

We use kinematic condition of steering to set the steering angles in such a way that the kinematic center of rotation of the vehicle sits at a desired point. At low speeds the ideal and actual paths of the vehicle are very close. With increase of forward speed the road and tire characteristics, along with the motion dynamics of the vehicle cause the vehicle to turn about time-varying points. By adjusting the steering angles, our algorithm controls the dynamic turning center of the vehicle so that it coincides with the road curvature center, hence keeping the vehicle on a given road autonomously. The position and orientation errors are used as feedback signals in a closed loop control to adjust the steering angles. The application of the presented autodriver algorithm demonstrates reliable performance under different driving conditions.

Keywords: Autonomous vehicle, Autodriver, Four-wheel-steering, Vehicle Dynamics.

1. INTRODUCTION

During recent decades, research and development of autonomous vehicles has been focused by many research teams worldwide. Various methodologies have been proposed for implementation of vehicles that are capable of collision-less navigation and intelligent path planning. Most of the works reported in the literature focus on performing a task autonomously. Examples of such tasks include integrated path planning and tracking, localization and parallel parking.

This work presents an introduction to, and extension of a mathematical theory of autodriver which was initially introduced by Jazar [1]. The proposed method is best suited to applications where a driver-less vehicle travels between cities. In such cases, we can define the road by mathematical equations. Such an application can reduce the driving stresses by simplifying the main function of a driver which is following the road. The algorithm will potentially increase the safety and productivity of ground transportation by reducing driving stresses as well as suggesting/controlling the proper speed at any point of the road. The algorithm can be easily applied to agricultural vehicles, to increase their productivity by eliminating human operators and related wasting times. Autodriver algorithm can be used in an extensive range of military, agricultural, and commercial applications.

The outline of the paper is as follows. In section 2, a four wheel steering vehicle is introduced, followed by its kinematic equations presented in section 3. For the paper to be self-contained, the mathematical relationships between global, local and wheel base coordinate systems and their corresponding variables of the vehicle motion are reviewed in section 4, based on which the principal idea behind the autodriver algorithm is described in section 5. Section 6 presents discussions on vehicle dynamics and stability followed by simulation results. Section 7 concludes this paper.

2. FOUR WEEL STEERING VEHICLE

Consider a vehicle and a road between points A and B. We assume that the path of motion is known mathematically with given center and radius of curvature. Let us call the center of curvature as *road center*. Figure 1(a) illustrates an example of a 2D road with equation $y = x^2$ and its road center, and Figure 1(b) depicts a 3D road with the following parametric equation.

$$\begin{aligned} x &= (a + b\sin\theta)\cos\theta \\ y &= (a + b\sin\theta)\sin\theta \\ z &= b + b\cos\theta \end{aligned}$$

$$a = 200\,\mathrm{m} \qquad b = 150\,\mathrm{m}$$

A vehicle is always in a turn on an instantaneous circle of curvature about an instantaneous curvature center. When the vehicle is moving straight, it is assumed to be turning about a point at infinity. A vehicle can follow a given path only if it turns about the road center at the correct distance of radius of curvature.

The center of rotation of a vehicle is kinematically at the intersection of perpendicular lines to the wheels. It is always on the rear axle of a vehicle if it is a front-wheel-steering (FWS) or on the front axle of a vehicle if it is a rear-wheel-steering (RWS) as shown in Figure 2. However, the kinematic center of rotation can be at any point on (x,y)-plane if the vehicle is equipped with four independent steering wheels (4WS). Figure 3 depicts a positive four-wheel steering vehicle.

4WS or all-wheel-steering (AWS), as is shown in Figures 3, is introduced to improve steering response, increase the stability at high speeds maneuvering, or decrease turning radius at low speeds. In a positive 4WS design the front and rear wheels steer in the same direction and in a negative 4WS situation the front and rear wheels steer opposite to each other. A negative 4WS has shorter turning radius than a FWS vehicle. For a FWS vehicle, the perpendiculars to the front wheels meet at a point on the extension of the rear axle. However, for a 4WS vehicle, the intersection point can be at any point in the horizontal plane. The point is the turning center of the car and its position depends on the steer angles of the wheels [2].

Most of the current roads can be easily defined mathematically. Using GPS, we are able to determine the position and orientation of a vehicle as exact as we need. The autodriver algorithm is based on pre-defined path of motion, and position/orientation of the vehicle which can be determined by GPS as the best route through an unknown area [3]. We define roads in a global coordinate frame attached to the ground.

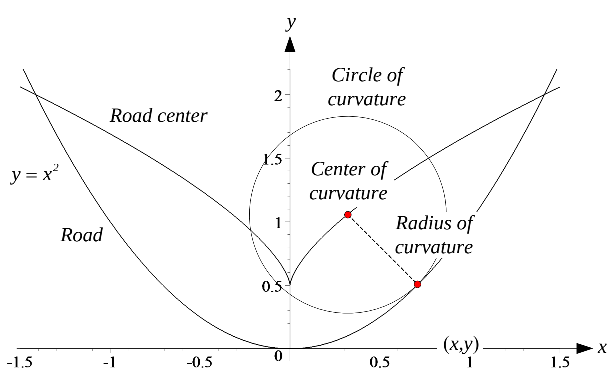


Figure 1(a). A 2D road and variations of its center.

