A Hybrid DWT-SVD Image-Coding System (HDWTSVD) for Color Images

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ABSTRACT

In this paper, we propose the HDWTSVD system to encode color images. Before encoding, the color components (RGB) are transformed into YCbCr. Cb and Cr components are downsampled by a factor of two, both horizontally and vertically, before sending them through the encoder. A criterion based on the average standard deviation of 8x8 subblocks of the Y component is used to choose DWT or SVD for all the components. Standard test images are compressed based on the proposed algorithm.

Keywords: coding color images using wavelets, singular value decomposition, HC-RIOT, discrete wavelets, vector quantization.

1. INTRODUCTION

The phenomenal increase in the generation, transmission, and use of digital images in many applications is placing enormous demands on the storage space and communication bandwidth. Data compression algorithms are a viable approach to alleviate the storage and bandwidth demands. They are key enabling components in a wide variety of information technology applications that require handling a large amount of information. From text and image representation in digital libraries to video streaming over the Internet, current information transmission and storage capabilities are made possible by recent advances in data compression.

In recent years, many compression techniques have been developed in different fields, specially in the subband coding (SBC) field - namely wavelets in applied mathematics, subband coding in digital signal processing, and multiresolution in computer vision that have converged to a unified theory. SBC is a powerful technique, which is efficiently implemented using filter banks to split and merge the image without distortion. In SBC the filtered images are downsampled to their respective Nyquist rates. The downsampling operation, performed by integer decimation for practical reasons, introduces distortions due to aliasing and filtering. Reconstruction theory for filter banks demonstrates that alias-free and distortion-free solutions exist [1], [2], [3].

There exist various techniques to construct wavelet bases, or to factor the filters into basic building blocks. One of these is lifting, which is known as the second generation wavelets. A construction using lifting, is completely spatial and is used when Fourier techniques are no longer available.

The basic idea of compression using the DWT is to exploit the local correlation that exists in most of the images for building an aproximation. In the first generation wavelets, the Fourier transform is used to build the space-frequency localization. However, in the second generation wavelets, this can be done in the spatial domain and can reduce the computational complexity of the wavelet transform by a factor of two [4]. Orthogonal transformations provide a good performance for signals with high correlation. However, they achieve a poor performance for signals with low correlations.

Together with the DWT, new algorithms to encode the resulting subbands have emerged. For example, the Embeded Zerotree Wavelet (EZW), introduced by Shapiro [5], the Set Partitioning in Hierarchical Trees (SPIHT) proposed by Said and Pearlman [6], and the HC-RIOT developed by Syed [7], belong to this category. The former is a quantization and coding strategy that incorporates some characteristics of the wavelet decomposition. It takes advantage of the fact that there are wavelet coefficients in different subbands that represent the same spatial location in the image. The second uses a partitioning of the trees in a manner that tends to cluster insignifficant coefficients together in a larger subset. The last algorithm combines techniques of the zerotree entropy (ZTE) algorithm introduced by Martucci [8] and a modified SPIHT to improve the quality of the images at low bit rates.

The Singular Value Decomposition (SVD) technique provides optimal energy-packing efficiency for any given image, but its application is very limited due to the computational complexity associated with the computation of eigenvalues and eigenvectors [9]. The best results for SVD image compression have been obtained by combining SVD and vector quantization (VQ) of the eigenvectors [10], [11]. In order to have a better performance of the transformation in the areas of the image where the correlation is low, a combination of SBC or DWT and SVD was proposed in [12]. This was called "A Hybrid DWT-SVD Image-Coding System" (HDWTSVD). In this research we propose a modification of this system for color images and discuss its performance.

