Cybernetic Control in a Supply Chain: Wave Propagation and Resonance

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ABSTRACT

The cybernetic control and management of production can be improved by an understanding of the dynamics of the supply chains for the production organizations. This paper describes an attempt to better understand the dynamics of a linear supply chain through the application of the normal mode analysis technique of physics. A model is considered in which an organization's response to a perturbation from the steady state is affected by the inertia which the company naturally exhibits. This inertia determines how rapidly an organization can respond to deviations from the steady state of its own inventories and those of the two organizations immediately preceding and following it in the chain. The model equations describe the oscillatory phenomena of the naturally occurring normal modes in the chain, in which waves of deviations from the steady state situation travel forward and backwards through the chain. It would be expected that the most effective cybernetic control occurs when resonant interventions cause either amplification or damping of the deviations from the steady state.

Keywords: cybernetic control, manufacturing supply chains, management control, government intervention, normal modes, temporal effects

1. INTRODUCTION

The cybernetic control and management of production can be improved by an understanding of the dynamics of the supply chains for the production organizations. This understanding can impact the responses of the organizations to external market forces and government regulations, the organization of control processes, and the design of automation. This paper describes an attempt to better understand supply chain dynamics through the application of the normal mode analysis technique of physics. It is an extension of an earlier statistical physics treatment of static phenomena in company behavior [1] to temporal phenomena.

We consider a cluster of a large number N of companies. We imagine that there is a long chain of suppliers, each supplier adding value to the output of the preceding supplier in the chain before passing its output on to the next supplier. At the top of the chain is the "exporting company".

This model is a special case of Gus Koehler's description of a "company cluster" in which several layers of supplier companies in the cluster provide goods and services to a limited number of exporting companies [2]. To simplify the model, in the current paper the assumption is made that each layer consists of only a single company. It is straightforward to remove this restriction, but this removal will be postponed to a future paper in order to not obscure the basic features of the temporal phenomena in the cluster.

The objective of this paper is to twofold: (1)to see how disturbances in the supply chain propagate from one supplier to the next, and (2) to begin to identify the most effective timing and amplitude dependences of management or government intervention measures to improve performance in a cluster

In Section 2, the parameters of the simple model for the N companies in a cluster are described.

In Section 3, the basic equations describing the cluster of N companies are derived.

In Section 4, oscillatory solutions to these equations are described, and propagating waves are found.

In Section 5, the nature of the propagating disturbances is considered, and resonances are identified associated with both the phase velocities and the group velocities of the disturbances.

Section 6 summarizes the results, and suggests next steps in further exploring the resonance phenomena and exploiting them for management control.

2. PARAMETERS FOR A LINEAR SUPPLY CHAIN

Consider N companies in a cluster. We imagine that the companies are all arranged in a chain such that the first N-1 companies in the chain are supplier companies, and the Nth company in the chain is the exporting company.

It is also possible to regard each "company" as a production organization within a large corporation, in which case the conclusions apply not to a cluster or companies, but rather to the behavior of the large corporation. Nevertheless, in the following, we shall continue to refer to each entity in the chain as a "company".

Each company is assumed to interact only with its closest neighbors in the chain. Specifically, the nth company receives the output of the(n-1)st company and after adding value to this input, it passes on its output to the (n+1)st company in the chain.

We denote the quantity of Company n's inventory at any time t by Q(n,t), with n = 1,2,…,N.

In general, the nature of the output of Company n will be different from that of the output of Company n-1 and Company n+1, since each company contributes its own "value-added" to the input that it receives.

In addition, the number of product units m_n that Company n produces for a given number of product units m_{n-1} that it receives from Company n-1 will in general not be the same as m_{n-1}.

As will be seen in the next Section, it will be convenient to define a "normalized inventory" for each company that specifically accounts for the difference between m_{n-1}, m_n. and m_{n+1}. Specifically, define

$$Y(n,t) = m_n Q(n,t) \quad [1]$$

and arbitrarily determine the normalization by designating that

$$m_1 = 1 \quad [2]$$

With this normalization, m_n now designates the number of units that the Company n in the chain could produce providing a single unit of output from Company 1 (the first company in the chain) goes through the value added processes of the first n companies in the chain.

In the following we shall see that oscillatory phenomena occur associated with the processing times t_n of the companies.