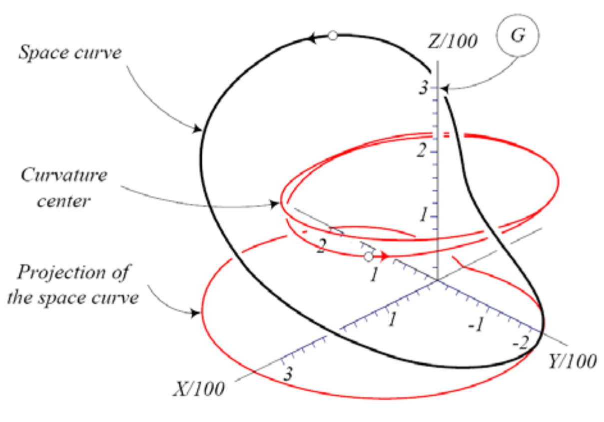


Figure 1(b). A 3D road and variations of its center.

3. KINEMATIC STEERING CONDITION

Consider a FWS vehicle that is moving at a very low speed. There is a kinematic condition between the inner and outer wheels of the vehicle that allows them to turn slip-free,

$$\cot\delta_o - \cot\delta_i = \frac{w}{l} \tag{1}$$

where, δ_i and δ_o are the steer angle of the inner and outer wheels respectively as is shown in Figure 2. The inner and outer wheels are defined based on the turning center O. Track w and wheelbase l are considered the kinematic width and length of the vehicle [2].

The kinematic radius of rotation of the vehicle, R, is measured as the distance between O and the car's mass center C

$$R = \sqrt{a_2^2 + l^2 \cot^2\delta} \tag{2}$$

where a_2 is the longitudinal distance between the rear axle and C, and δ is the cot-average of the inner and outer steer angles.

$$\cot\delta = \frac{\cot\delta_o + \cot\delta_i}{2} \tag{3}$$

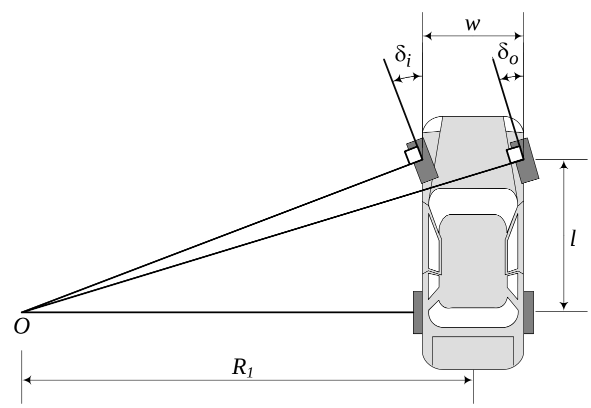


Figure 2. A front-wheel-steering (FWS) vehicle and the Ackerman condition.

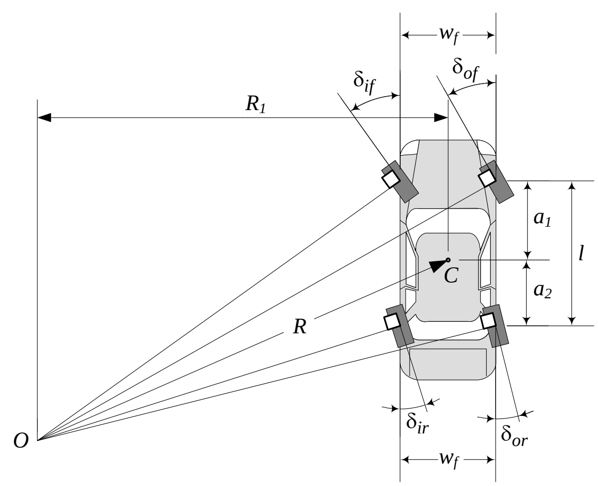
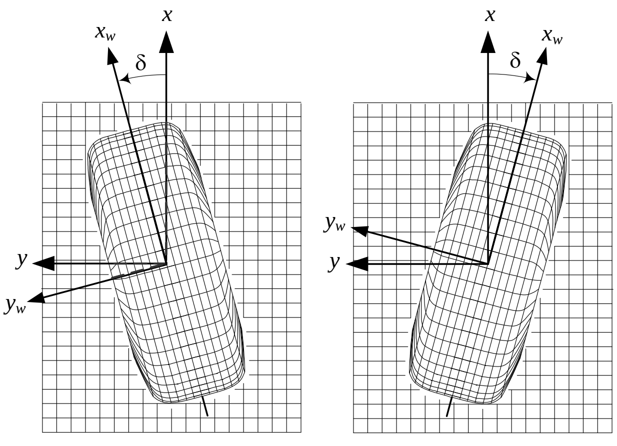


Figure 3. A positive four-wheel steering vehicle.



(*Positive steer angle*) (*Negative steer angle*)

Figure 4. Sign convention for steer angles.

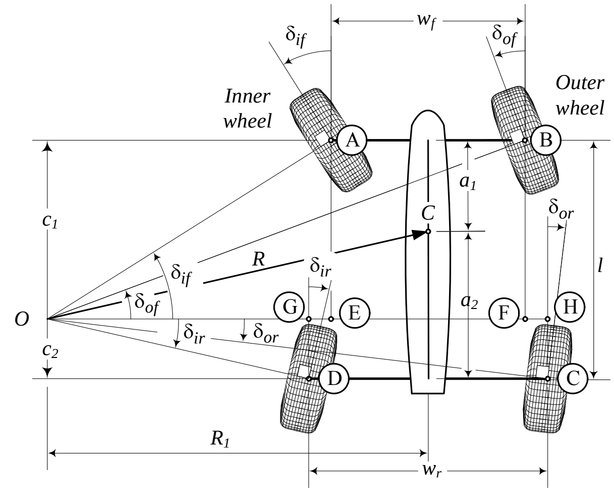


Figure 5. Illustration of a positive four-wheel steering vehicle in a left turn.

