# Self-Organized Synchronization Phenomena in Spatiotemporal Coupled Oscillator Model for Emergent Systems

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# ABSTRACT

The author proposes a spatiotemporal coupled Lorenz model with an excitatory-excitatory connection or an excitatoryinhibitory connection, which consists of three temporal coupling coefficients  $c_{1,2,3}$  and three spatial coupling coefficients  $d_{1,2,3}$ . This model is an emergent device that has synchronized three nonlinear oscillators. In this study, the author discovers that selforganized various phase transition phenomena appear in this model in changing the values of  $c_{1,2,3}$  and  $d_{1,2,3}$  in the case of using the excitatory-inhibitory connection. Proposed model also concerns the neural population model for autonomous agent.

Keywords: emergence, coupled oscillator, on-off intermittency, synchronization, spatiotemporal, chaos, neuron, brain

#### 1. INTRODUCTION

The phenomena of synchronization in coupled oscillator models have generated much interest in many areas of mathematical physics [1][2], secure communications [3][4], and systems biology [5]. Especially, a matter of great interest is the discovery of an evidence of synchronization phenomena of neurons in perceptive processes in the mammal's brain [6]. In addition, Inoue et al. [7] proposed a model of the processes in cognitive interpretation using the on-off intermittency [8][9] as appears in a coupled chaos oscillator. Further possibilities of the coupled nonlinear oscillator models are expected from mathematical physics but also brain sciences [10], neural physiology [11], and neural computations [12]. Moreover, neural populations model [13] has been attracted in the neural computations, and is helping us understand the properties of nonlinear dynamical systems. And, in the recent decade, chaos study has been shifted to the spatiotemporal systems [14], although the properties of these systems remain more elusive.

#### 2. DEVICE for EMERGENT SYSTEMS

#### The Coupled Lorenz Model

Two continuous-time autonomous dynamical systems  $X_a$  and  $X_b$  are considered in *n*-dimensional Euclidean space.

$$\dot{\mathbf{X}}_{\mathbf{a}} = \mathbf{F}(\mathbf{X}_{\mathbf{a}}), \quad \dot{\mathbf{X}}_{\mathbf{b}} = \mathbf{F}(\mathbf{X}_{\mathbf{b}}) \quad \cdots (1)$$

Here, **F** is considered to be well-known the Lorenz system [15] for both with n=3, where individual vector components are

$$\mathbf{X}_{\mathbf{a}} = [x_1, x_2, x_3], \quad \mathbf{X}_{\mathbf{b}} = [x_4, x_5, x_6] \quad \cdots (2)$$

These are bi-directionally coupled and indicated as below Equation (3). Here,  $0 \le c \le 1$  is a coupling coefficient.

$$\begin{pmatrix} \dot{x}_{1,4} \\ \dot{x}_{2,5} \\ \dot{x}_{3,6} \end{pmatrix} = \begin{pmatrix} \sigma(x_{2,5} - x_{1,4}) \\ x_{1,4} (r - x_{3,6}) - x_{2,5} \\ x_{1,4} x_{2,5} - b x_{3,6} \end{pmatrix} \pm c \begin{pmatrix} x_4 - x_1 \\ x_5 - x_2 \\ x_6 - x_3 \end{pmatrix} \quad \dots (3)$$

In a mutually coupled oscillator system is shown as Equation (3), at least one of the maximum Lyapunov exponent for **F** is positive value and the coupling coefficient *c* is sufficiently small for  $X_a$  and  $X_b$ , then  $X_a$  and  $X_b$  in depict independent trajectories, although when the coupling coefficient *c* is grater than certain value even if  $|X_a(0)-X_b(0)|>0$ , the two trajectories are entrained bi-directionally and synchronized after a moment; the two then depict exactly the same trajectory. This means that the coupled system for Equation (3) is in six-dimensional space, although the two trajectories are constrained to three dimensional invariant manifold [16]; and near this synchronous/desynchronous boundary, on-off intermittent chaos where the laminar phase and burst phase appear intermittently is observed at  $X_a(t)-X_b(t)$ .