To summarize, the parameters in the model are as stated in the following table.

Table 1. Parameters for simple linear-chain model of an N-company cluster

Inventory of nth company at time t	Q(n,t)
Number of units producible by nth company after a single unit of Company 1 passes through the first n companies	m_n
Normalized inventory of nth company at time t	$Y(n,t) = m_n Q(n,t)$
Processing time of nth company	t_n

It is apparent that many other parameters and assumptions could be incorporated into a model of a cluster of N companies, but this simple model is sufficient to illustrate the type of wave and resonance phenomena that can occur.

3. EQUATIONS FOR TIME-DEPENDENT INVENTORIES

Consider a steady state situation in which everything is working smoothly in the cluster, and each company in the chain receives just what it needs from the preceding company in the chain to produce its desired output. Designate this steady state condition with the superscript 0. Then:

$$m_{n-1} Q^0(n-1) = m_n Q^0(n) \quad [3]$$

In terms of the normalized inventories

$$Y^0(n-1) = Y^0(n) \quad [4]$$

i.e. in the ideal steady state condition, all of the normalized inventories have the same value, which we can designate by Y^0:

$$Y^0(n) = Y^0 \quad [5]$$

Now, suppose the normalized inventories do not satisfy the ideal condition of eq. [5], but differ from that condition by small quantities $y_n(t)$:

$$Y(n,t) = Y^0 + y(n,t) \quad [6]$$

Then each company will try to respond to this situation by changing its rate of production to return to a steady state condition.

Note: This does not preclude attaining a steady state at a higher level of production, since m_1 could change to a larger value. We shall postpone studying the transient phenomena associated with this change to future papers.

How will the nth company respond?

Suppose that the inventory of the (n-1)st company drops below the desired Y^0 value. Then (with just-in-time cybernetic control operating), the nth company will not receive all it needs to achieve its desired production, and might have to decrease its rate of production. The amount it must change its rate of production, however, depends on what its current normalized inventory is, as this determines how much it can buy from the (n-1)st company. If it is lower than the current normalized inventory of the (n-1)st company, it could actually increase its rate of production. Thus, the rate at which it will change its rate of production depends on the difference between its own current inventory and the current inventory of the (n-1)st company.

In the same manner, the rate of production of the nth company is affected by the current inventory of the (n+1)st company. If the latter is very small, then it will not have enough capital to buy the output of the nth company, and the nth company will have to decrease its rate of production. Again, however, the rate at which it changes its rate of production will have to depend on the difference between its own current inventory and that of the (n+1)st companies.

To account for these two effects we write the approximate equation

$$d^2y(n,t)/dt^2 = (1/t_n^2)[y(n-1,t) - y(n,t)] + (1/t_n^2)[y(n+1, t) - y(n,t)] \quad [7a]$$

In this equation, the LHS describes the rate at which the nth company is accelerating (or decelerating) its rate of production dy(n.t)/dt of inventory.

The first term on the RHS describes the interaction of the nth company with the (n-1)st company and the second term on the RHS describes the interaction with the (n+1)st company.

The $(1/t_n^2)$ factors account for the fact that any rate of change of production rate of the nth company must always be on the time scale of the company's processing time t_n.

The linear dependences on the differences between the inventories of adjacent companies in the chain can be regarded as the first terms in a Taylor expansion of any more realistic production rate responses.

Admittedly, eq. [7] is a somewhat crude approximation to the actual situation, but it will serve to demonstrate the desired propagation and resonance phenomena.

The two y(n,t) terms on the RHS of eq. [7a] can be combined, giving

$$d^2y(n,t)/dt^2 = (1/t_n^2)[y(n-1,t) - 2y(n,t) + y(n+1, t)] \quad [7b]$$

as the desired wave propagation equation.

To be precise, there are two special cases to which eq. [7] does not apply: n=1 and n=N, since each of those companies have only 1 adjacent company in the chain:

We can use as the equation for n=1:

$$d^2y(1,t)/dt^2 = (1/t_1^2)[y(2, t) - y(1,t)] \quad [8]$$

while for n = N, we can use

$$d^2y(N,t)/dt^2 = (1/t_N^2)[y(N-1,t) - y(N,t)] \quad [9]$$

i.e. for each of the two companies at the end of the chain, the acceleration or deceleration of production rate is determined only by how the inventory of the nearest neighbor in the chain compares with its own inventory.