The angle δ is the equivalent steer angle of a bicycle model of the vehicle having the same wheelbase l and radius of rotation R. Substituting a vehicle with an equivalent bicycle model simplifies the mathematical equations and is a standard method in vehicle dynamics [2].

Employing the wheel coordinate frame (x_w, y_w, z_w), we define the steer angle as the angle between the vehicle's longitudinal x-axis and the wheel x_w-axis, measured about the z-axis. Using this convention, the kinematic steering condition for 4WS vehicles is

$$\cot\delta_{fr} - \cot\delta_{fl} = \frac{w_f}{l} - \frac{w_r}{l}\frac{\cot\delta_{fr} - \cot\delta_{fl}}{\cot\delta_{rr} - \cot\delta_{rl}} \tag{4}$$

where, δ_{fl} and δ_{fr} are the steer angles of the front left and front right wheels, and δ_{rl} and δ_{rr} are the steer angles of the rear left and rear right wheels. Using the wheel-steering sign convention, Equation (4) expresses the kinematic condition for both, positive and negative 4WS systems.

Figure 5 illustrates a 4WS vehicle in a left turn. The front inner and outer steer angles δ_{if} and δ_{of} may be calculated from the triangles ΔOAE and ΔOBF, while the rear inner and outer steer angles δ_{ir} and δ_{or} may be calculated from the triangles ΔODG and ΔOCH.

$$\tan\delta_{if} = \frac{c_1}{R_1 - \frac{w_f}{2}} \qquad \tan\delta_{of} = \frac{c_1}{R_1 + \frac{w_f}{2}} \tag{5}$$

$$\tan\delta_{ir} = \frac{c_2}{R_1 - \frac{w_r}{2}} \qquad \tan\delta_{or} = \frac{c_2}{R_1 + \frac{w_r}{2}} \tag{6}$$

Combining the equations provides two conditions between the front and rear, and left and right wheels. Eliminating R_1

$$R_1 = \frac{w_f}{2} + \frac{c_1}{\tan\delta_{if}} = -\frac{w_f}{2} + \frac{c_1}{\tan\delta_{of}} = \frac{w_r}{2} + \frac{c_2}{\tan\delta_{ir}} = -\frac{w_r}{2} + \frac{c_2}{\tan\delta_{or}} \tag{7}$$

between (5) and (6) provides the kinematic condition between the front steering angles δ_{if} and δ_{of} or between the rear steering angles δ_{ir} and δ_{or}.

$$\cot\delta_{of} - \cot\delta_{if} = \frac{w_f}{c_1} \qquad \cot\delta_{or} - \cot\delta_{ir} = \frac{w_r}{c_2} \tag{8}$$

The longitudinal distance between point O and the axles of the car are indicated by c_1, and c_2 measured in the body coordinate frame.

$$c_1 = \frac{w_f}{\cot\delta_{of} - \cot\delta_{if}} \qquad c_2 = \frac{w_r}{\cot\delta_{or} - \cot\delta_{ir}} \tag{9}$$

$$c_1 - c_2 = l \tag{10}$$

Based on the above analysis, it is shown that the kinematic steer angles of a 4WS vehicle can be calculated from four functions of two variables of c_1 and R_1.

$$\begin{aligned} \delta_{if} &= \delta_{if}(c_1, R_1) \qquad & \delta_{of} &= \delta_{of}(c_1, R_1) \\ \delta_{ir} &= \delta_{ir}(c_2, R_1) \qquad & \delta_{or} &= \delta_{or}(c_2, R_1) \end{aligned} \tag{11}$$

4. VEHICLE-GLOBAL COORDINATE FRAMES

To determine the position and orientation of a vehicle, we attach a global-fixed coordinate frame $G(OXYZ)$ on the flat ground and a body-fixed coordinate frame $B(Cxyz)$ at the mass center of the vehicle C as is shown in Figure 6.

The equations of motion of the vehicle are usually expressed in the set of vehicle coordinate frame $B(Cxyz)$. The x-axis is a longitudinal axis passing through C and directed forward. The y-axis goes laterally to the left from the driver's viewpoint. The z-axis makes the coordinate system a right-hand triad. When the car is parked on a flat horizontal road, the z-axis is perpendicular to the ground, opposite to the gravitational acceleration $\mathbf{g}$.

The position and orientation of the vehicle coordinate frame $B(Cxyz)$ is measured with respect to the grounded fixed coordinate frame $G(OXYZ)$. The vehicle coordinate frame is called the *body frame* or *vehicle frame*. Analysis of the vehicle motion is equivalent to expressing the position and orientation of $B(Cxyz)$ in $G(OXYZ)$.

The turning center of a vehicle should be the road center at the correct distance to follow the road ideally. If the road is known mathematically, then at any point of the road, the kinematic turning center in the vehicle body coordinate frame, is at a point with coordinates (x_c, y_c).

$$x_c = -a_2 - c_2 = -a_2 - \frac{w_r}{\cot\delta_{or} - \cot\delta_{ir}} \tag{12}$$

$$y_c = R_1 = \frac{l + \frac{1}{2}\left(w_f \tan\delta_{if} - w_r \tan\delta_{ir}\right)}{\cot\delta_{if} - \cot\delta_{ir}} \tag{13}$$

Equations (12) and (13) can be discovered by substituting c_1 and c_2, and defining y_c in terms of δ_{if} and δ_{ir}. These equations define the coordinates of the kinematic turning center for both, positive and negative 4WS systems.