A sample of the on-off intermittency of the model at the initial conditions:  $x_1(0)=x_2(0)=x_3(0)=1.00$ ,  $x_4(0)=x_5(0)=x_6(0)=1.01$  and the parameters:  $\sigma = 10$ , b=8/3, r=28, c=0.4 is indicated in Figure 1 and the attractor at the same conditions is indicated in Figure 2. With in the laminar phase where the two attractors are completely synchronized, the attractor is constrained to one plane of  $(x_3, x_1)$ ; identically,  $x_1$ - $x_4$  becomes zero. These hyperplanes where attractor is constrained at the laminar phase are the  $(x_1, x_2)$  plane and  $(x_2, x_3)$  plane in addition to the  $(x_3, x_1)$  plane. In the c=0.4 in the same manner as before, when the trajectory for  $(x_1$ - $x_4$ ,  $x_2$ - $x_5$ ,  $x_3$ - $x_6$ ) space is indicated graphically, an attractor like that in Figure 3 is depicted.



Figure 1 On-off intermittency of the coupled Lorenz model,  $x_1-x_4$  versus  $t, t=0-1000, \sigma=10, b=8/3, r=28, c=0.4, x_1(0)=x_2(0)=x_3(0)=1.00, x_4(0)=x_5(0)=x_6(0)=1.01.$ 



Figure 2 Trajectory of the coupled Lorenz attractor,  $x_1$ - $x_4$  versus  $(x_3, x_1)$  plane, t=0~250,  $\sigma=10$ , b=8/3, r=28, c=0.4,  $x_1(0)=x_2(0)=x_3(0)=1.00$ ,  $x_4(0)=x_5(0)=x_6(0)=1.01$ .



Figure 3 Trajectory of the coupled Lorenz attractor,  $x_1$ - $x_4$  versus  $x_2$ - $x_5$  versus  $x_3$ - $x_6$ , t=0~250,  $\sigma$ =10, b=8/3, r=28, c=0.4,  $x_1(0)$ = $x_2(0)$ = $x_3(0)$ =1.00,  $x_4(0)$ = $x_5(0)$ = $x_6(0)$ =1.01.

### **A Proposed Model**

In previous model, where the two attractors are completely synchronized, the attractors are constrained to the hyperplanes  $(x_1, x_2), (x_2, x_3), (x_3, x_1)$ , and converge to one point of  $(x_1-x_4, x_2-x_5, x_3-x_6)=(0, 0, 0)$ , although when on-off intermittent chaos occurs, they repeatedly have irregular and unpredictable intermittency with wandering into a three dimensional space  $(x_1-x_4, x_2-x_5, x_3-x_6)$  from one point of (0, 0, 0) like Figure 3. This wandering in the three dimensional space on the burst phase with seeking and gathering of valuable information from this, and synchronized stabilization in the laminar phase of (0, 0, 0) can be interpreted as a process that intermittently and irregularly repeats.

The coupled Lorenz model is one device, although the Lorenz model itself is a model of the smooth manifold in threedimensional space. When the coupled Lorenz attractor leaves invariant manifold, it simultaneously leaves the three hyperplanes. That is, as like Figure 4, phase transition between the laminar phase and burst phase must simultaneously occur in three dimensions.

Although one dimensional array models of the Equation (3) are already cited [4][17], the author has considered this to be a model of the coincidence detectors [18] of the neural populations; a device with three nonlinear oscillators { $x_1$ - $x_4$ ,  $x_2$ - $x_5$ ,  $x_3$ - $x_6$ } is coupled to each of the three by coupling of the coupled Lorenz model in the Equation (3) spatially as well and a new device was considered. This is indicated in Equation (4), where  $0 < c_{1,2,3} < 1$  are temporal coupling coefficients and  $0 < d_{1,2,3} < 1$  are spatial coupling coefficients.