In the next Section, we consider oscillatory and wave propagation solutions to these equations.

4. OSCILLATORY SOLUTIONS AND WAVES

Assume a time dependence of the form

$$y(n,t) = y(n)\exp(-i\omega t) \quad [8]$$

Then eq. [7] becomes a second order difference equation in y(n).

$$y(n-1) - 2y(n) + y(n+1) + (\omega t_n)^2 y(n) = 0 \quad [9]$$

Since the last term in this equation has a positive coefficient [i.e. since $(\omega t_n)^2 > 0$], this equation describes solutions that are also oscillatory in n.

By definition, the solutions of eq. [9] are the normal modes of the system.

Special case of uniform processing times

To get some feel for the type of solutions that eq. [9] has, consider the case where all of the processing times in the companies in the chain are the same:

$$t_n = T \qquad n = 1,2,\ldots,N \quad [10]$$

This is certainly not an essential assumption and can easily be modified without changing the essential nature of the solution.

Then eq. [9] simplifies to:

$$y(n-1) - 2y(n) + y(n+1) + (\omega T)^2 y(n) = 0 \quad [11]$$

Since this equation has constant coefficients – i.e. coefficients that are independent of n - we can in general write the solution as a Fourier series

$$y(n) = \Sigma\, y(p)\exp[i2\pi pn/N] \quad [12]$$

where the summation is over all integer p from $-\infty$ to $+\infty$.

On substituting eq. [12] into eq. [11] and using the orthogonality properties of the exponentials, we find the necessary condition:

$$\exp[-i2\pi p/N] - 2 + \exp[i2\pi p/N] + (\omega T)^2 = 0 \quad [13]$$

Equation [13] is the dispersion relation for a wave propagating along the chain.

This equation can be rewritten:

$$2[\cos(2\pi p/N)-1] + (\omega T)^2 = 0 \quad [14]$$

or since

$$\cos(2\pi p/N)-1 = -2\sin^2(\pi p/N) \quad [15]$$

eq. [14] can be rewritten

$$-4\sin^2(\pi p/N) + (\omega T)^2 = 0 \quad [16]$$

i.e.

$$\omega = \pm (2/T)\sin(\pi p/N) \quad \text{where p is any integer} \quad [17]$$

This dispersion relation gives the allowable oscillation frequencies for disturbances in the chain of N companies in the cluster.

Comparison with dispersion relation for acoustic phonons in a solid

It is interesting to compare the dispersion relation of eq. [17] with that for acoustic phonons in a monatomic lattice [3]

$$\omega = \pm\, 2(K/M)^{1/2} \sin(ka/2) \qquad [18]$$

In eq. [18], K is the force constant between adjacent atoms, M is the mass of the atom, k is the wave number of an acoustic wave, and a is the dimension of the unit cell in the lattice.

The phase velocity of the acoustic wave (which describes how the phase of a perfect wave moves through space) is

$$V_{phase} = \omega/k = 2(K/M)^{1/2} \sin(ka/2)/k \qquad [19a]$$

and the group velocity of the acoustic wave (which describes how a localized disturbance moves) is

$$V_{group} = d\omega/dk = (Ka^2/M)^{1/2} \cos(ka/2) \qquad [19b]$$

In the limit where ka<<1 – i.e. where the wavelength is long compared to the lattice spacing,

$$V_{phase} = V_{group} \Rightarrow (Ka^2/M)^{1/2} \qquad [20]$$

i.e. the two velocities become equal, and become independent of the wave number k.

The physically significant wave numbers k are all less than

$$k_{max} = \pm\pi/a \quad \text{[boundary of first Brillouin zone]} \qquad [21]$$

since larger k give redundant information due to the periodicity length a.

Because of the similarity between the dispersion relations of eqs. [17] and [18], we can expect the same sort of behavior for waves in a cluster of companies. This is discussed in the next Section.