We may define and employ an equivalent bicycle models as shown in Figure 7 to determine the vehicle's kinematic turning radius R.

$$R = \sqrt{\left(a_2 + c_2\right)^2 + c_1^2 \cot^2\delta_f} \tag{14}$$

Consider a road with a given equation $Y=f(X)$ in a global coordinate frame G, as is shown in Figure 8. Point c_c is the road center at the moment. Road center is supposed to be the turning center of the car at the instant of consideration. The road radius of curvature R_κ at point X is:

$$R_\kappa = \frac{\left(1 + Y'^2\right)^{3/2}}{Y''} \qquad Y' = \frac{dY}{dX} \qquad Y'' = \frac{d^2Y}{dX^2} \tag{15}$$

The z and Z axes are assumed to be parallel and the heading angle ψ between x and X axes, indicates the orientation of B in G. If (X_c, Y_c) indicate the coordinates of the road center c_c in G then, the coordinates of c_c in B would be:

$${}^B r_c = R_{z,\psi}\left({}^G r_c - {}^G\mathbf{d}\right) \tag{16}$$

$$\begin{aligned}\begin{bmatrix} x_c \\ y_c \\ 0 \end{bmatrix} &= \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} X_c \\ Y_c \\ 0 \end{bmatrix} - \begin{bmatrix} X \\ Y \\ 0 \end{bmatrix}\right) \\ &= \begin{bmatrix} \left(X_c - X\right)\cos\psi + \left(Y_c - Y\right)\sin\psi \\ \left(Y_c - Y\right)\cos\psi - \left(X_c - X\right)\sin\psi \\ 0 \end{bmatrix}\end{aligned} \tag{17}$$

Having the coordinates of the road center c_c in the vehicle coordinate frame is enough to determine the kinematic characteristics R_1, c_1, and c_2.

$$R_1 = y_c = \left(Y_c - Y\right)\cos\psi - \left(X_c - X\right)\sin\psi \tag{18}$$

$$\begin{aligned} c_1 &= c_2 + l = -\left(X_c - X\right)\cos\psi - \left(Y_c - Y\right)\sin\psi + a_1 \\ c_2 &= -a_2 - x_c = -\left(X_c - X\right)\cos\psi - \left(Y_c - Y\right)\sin\psi - a_2 \end{aligned} \tag{19}$$

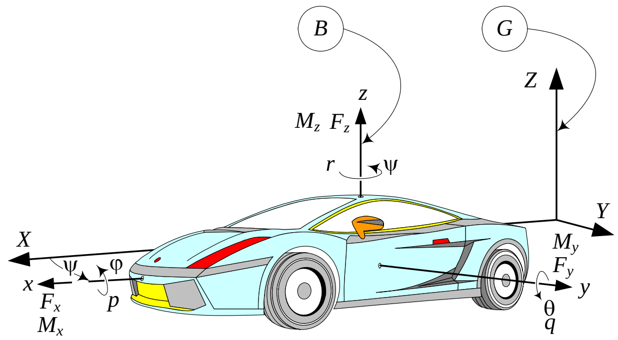


Figure 6. Illustration of a moving vehicle, indicated by its body coordinate frame B in a global coordinate frame G.

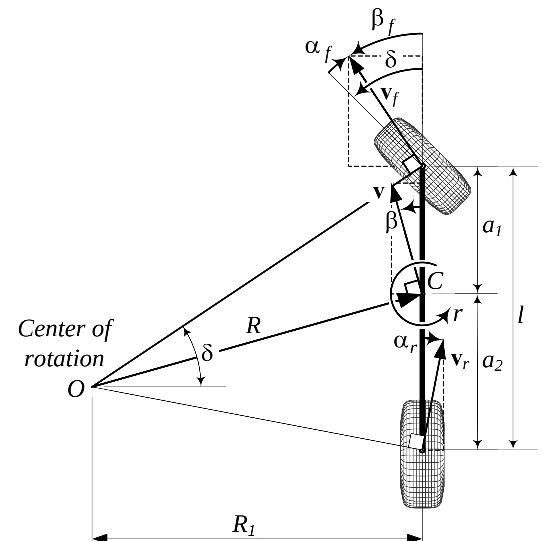


Figure 7. A two-wheel model for a vehicle.

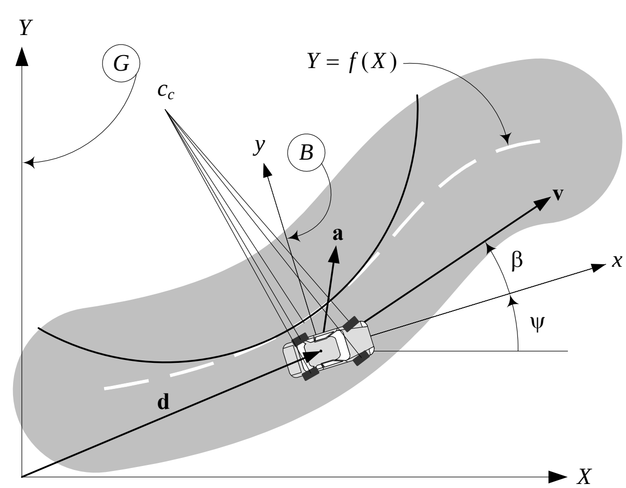


Figure 8. Illustration of a car that is moving on a road at a point that c_c is the center of curvature.

Then, we can adjust the required steer angles of the wheels to have the same kinematic radius as the road radius of curvature.