Figure 4 Synchronization of the three-dimensional on-off intermittencies, top: $(x_1-x_4)$ , middle: $(x_2-x_5)$ , bottom: $(x_3-x_6)$ versus *t*, *t*=760~860,  $\sigma$ =10, *b*=8/3, *r*=28, *c*=0.45,  $x_1(0)=x_2(0)=x_3(0)=1.00, x_4(0)=x_5(0)=x_6(0)=1.01.$ 

$$\begin{pmatrix} \dot{x}_{1,4} \\ \dot{x}_{2,5} \\ \dot{x}_{3,6} \end{pmatrix} = \begin{pmatrix} \sigma(x_{2,5} - x_{1,4}) \\ x_{1,4}(r - x_{3,6}) - x_{2,5} \\ x_{1,4}x_{2,5} - bx_{3,6} \end{pmatrix} \pm \mathbf{D}^* \begin{pmatrix} x_4 - x_1 \\ x_5 - x_2 \\ x_6 - x_3 \end{pmatrix} \quad \dots (4)$$
$$\mathbf{D}^* = \mathbf{D} = \begin{pmatrix} c_1 & d_2 & d_3 \\ d_1 & c_2 & d_3 \\ d_1 & d_2 & c_3 \end{pmatrix} \quad :$$

excitatory - excitatory connection,

$$\mathbf{D}^* = \tilde{\mathbf{D}} = \begin{pmatrix} c_1 & d_2 & 1 - d_3 \\ 1 - d_1 & c_2 & d_3 \\ d_1 & 1 - d_2 & c_3 \end{pmatrix} :$$

excitatory - inhibitory connection.

### **3. NUMERICALLY SIMULATION**

In the previous paper [19], the author has presented that the c and d control on-off intermittent chaos, although they have no direct effect on individual vectors and the c and d work as independent parameters without providing internal disturbance. In this paper, the difference in behavior of the model with the

case where the excitatory-excitatory connection matrix or the excitatory-inhibitory connection matrix in Equation (4) is used are shown in Figures 5 and 6, when the uniform spatial coupling coefficients  $d_1=d_2=d_3=d$  and the uniform temporal coupling coefficients  $c_1=c_2=c_3=c$  are considered.

These figures show the behaviors of  $\{X, Y, Z\} = \{x_1-x_4, x_2-x_5, x_3-x_6\}$  to change of the values of *d* at the value of certain *c*. Then, only X is illustrated in these figures. Each figure is plotted in *t*=0~100000, *d*=0~1. The *d* is changing linearly with *t*, where *d*=0.00001*t*.



Figure 5  $x_1$ - $x_4$  versus *d*, excitatory- excitatory connection, top: *c*=0.2, middle: *c*=0.3, bottom: *c*=0.4,  $\sigma$ =10, *b*=8/3, *r*=28,  $x_1(0)=x_2(0)=x_3(0)=1.00, x_4(0)=x_5(0)=x_6(0)=1.01.$ 

When the excitatory-excitatory connection matrix is used, as shown in Figure 5, if the value of *c* becomes large, the value of *d* becomes small with which the  $\{x_1\text{-}x_4, x_2\text{-}x_5, x_3\text{-}x_6\}$  synchronizes, and an on-off intermittent domain also becomes narrow, so the spatial coupling coefficients *d* works as an effect like a switch. And as shown in Figure 6, when the excitatory-inhibitory connection matrix is used, the domain of *d* separates to two places where the  $\{x_1\text{-}x_4, x_2\text{-}x_5, x_3\text{-}x_6\}$  does desynchronize. Furthermore, it should mention especially as shown in the bottom figure of Figure 6, in the domain of certain *c*, when only the value of *d* is changed, various phase transition phenomena such as chaos→limit cycle→intermittent chaos→laminar phase appear in this model as shown in Figure 7.







Figure 7  $x_{1}-x_{4}$  versus t=0~10000 in the bottom figure of Figure 6, top to bottom: d=0.03, d=0.18, d=0.33, d=0.34.

Next, in the case of using excitatory-inhibitory connection, when c and d differ in order to examine the influence of c and d that affect the flow of information between the three information codes {X, Y, Z}={ $x_1$ - $x_4$ ,  $x_2$ - $x_5$ ,  $x_3$ - $x_6$ } when the intermittency observed is almost the same, the ratio of c terms and d terms is each plotted in Figures 8 and 9.

With regard to terms including the value of c and d like Figure 8, the information flow between channels is greater with a larger d/c, although when plotting the same data without the value of c and d like Figure 9, this difference is not noted for the most part. That is, the c and d control on-off intermittent chaos, although they have no direct effect on individual vectors.