5. RESONANCES IN THE CLUSTER ASSOCIATED WITH PHASE AND GROUP VELOCITIES

Management and government intervention in the cluster can be envisioned on several time scales. In general, we can classify the intervention in terms of how the time scale t_G of the intervention compares with the natural time scales $(1/\omega)$ of the cluster. Three cases are apparent:

Case 1. Gradual background (infrastructure) modification: $\omega t_G >> 1$

Case 2. Resonant intervention $\omega t_G = O(1)$

Case 3. Noise $\omega t_G << 1$

In the first case, the intervention results in a slow change in the cluster's parameters. This case can be described by treating the equations adiabatically, using a multi-time scale formalism that describes how a gradual change affects the much faster natural changes in the cluster [4].

In the third case, the intervention can average out over the cluster's natural time scale. However, the presence of the rapidly changing interventions can result in a thermal-like effect on the cluster's behavior. This can cause an increased spread in the likely performance of the sector. The details of the effect can be treated via a Fokker-Planck equation [5].

Both the first and third cases will be treated in subsequent papers.

Our focus here is on Case 2, the resonant intervention case. We shall postpone a detailed treatment of this case, also, since it involves developing a Liouville-type equation for an appropriate distribution function for the cluster and using this distribution function to examine the phenomenon of Landau damping and amplification [6]. The latter involves disturbances that travel at the same velocities as the natural wave propagation velocities of the system. This allows a continual resonant interaction that does not suffer phase cancellation.

Here we restrict our attention to simply identifying what the natural velocities in the system are.

In the acoustic case, the phase and group velocities have dimensions of

$$[V] = [\text{distance/time}] \qquad [22]$$

In the case of a large number N of companies in the cluster, the phase and group velocities simply measure how fast disturbances move from one company to the next in the supplier chain. Thus, the phase and group velocities for the N-company cluster simply have the dimensions of

$$[V\} = [1/\text{time}] \qquad [23]$$

Accordingly, the velocities here are essentially the inverse of the time that it takes for a disturbance to move from a company to an adjacent company.

By comparison with the acoustic case, the phase velocity is

$$V_{phase} = \omega/(2\pi p/N) \qquad [24]$$

since the unit cell dimension here is simply $\Delta n = 1$.

The phase velocity describes how fast any particular phase of a sine (or cosine)wave moves from one company to an adjacent company.

The definition of the group velocity is

$$V_{group} = d\omega/\, d(2\pi p/N) \qquad [25]$$

The group velocity describes how fast any localized disturbance [which is made up of a superposition of a number of sine (and cosine) waves] moves from one company to an adjacent company.

From eq. [17], we see that

$$V_{phase} = \omega/(2\pi p/N) = \pm\, (N/\pi T) \sin(\pi p/N)/p \qquad [26]$$

whereas

$$V_{group} = d\omega/d(2\pi p/N) = \pm\, (1/T) \cos(\pi p/N) \qquad [27]$$

When

$$\pi p/N << 1 \qquad [28]$$

both velocities reduce to

$$V_{phase} = \quad V_{group} => \pm (1/T) \qquad [29]$$

This is quite reasonable, since the processing time in each step $\Delta n = 1$ is $t_n = T$.

The maximum value of p that is nonredundant is

$$p_{max} = N/2 \qquad [30]$$

[by analogy to eq. [20] that defines the boundary of the first Brillouin zone for acoustic phonons].

When

$$p => p_{max} \qquad [31]$$

then

$$V_{phase} = \omega/(2\pi p/N) = \pm (N/\pi T) / \{N/2\} = 2/[\pi T] \qquad [32]$$

$$V_{group} => 0 \qquad [33]$$

Thus, over the whole range of p, the magnitude of the phase velocity is of O(1/T), but as p gets large the magnitude of the group velocity drops from O(1/T) to 0.

The significance of these findings for resonant interactions can be summarized in three statements:

(1) Resonance effects can occur for interventions that start at the top of the supply chain and move down the supply chain as well as for those that start at the bottom of the supply chain and move upwards.

(2) Resonant interactions with the phase of a propagating wave will always occur with a phase velocity of O(1/T)

(3) Resonant interactions with a local disturbance must have velocities that are adjusted to the dominant wave number of the local disturbance. For very localized disturbances, the velocity will be very small, whereas for mildly localized disturbances, the velocity will be of O(1/T).

6. SUMMARY AND DISCUSSION

Principal results

In this paper, we have considered resonance phenomena in a cluster of a large number N of companies.