If (XO, YO) are the coordinates of O in the global coordinate frame G then, the coordinates of C in B would be

$$ {}^{B}\mathbf{r}_O = R_{z,\psi}\,{}^{G}\mathbf{r}_C + \mathbf{d} \tag{20} $$

$$ \begin{aligned} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} &= \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_C \\ Y_C \\ 0 \end{bmatrix} + \begin{bmatrix} X \\ Y \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} X\cos\psi + Y\sin\psi \\ Y\cos\psi - X\sin\psi \\ 0 \end{bmatrix} + \begin{bmatrix} X \\ Y \\ 0 \end{bmatrix}. \end{aligned} \tag{21} $$

Having coordinates of C in the vehicle coordinate frame is enough to determine R_1, c_1, and c_2.

$$ \begin{aligned} R_1 &= y_C \\ &= Y_C\cos\psi - X_C\sin\psi + Y \end{aligned} \tag{22} $$

$$ \begin{aligned} c_2 &= -a_2 - x_C + y \\ &= X\cos\psi + Y\sin\psi - a_2 + X \end{aligned} \tag{23} $$

$$ \begin{aligned} c_1 &= c_2 + l + X \\ &= X_C\cos\psi + Y_C\sin\psi + a_1 + X \end{aligned} \tag{24} $$

For example, an elliptic road with parametric equations of

$$ X = a\cos\theta \qquad Y = b\sin\theta \tag{25} $$

where $a>>l$ as the semi-major and $b>>l$ as the semi-minor axes provides the following global and body coordinates of the curvature center c_c of the road.

$$ \begin{aligned} X_c &= X - \frac{Y'\left(X'^2+Y'^2\right)}{X'Y''-Y'X''} = \frac{a^2-b^2}{a}\cos^3\theta \\ Y_c &= Y + \frac{X'\left(X'^2+Y'^2\right)}{X'Y''-Y'X''} = -\frac{a^2-b^2}{b}\sin^3\theta \end{aligned} \tag{26} $$

$$ \begin{aligned} x_c &= \left(\frac{a^2-b^2}{a}\cos^3\theta - a\cos\theta\right)\cos\psi \\ &\quad + \left(-\frac{a^2-b^2}{b}\sin^3\theta - b\sin\theta\right)\sin\psi \\ y_c &= \left(-\frac{a^2-b^2}{b}\sin^3\theta - b\sin\theta\right)\cos\psi \\ &\quad - \left(\frac{a^2-b^2}{a}\cos^3\theta - a\cos\theta\right)\sin\psi \end{aligned} \tag{27} $$

When the vehicle is at the coordinates (X, Y), the parameters c_1, c_2 and R are:

$$ \begin{aligned} c_2 = -a_2 - x_c &= -a_2 - \left(\frac{a^2-b^2}{a}\cos^3\theta - a\cos\theta\right)\cos\psi \\ &\quad + \left(-\frac{a^2-b^2}{b}\sin^3\theta - b\sin\theta\right)\sin\psi \end{aligned} \tag{28} $$

$$ \begin{aligned} c_1 = c_2 + l &= a_1 - \left(\frac{a^2-b^2}{a}\cos^3\theta - a\cos\theta\right)\cos\psi \\ &\quad + \left(-\frac{a^2-b^2}{b}\sin^3\theta - b\sin\theta\right)\sin\psi \end{aligned} \tag{29} $$

$$ \begin{aligned} R_1 = y_c &= \left(-\frac{a^2-b^2}{b}\sin^3\theta - b\sin\theta\right)\cos\psi \\ &\quad - \left(\frac{a^2-b^2}{a}\cos^3\theta - a\cos\theta\right)\sin\psi \end{aligned} \tag{30} $$

$$ \theta = \cos^{-1}\frac{X}{a} \tag{31} $$

Therefore the required steer angles of such a vehicle is:

$$ \begin{aligned} \delta_{if} &= \tan^{-1}\frac{c_1}{R_1 - \frac{w_f}{2}} \\ &= \frac{a_1 - \left(\frac{a^2-b^2}{a}\cos^3\theta - a\cos\theta\right)\cos\psi + \left(-\frac{a^2-b^2}{b}\sin^3\theta - b\sin\theta\right)\sin\psi}{-\frac{w_f}{2} - \left(-\frac{a^2-b^2}{b}\sin^3\theta - b\sin\theta\right)\cos\psi - \left(\frac{a^2-b^2}{a}\cos^3\theta - a\cos\theta\right)\sin\psi} \end{aligned} \tag{32} $$

5. AURODRIVER ALGORITHM

Having the position of the road center at any moment, we are able to calculate the required steering angles to coincide the kinematic center of a given vehicle on the road center. So, the vehicle will ideally turns about the road center. This simple strategy will work only at no speed condition because when the vehicle is moving, its dynamic will cause the vehicle to turn about an actual center of rotation which generally differs from both the kinematic and the road center. The actual rotation center is the point that the vehicle will turn about depending on the velocity, road-wheel interaction, and the steer angles. At a given speed, the only controllable variable is the steer angles which control the position of the kinematic rotation center. However, we may move the position of the actual rotation center by moving the kinematic rotation center. The autodriver algorithm is to coincide the actual rotation center on the road center by adjusting the kinematic rotation center.

Because the actual rotation center depends on dynamics of the vehicle, we need to employ the dynamic equations of motion of the vehicle to calculate the position of the actual rotation center. Using the equations of motion as the relationship between the steering angles as the input, and the coordinates of the actual rotation center as the output, we may apply the autodriver algorithm.