Therefore, the *c* and *d* work as independent parameters without providing internal disturbance. Thus, the *c* and *d* are not only constant values and can be incorporated as coefficients that changing with time and as functions of nonlinear oscillator  $\{X, Y, Z\}$  itself. These facts imply that they can be used as appropriate emergent parameters from the inside.



Figure 8 Information distribution, vertical (dX+(1-d)Y)/cZ versus horizontal (dZ+(1-d)X)/cY versus (dY+(1-d)Z)/cX, left: *d/c*=0.44/0.2, right: *d/c*=0.32/0.4.



Figure 9 Information distribution, vertical (X+Y)/Z versus horizontal (Z+X)/Y versus (Y+Z)/X, left: *d/c*=0.44/0.2, right: *d/c*=0.32/0.4.

## 4. DISCUSSION

## **Creation and Cognition**

A great deal of research concerning clarification of characteristics of creativity or act of creation, in which things that had hitherto not existed are created, or a description of that process had been conducted, although research was conducted in particular primarily with a focus on the field of psychology.

Psychologists [20][21] have clarified the existence of a process controlled by imagination that precedes what is called design act, which are deductive logic operations, for act of creation, in which things that had hitherto not existed are created; a creative act absent this process in mental space is not possible. In other words, a process that could be perceived as what is called conception or idea generation before deductive logic operations substantially controls the creative process. Numerous papers have been published concerning mathematical models of memory and learning of human brain activity [22], although there is a great deal of difficulty in describing the process of conception or idea generation, which lacks objectivity in an empirical sense, so research proposing mathematical models for the creative cognition process for linkage to creativity has seldom been conducted.

Concerning subject of creativity, among the varied fields of creation, especially, creation of arts is characterized by an extremely close relationship between the creator and his/her creating object. Taking the creation of musical works, for example, creative process for music as an expressive art begins with the desire for individual expression of the composer himself/herself to create a piece of music. The desire produced by the environment of the creating individual and the expression required by the individual as well are determined by the composer's environment and his/her experiences, so if the environment differs, the expression also differs. A work that is a work on account of the composer is a characteristic of music as an expressive art.



Figure 10 Creation of musical works in topological space

## Symbolic Systems versus Dynamical Systems

Concerning of creation of musical works, Figure 10 shows creation process of musical works in topological space [23][24]. In this figure, the process from a sound image X1' in the mental space, which is in the brain of the composer, to a requirement X1 of expression, is as shown below when indicated as a convergent process through the combination of deduction and abduction [25].

$$\frac{X1' \quad X1 \rightarrow X1'}{X1} : \text{abduction}$$

$$\downarrow$$

$$\frac{X1 \quad X1 \rightarrow X1'}{X1'} : \text{deduction}$$

This schema indicates requirement X1 using continuous

mapping  $\xi^{-1}:X1 \rightarrow X1'$  through addition of the composer's individual thought in the stage of abduction where the requirement X1 is postulated from the sound image X1' in the mental space of the creator himself/herself. This is indicated as a process that determines requirement X1 through verification of X1' deduced from requirement X1 with X1' that the creator previously held. However, characteristic of the creative process for musical works [26] is that the sound image X1' previously held might change due to X1' that appears after deduction.

In other words, a convergent process through the combination of abduction and deduction is a feedback system in a systems theory sense, although the input X1' may change because of the X1' to verify, which means that what determines the music required changes itself during creation. If mentioned in a design theory sense, the design environment itself is a dynamical system that changes in the design process as well rather than something designed based on determined specifications that converge to target values within extremely limited tolerances and this cannot be processed with an algorithm that sequentially executes previously programmed statements like the symbolic systems [27].

Of course, numerous methods actively using computers have been researched in order to aid human creative activity. These have been used in the creation of musical works and Computerassisted Composition, a support environment for musical work creation, currently exists, and these are mostly providing a fixed melody and harmonization with it.

However, when analyzing a musical work's structure as in Figure 11 like this, for example, a melody like the motive is present here, although the melody and harmony are inseparable; there is absolutely no way to first have the melody and then harmonization with it. If melody and harmony do not exist simultaneously in the brain of the composer as a sound image, then creation of the works like these would be close to impossible.