Equations have been derived for the inventories in a simple model in which the nth company in the supply chain interacts only with the (n-1)st and (n+1)st companies. The equations were found to simplify when a normalized inventory Y(n,t) was introduced [eq.(3)] in terms of the actual inventory Q(n,t). In particular, the steady state Y(n) is the same for all companies in the chain.

A second order differential equation in time has been developed for the rate at which Company n accelerates or decelerates its rate of production in response to how its current inventory differs from that of its two nearest neighbors [eq. (7)]. A wave equation was shown to result when an oscillatory solution was assumed for the deviations y(n,t) of the normalized inventories from the steady state [eq. (9)]. The wave equation defines the normal modes of the system.

A dispersion relation {eq. (17) was derived for the case where the processing times t_n are independent of n: i.e. $t_n = T$. The dispersion relation f eq. (17) is quite similar to that for acoustic phonons in a monatomic lattice [eq. (18)], allowing an intuitive understanding of the resulting resonance phenomena.

Management or government interventions were divided into three classes, depending on whether their time scales were longer than [Case 1], comparable to {Case 2}, or shorter than {Case 3}, the naturally occurring oscillation frequencies given by the dispersion relation of eq. (17). Case 1 can be treated by adiabatic multi-time scale formalism. Case 2 can be described by a Liouville equation/Landau resonance formalism. Case 3 can be addressed by a Fokker-Planck operator. All three treatments will be examined in more detail in future papers.

The phase velocity [eq. (26)] and group velocity [eq. (27)] corresponding to the dispersion relation of eq. (17) were obtained. The former describes how fast the phase of a disturbance moves from one company to the next, while the latter describes how fast the peak of a localized disturbance moves from one company to the next. It was shown that phase velocities are always of O(1/T) [eqs. (29) and (32)] while group velocities can vary from O(1/T) for mildly localized disturbances to O(0) for very localized disturbances [eqs. (29) and (33)].

Although not treated in detail here, it was pointed out that the largest intervention effects correspond to those that are applied to the cluster at the naturally occurring propagation velocities. Thus, effective interventions can occur (1) in both top/down and bottom/up modes, (2) by resonating with phase propagation, and (3) by resonating with peak amplitude motion.

Discussion

The results obtained for this simple treatment of a cluster of N companies forming a linear supply chain appear to be quite promising. They provide a good starting point for a quantitative analytical treatment of resonance phenomena in a cluster. In particular, the model provides a basis for analyzing the propagation of disturbances in the cluster, and for analyzing how management and government can most effectively interact with these naturally propagating disturbances.

Future papers can extend the treatment in several ways:

In a typical supply chain, it is probably not realistic to assume that each company in the chain requires the same processing time to add its value. More realistic distributions of processing times should be considered, e.g., a chain where the processing times increase the higher up the chain a company is situated.

When intervention time scales are long compared to the naturally occurring oscillation times of the cluster, Bogoluibov's multi-time scale formalism can be employed to describe adiabatic changes in the cluster behavior. When intervention time scales are comparable to the naturally occurring oscillation times of the cluster, a Wigner distribution function can be introduced, and the effect of phase velocity resonance on the cluster's Wigner distribution function can be obtained by a quasilinear treatment that has proven effective in plasma physics. When intervention time scales are short compared to the naturally occurring oscillation times of the cluster, scattering phenomena occur. This can be treated by introducing a Fokker-Planck operator. The result should be an effective "temperature" for the cluster.

The preceding considerations are fairly general and independent of the specific form of management and government intervention. Nevertheless, it will be of interest to consider different specific models of how government intervention affects the different parameters of the cluster model.

We have not incorporated costs and pricing in this simple model. These additional elements will introduce the possibility of optimizing system profits. Also, the manufacturing capabilities of each company have not been taken into account in the model. The effect of a weakest link needs to be investigated.

We note that the considerations here are not limited to the manufacture of solid goods, but with some modification should be applicable to organizations that provide services as well. In those situations, too, there may appear resonance phenomena that can be used to optimize the services provided by the entire service enterprise.

In the model described here, the companies in the cluster are all arranged in a linear chain. Gus Koehler's cluster model is more general, allowing many companies in each layer of a supplier chain. This generalization should be incorporated.

It is hoped that the application of the normal mode technique of analysis to manufacturing supply chains can provide insight into the dynamics of the chains, and that this insight can indeed result in better cybernetic control of the supply chain processes.

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