6. VEHICLE DYNAMICS

Figure 9 illustrates a vehicle in a planar motion. The global position vector of the mass center is denoted by ${}^{G}\mathbf{d}$. The vehicle is assumed to have a planar motion with three degrees of freedom: translation in the x and y directions, and a rotation about the z-axis. The equations of motion of the vehicle in the body coordinate frame B are [2]:

$$
\begin{aligned}
\dot{v}_x &= \frac{F_x}{m} + r v_y \\
\dot{v}_y &= \frac{1}{m v_x}\left(-a_1 C_{\alpha f} + a_2 C_{\alpha r}\right) r - \frac{1}{m v_x}\left(C_{\alpha f} + C_{\alpha r}\right) v_y \\
&\quad + \frac{1}{m} C_{\alpha f}\delta_f + \frac{1}{m} C_{\alpha r}\delta_r - r v_x \\
\dot{r} &= \frac{1}{I_z v_x}\left(-a_1^2 C_{\alpha f} - a_2^2 C_{\alpha r}\right) r - \frac{1}{I_z v_x}\left(a_1 C_{\alpha f} - a_2 C_{\alpha r}\right) v_y \\
&\quad + \frac{1}{I_z} a_1 C_{\alpha f}\delta_f - \frac{1}{I_z} a_2 C_{\alpha r}\delta_r
\end{aligned}
\tag{25}
$$

where $r = \dot{\psi} = \omega_z$ is the yaw rate of the car, and steer angle of the front and rear wheels are cot-average of the associated left and right wheels.

$$
\begin{aligned}
\cot\delta_f &= \frac{\cot\delta_{fR} + \cot\delta_{fL}}{2} \\
\cot\delta_r &= \frac{\cot\delta_{rR} + \cot\delta_{rL}}{2}
\end{aligned}
\tag{26}
$$

To analyze the autodriver algorithm, we assume a constant forward speed $v_x = const$ and simplify the second and third equations to

$$
\begin{aligned}
\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} &= \begin{bmatrix} -\dfrac{C_{\alpha f} + C_{\alpha r}}{m v_x} & \dfrac{-a_1 C_{\alpha f} + a_2 C_{\alpha r}}{m v_x} - v_x \\ -\dfrac{a_1 C_{\alpha f} - a_2 C_{\alpha r}}{I_z v_x} & \dfrac{-a_1^2 C_{\alpha f} - a_2^2 C_{\alpha r}}{I_z v_x} \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} \\
&\quad + \begin{bmatrix} \dfrac{C_{\alpha f}}{m} & \dfrac{C_{\alpha r}}{m} \\ \dfrac{a_1 C_{\alpha f}}{I_z} & -\dfrac{a_2 C_{\alpha r}}{I_z} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix}
\end{aligned}
\tag{27}
$$

Considering Equation (27) as a dynamic system

$$
\dot{\mathbf{q}} = [A]\mathbf{q} + [B]\boldsymbol{\delta} \tag{28}
$$

$$
\mathbf{q} = \begin{bmatrix} v_y \\ r \end{bmatrix} \qquad \boldsymbol{\delta} = \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix} \tag{29}
$$

we may assume that the steer angles are the input of the dynamic system and the lateral speed v_y and yaw rate r are the outputs. Having the function of the steering angles and starting from an initial condition for $v_y(0)$ and $r(0)$, we determine v_y and r in future times. By integrating v_x , v_y and r, we calculate the position and orientation of the vehicle in both, body and global coordinate frames

$$
\psi = \int_0^t r\, dt \tag{30}
$$

$$
\begin{aligned}
{}^G\mathbf{v}_c &= {}^G R_B\, {}^B\mathbf{v}_c \\
\begin{bmatrix} v_X \\ v_Y \end{bmatrix} &= \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}
\end{aligned}
\tag{31}
$$

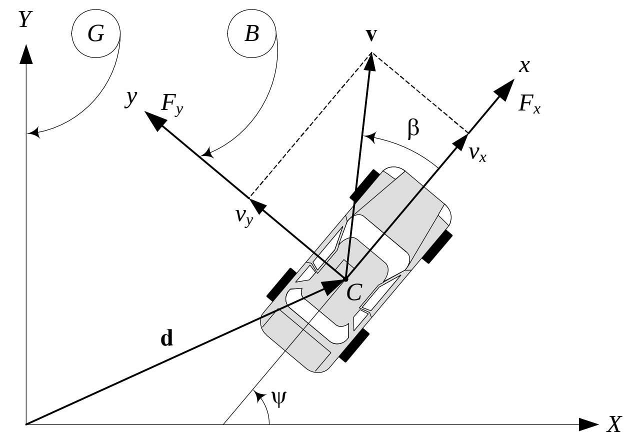


Figure 9. A rigid vehicle in a planar motion.

$$
\begin{aligned}
{}^G\mathbf{d} &= {}^G\mathbf{d}_0 + \int {}^G\mathbf{v}\, dt \\
\begin{bmatrix} X \\ Y \end{bmatrix} &= \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} + \begin{bmatrix} \int_0^t \left(v_x\cos\psi - v_y\sin\psi\right) dt \\ \int_0^t \left(v_x\sin\psi + v_y\cos\psi\right) dt \end{bmatrix}
\end{aligned}
\tag{32}
$$

The dynamic radius of curvature for such a vehicle and its position in the body and global coordinate frames are:

$$
R = \frac{v_x}{r} \tag{33}
$$

$$
{}^B\mathbf{r}_c = \begin{bmatrix} 0 \\ R \end{bmatrix} \tag{34}
$$

$$
\begin{aligned}
{}^G\mathbf{r}_c &= {}^G\mathbf{d} + {}^G R_B\, {}^B\mathbf{r}_c \\
\begin{bmatrix} X_c \\ Y_c \end{bmatrix} &= \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ R \end{bmatrix} = \begin{bmatrix} X + R\sin\psi \\ Y + R\cos\psi \end{bmatrix} \\
&= \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} + \begin{bmatrix} \int_0^t \left(v_x\cos\left(\int_0^t r\, dt\right) - v_y\sin\left(\int_0^t r\, dt\right)\right) dt \\ \int_0^t \left(v_x\sin\left(\int_0^t r\, dt\right) + v_y\cos\left(\int_0^t r\, dt\right)\right) dt \end{bmatrix}
\end{aligned}
\tag{35}
$$