Moreover, individual motives are allocated to individual instruments such as woodwinds, brass, and strings for respective sounds and with harmonic progression are changed to be extremely effective as motives; if changes in both harmonic progression and "timbre" in the process of creation do not exist simultaneously in the brain of the composer as a sound image, creation of a musical work like these would be impossible.

That is, harmony, melody, and timbre are in one mode where they are blended into one another and creation must be interpreted to progress with simultaneous processing of these in parallel in the brain. The reality of creation process is not a sequence process of the symbolic systems.

#### Mental Space Model as Network Model

Therefore from the above context and in order to build creative emergent systems, in this paper, the author proposed a spatiotemporal coupled oscillator model that is a network modelbased device, which is synchronized three nonlinear oscillators  $\{X, Y, Z\}=\{x_1-x_4, x_2-x_5, x_3-x_6\}$ . And a new type of emergence, which has not been expected until now, can be realizable by regarding this model as a functional subsystem, for example, mounting in the recurrent neural networks [29] or the autonomous agent systems. Then, each of  $\{X, Y, Z\}$  corresponds to each state space that governs the creation process of the creating object.



Figure 12 Spatiotemporal coupled oscillator model as a network model-based subsystem for emergent systems

Numerous results for method of artificial neural networks that imitate large-scale neural structure have been cited, although in actuality these results are substantially controlled by how synapses that link neurons are setting; when the quantity of information increases, the handling time for these setting increases tremendously. The reality is that neural networks manifest certain functions even on a small-scale like a functional device as a proposed model as well as functions produced by a large-scale structure.

Incidentally, in this paper, only the instance where  $r_a=r_b$ , which are parameters of each of  $X_a$  and  $X_b$  of the Lorenz model, was considered, although assuming  $|r_a r_b| > 0$ , the two attractors will not completely synchronize even if  $X_a(0)-X_b(0)=0$  and c=1 and a slight amount of noise will remain in  $X_a(t)-X_b(t)$ . In addition, an analogous attractor exists even for this noise as well and more varied emergency is possible. These topics will be reported in the further papers.

#### 5. CONCLUSION

This paper proposed a new spatiotemporal coupled Lorenz model with an excitatory-excitatory connection or an excitatory-inhibitory connection, which consists of three temporal coupling coefficients  $c_{1,2,3}$  and three spatial coupling coefficients  $d_{1,2,3}$ . This model is a network model-based device that has synchronized three nonlinear oscillators. In this study, the results indicate that self-organized various phase transition phenomena appear in this model in changing the values of  $c_{1,2,3}$  and  $d_{1,2,3}$  in the case of using the excitatory-inhibitory connection, and this device can be used as an emergent subsystem for three channels through control of on-off intermittent chaos as observed in this model with the  $c_{1,2,3}$  and  $d_{1,2,3}$  as parameters.