We calculate the slip angle of the car, as well as the required traction force that keeps the forward speed constant.

$$
F_x = -m\frac{v_x}{r} \tag{36}
$$

$$
\beta = \tan^{-1}\frac{v_y}{v_x} \tag{37}
$$

Let us consider a car with the following characteristics as an example:

$$
\begin{aligned}
&C_f = 57296 \text{ N/rad} \qquad C_r = 52712 \text{ N/rad} \\
&m = 917 \text{ kg} \qquad I_z = 1128 \text{ kg m}^2 \\
&a_1 = 0.91 \text{ m} \qquad a_2 = 1.64 \text{ m} \\
&w_f = 1.3 \text{ m} \qquad w_r = 1.4 \text{ m} \\
&l = 2.55 \text{ m}
\end{aligned}
\tag{38}
$$

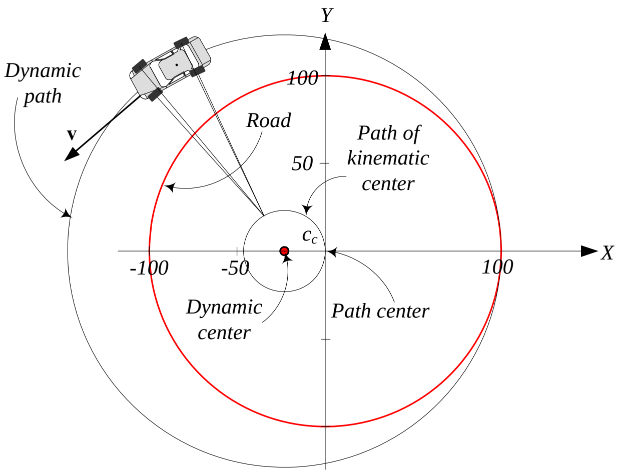


Figure 10. Ideal and actual paths of a car at higher speed.

The car is supposed to turn on a circle with a radius of $R=100$ m. Using the car's numerical data, we calculate the front and rear steer angles

$$\delta_f = 0.0090997486 \text{ rad} \qquad \delta_r = 0.016398530 \text{ rad} \qquad (39)$$

to have a kinemaric radius equal to $R=100$ m. By setting the front and rear steer angles equal to (39), and run the car with very low speed, it turns on a circle with $R=100$ m.

When the forward speed increases, the kinematic steering condition cannot keep the car on the presumed road and a constant steer angle is not desirable. Because of road and tire conditions, as well as the vehicle's velocity and acceleration, the dynamic path of motion differs from the kinematic one.

We distinguish three centers: *kinematic center*, *road center*, and *dynamic center*. Consider the car in the circle path moving with $v_x=20$ m/s as shown in Figure 10. The road is a circle with a center at the origin of G, however the dynamic path of the car at the constant forward speed is a circle with a dynamic center on the x-axis. While the car is turning about the dynamic path, the kinematic turning center of the car is moving on a circular path around the dynamic turning center.

To make the car turning on the desired road, the steer angles must be increased to have a shorter kinematic radius of curvature than the road radius curvature such that the dynamic radius of turning becomes equal to the road curvature radius.

7. EXPERIMENTATION

Path planning is a critical problem in the design of an autonomous vehicle as it is fully analyzed here. We could approach practical part of the research in autonomous vehicles by designing simulation systems, small models, or by experimenting with the real vehicles on the road. The last one, or the real life scenario, will be performed in the final stages of the research. We basically need to deal with the 4WS/AWS vehicle motion control taking into account all autodriver algorithm specifications, given here, with obstacle avoidance and position feedback systems included.

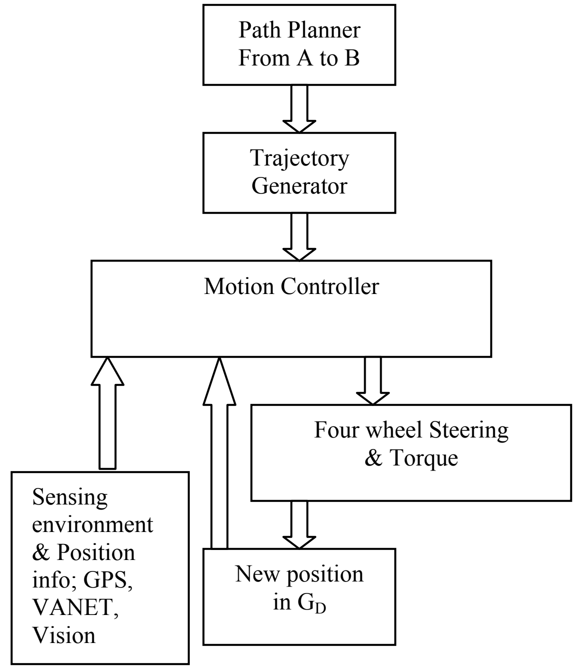


Figure 11. Comprehensive motion control block diagram.

Path following and similar problems are also studied in the area of mobile robotics and other autonomous systems, i.e. vehicles. We made decision about the parameters and conditions, in out simulation / experiment, in order to simplify realization and to concentrate mainly onto autodriver algorithm testing and application development.

Our research approach is based on modeling the system in controlled campus environment. Selected development environment, lab and small vehicle (scaled) models, or outdoor and real vehicles, dictates decision about position feedback systems that could be applied. Nevertheless, the general story i.e. conclusions about the applicability, reliability and efficiency of autodriver algorithm for autonomous vehicles is the same.

Important problem to be solved is the motion control in the real time that includes sensing, task planning and actuation. All those components of the feedback motion control are shown in Figure 11. The ideal path, known and generated mathematically, is input to the trajectory generator. Output of the generator is a time dependent function that feeds the controller together with the position and direction feedback data i.e. information about the actual path. Analysis of the vehicle motion is equivalent to expressing the position and orientation of $B(Cxyz)$ in $G(OXYZ)$. Based on the algorithm and the feedback, controller performs PID control. It calculates steering angles and the torques for all 4 wheels as well as brake action.

Environment awareness and position tracking could be performed using variety of sensors, or sensor networks. First of all, one of the obvious solutions for the position tracking, on the road, is by the use of General Positioning System (GPS), as already mentioned. In the cases of signal interferences, or missing GPS signals, like in the tunnel, we need additional position feedback mechanisms. They could be based on the sensor networks, along the road, or sensors attached to the vehicle. Variety of sensing devices can be used: ultrasonic, infrared, radar, or laser. The vehicle could also track its own position using measuring wheels and perform corrections of accumulated errors, when needed. Feedback system should be accurate, simple and reliable in the measurements of the vehicle position and the direction of motion. Traveling course should be easily implemented and all motion control should take place in the real time, i.e. within defined time constraints. Real time operation requirements, like system response time, directly depend on the vehicle speed.

8. THE ROLE OF COMMUNICATION SYSTEM

In addition to all, both position feedback information and environment awareness data could be received from the communication systems. Vehicle to Infrastructure (V2I) communication is probably one of the best solutions, although the Vehicle to Vehicle (V2V) communication could be used. If V2V is applied, it will imply application of V2I. Communications systems, on the road, will soon play one of the major roles in the development of the modern Intelligent Transport systems. Vehicles and the infrastructure units, all called network devices, are network nodes in a Vehicle Ad Hoc Network (VANET).

Another communication system, Dedicated Short Range Communications (DSRC) is a medium range wireless communication system currently developed and designed for automotive applications. Spectrum that can be used, in the range of 5GHz, is allocated by local government telecommunications authorities. By its physical characteristics it is suitable for the vehicle on the road applications, securing reliable communications, on the reasonable distances.

Radio measurement techniques could be applied for distance estimation, independently, in collaboration other sensors' data, or with GPS data. Apart from this the main purpose of establishing DSRC system are applications like:

- Intersection Collision Avoidance,
- Cooperative Forward Collision Warning,
- Cooperative Adaptive Cruise Control,
- Emergency Warning Systems.

There are many other applications but the four mentioned here should probably be combined with the Autodriver application when the comprehensive testing of the algorithm is completed and application developed. Some other, like vision based warning systems are currently being tested and they might be included in our system as environment awareness subsystems that supply feedback to motion controller as shown in Figure 12.

The model car that will be used for algorithm testing is an electrical vehicle designed by RMIT University students. A large number of model vehicles were already designed in the School of School of Aerospace, Mechanical, and Manufacturing Engineering: Solar car, which was another electrical vehicle, Hydrogen car with combustion engine, Electric car and many versions of racing cars run by petrol fuel combustion engine. In our case we have All Wheel Drive (AWD) capabilities realized through the use of 4 Brushless DC (BLDC) motors. All Wheel Steering is enabled by attaching actuators on all 4 wheels. Finally brake pedal is attached to an actuator as well.

Vehicle can be controlled through a wireless LAN (WLAN) connection to the control computer, or can behave as a truly autonomous system / vehicle when onboard controller is used. WLAN connection is used to download motion controller program, apply changes to it and can also be used to simulate inputs from the communications systems on the road, as discussed above Finally it can also be used for the vehicle data acquisition Speed, acceleration, torque and numerous of other parameters can be monitored remotely and easily. Onboard camera is used just for system motion monitoring but it can be made as a part of feedback systems for the motion control.

6. CONCLUSIONS

We assume a road is mathematically expressed by a given function in a global coordinate frame attached to the ground. There is also a local coordinate frame attached to the car at its mass center. Detecting and using the global coordinates of the vehicle we can determine the instantaneous curvature center of the road, known as road center. The road center must be the dynamic center of rotation of the vehicle. Using the kinematic steering conditions, the steer angle of the vehicle's wheel can be set such that the kinematic center of the vehicle coincide with the road center. Such a steered vehicle can trace the road at a very low speed. We calculate the dynamic center of rotation of a vehicle by solving a set of coupled ordinary differential equations. Analyzing the difference between the three centers and using a feedback signal, it is possible to adjust the steer angles such that the dynamic centers of rotation coincide with the road center. When the vehicle is turning about the road center dynamically, the kinematic center of rotation, that is the intersection of the perpendicular lines to the wheels, will move on a steering circle about the dynamic turning center.

9. REFERENCES

[1] Jazar, Reza N., Mathematical Theory of Autodriver for Autonomous, Vehicles, **Journal of Vibration and Control**, 16(2), 253-279, 2010.

[2] Jazar, Reza N., **Vehicle Dynamics: Theory and Application**, Springer, New York, 2009.

[3] Bishop, R., **Intelligent Vehicle Technology and Trend**, Artech House, Norwood, MA, 2005.