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## REFERRENCES

- Fujisaka, H. and Yamada, T., Stability Theory of Synchronized Motion in Coupled Oscillator Systems, Progress of Theoretical Physics, Vol. 69 (1983), 32.
- [2] Ott, E. and Sommerer, J.C., Blowout Bifurcations; The Occurrence of Riddled Basins and On-off Intermittency, Physics Letters A, Vol. 188 (1994), 39.
- [3] Pecora, L.M. and Carroll, T.L., Synchronization in Chaotic Systems, Physical Review Letters, Vol. 64 (1990), 821.
- [4] Kocarev, L. and Parlitz, U., Synchronizing Spatiotemporal Chaos in Coupled Nonlinear Oscillators, Physical Review Letters, Vol. 77 (1996), 2206.
- [5] Han, S.K., Kurrer, C., and Kuramoto, K., Dephasing and Bursting in Coupled Neural Oscillators, Physical Review Letters, Vol. 75 (1995), 3190.
- [6] Gray, C.M. and Singer, W., Stimulus-specific Neuronal Oscillations in Orientation Columns of Cat Visual Cortex, Proceedings of the National Academy of Sciences of the United States of America, Vol. 86 (1989), 1698.
- [7] Inoue, M. and Nakamoto, K., Dynamics of Cognitive Interpretations of a Necker Cube in a Chaos Neural Networks, Progress of Theoretical Physics, Vol. 92 (1994), 501.
- [8] Fujisaka, H. and Yamada, T., A New Intermittency in Coupled Dynamical Systems, Progress of Theoretical Physics, Vol. 74 (1985), 918.
- [9] Platt, N., Spiegel, E.A., and Tresser, C., On-off Intermittency; A Mechanism for Bursting, Physical Review Letters, Vol. 70 (1993), 279.
- [10] Skarda, C.A. and Freeman, W.J., How Brains Make Chaos in order to Make Sense of the World, Behavioral and Brain Sciences, Vol. 10 (1987), 161.
- [11] Wilson, C.J. and Callaway, J.C., A Coupled Oscillator Model of the Dopaminergic Neuron of the Substantia Nigra, Journal of the Neurophysiology, Vol. 83 (2000), 3084.
- [12] König, P. and Schilen, T.B., Stimulus-dependent Assembly Formation of Oscillatory Responses: I. Synchronization, II. Desynchronization, Neural Computation, Vol. 3 (1991), 155.
- [13] Freeman, W.J., **How Brains Make Up Their Mind**, Columbia University Press (2000)
- [14] Kocarev, L., Tasev, Z., Stojanovski, T., and Parlitz, U., Synchronizing Spatiotemporal Chaos, Chaos, Vol. 7 (1997), 635.
- [15] Lorenz, E.N., Deterministic Non-periodic Flow, Journal of the Atmospheric Sciences, Vol. 20 (1963), 130.
- [16] Josic, K., Invariant Manifolds and Synchronization of Coupled Dynamical Systems, Physical Review Letters, Vol. 80 (1998), 3053.
- [17] Mirus, K.A. and Sprott, J.C., Controlling Chaos in a High Dimensional System with Periodic Parametric Perturbations, Physics Letters A, Vol. 254 (1999), 275.
- [18] König, P., Angel, A.K., and Singer, W., Integrator or Coincidence Detector? The Role of the Cortical Neuron Revisited, Trends in Neurosciences, Vol. 19 (1996), 130.
- [19] Emura, T., A Nonlinear Coupled Oscillator Drives the Emergent Creative Process, Proceedings of the 18th

International Congress on Acoustics, Vol. 4 (2004), 2495.

- [20] Boden, M.A., **The Creative Mind: Myths & Mechanisms**, Basic Books, New York (1990)
- [21] Finke, R.A., Ward, T.B., and Smith, S.M., Creative Cognition, Theory: Research, and Applications, MIT Press (1992)
- [22] Amit, D.J., **Modeling Brain Function**, Cambridge University Press (1989)
- [23] Emura, T., Modeling of Music Creation Process As an Application to Creative Arts Field of Yoshikawa's General Design Theory, 1st Report, Transactions of the Japan Society of Mechanical Engineers C, Vol. 66 (2000), 3805.
- [24] Emura, T., Modeling of Music Creation Process As an Application to Creative Arts Field of Yoshikawa's General Design Theory, 2nd Report, Transactions of the Japan Society of Mechanical Engineers C, Vol. 69 (2003), 2818.
- [25] Peirce, C.S., Selected Papers of Charles Sanders Peirce, Harvard University Press (1978)
- [26] Sloboda, J.A., The Musical Mind: The Cognitive Psychology of Music, Oxford University Press (1985)
- [27] Lerdahl, F. and Jackendoff, R., A Generative Theory of Tonal Music, MIT Press (1983)
- [28] Wargner, R., Tristan und Isolde, Peters, Leipzig (1910)
- [29] Hopfield, J.J., Neural Networks and Physical Systems with Emergent Collective Computational Abilities, Proceedings of the National Academy of Sciences of the United States of America, Vol. 79 (1982), 2554.
- [30] Emura, T., Les Papillons de Lorenz pour orchestre, Gérard Billaudot Editeur, Paris (1999)

## SUPPLEMENTARY NOTE

The goal of this research is clarification of characteristics of creativity or act of creation, in which things that had hitherto not existed are created, and a description of that process. However, the author has implemented this device to the MATLAB/Simulink, and actually tried to compose of the orchestral works by partly assisted from them. An actual example of these works has been publishing [30]. On the musical composition using proposed model, a set of {X, Y, Z} means only a set of state space that governs the creation process of the piece, do not mean a set of individual musical note of the piece. These topics of the actual applications will be also reported in the further articles.