2. SVD CODING

The SVD is a transform suitable for image compression because it provides optimal energy compaction for any given image [9]. A good representation of the image can be achieved by taking only a few largest eigenvalues and corresponding eigenvectors. A (NxN) matrix $\boldsymbol{A}$ is decomposed to form two orthogonal matrices, $\boldsymbol{U}$ and $\boldsymbol{V}^T$, representing the eigenvectors, and a diagonal matrix $\boldsymbol{\Sigma}$ representing the eigenvalues.

$$\boldsymbol{A} = \boldsymbol{U \Sigma V}^T \tag{1}$$

where r is the rank of $\boldsymbol{A}$

$$\begin{aligned} \Sigma^T \Sigma &= diag(\lambda_1, \lambda_2 ......\lambda_r, 0,0,0,0) \\ &= diag(\sigma_1^2, \sigma_2^2, ..., \sigma_r^2, 0,0,0,0) \\ \lambda_1 \geq \lambda_2 ... \geq \lambda_r &> \lambda_{r+1} = ... = \lambda_N = 0 \end{aligned} \tag{2}$$

We can calculate the columns $\boldsymbol{v}(n)$ of $\boldsymbol{V}$ by solving

$$(\boldsymbol{A}'-\lambda(n)\boldsymbol{I})\, \boldsymbol{v}(n)=0 \qquad n=1,.....,r \tag{3}$$

where $\boldsymbol{A}' = \boldsymbol{A}^T\boldsymbol{A}$. The columns of $\boldsymbol{U}$ are

$$\boldsymbol{u}(n) = \frac{1}{\sqrt{\lambda(n)}} \boldsymbol{A}\boldsymbol{v}(n) \qquad n=1,.....,r \tag{4}$$

The original block can be estimated by retaining the q largest eigenvalues and corresponding eigenvectors

$$\hat{A} = \sum_{n=1}^{q} \sqrt{\lambda(n)}\, \boldsymbol{u}(n)\, \boldsymbol{v}^T(n) \qquad q \leq r \tag{5}$$

The square error is equal to the sum of the discarded eigenvalues

$$\sum_{m=1}^{r} \sum_{n=1}^{r} \left| X(m,n) - \hat{X}(m,n) \right|^2 = \sum_{n=q+1}^{r} \lambda(n) \tag{6}$$

where $X(m,n)$ is the original sample and $\hat{X}(m,n)$ is the reconstructed sample of a subblock. The energy contained in q retained eigenvalues is

$$Energy = \sum_{n=1}^{q} \lambda(n) \tag{7}$$

SVD yields two matrices of eigenvectors and one of eigenvalues. VQ techniques have been used successfully to encode the eigenvectors and scalar quantization techniques (SQ) to encode the eigenvalues [11] [12].

3. THE PROPOSED ALGORITHM

Figure 1 shows the Hybrid DWT-SVD system proposed in [12]. Figure 2 shows the proposed system to encode color images based on this system.

A. Encoder

Essentially, the HDWTSVD encoder remains the same as in [12]. The modifications are done before and after encoding. Before encoding, the 512x512, 8 bit PCM, RGB color components are transformed to a 512x512 luminance-chrominance components (YCbCr) by using Eq. (8) [13]. The chrominance (Cb, Cr) components are then downsampled by using the decimation filter of Eq. (9) [14], decimation along rows is followed by decimation along columns. The downsampled color components are of size 256 x 256.

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.257 & 0.504 & 0.098 \\ -0.148 & -0.291 & 0.439 \\ 0.439 & -0.368 & -0.071 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} + \begin{bmatrix} 16 \\ 128 \\ 128 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} 1 & 3 & 3 & 1 \end{bmatrix} // 8\,; \tag{9}$$

where // denotes the integer division with rounding to the nearest integer away from zero.

The luminance component is divided into 64x64 blocks (tiles) and the chrominance components into 32 x 32. In order to be consistent with the spatial resolution, the maximum number of levels of subband decomposition for each Y tile is 3, while for each chrominance tile is one level less than that applied to the corresponding luminance tile (see Fig. 3).

To decide which transform (DWT or SVD) to use, the average standard deviation of the luminance tile is calculated. If it is above a threshold [12], then all the three components are encoded using SVD and otherwise DWT.

B. Decoder

After decoding all the tiles, the original size of the chrominance components is restored by using the interpolation filter of Eq. (10) [14] , upsampling along columns is followed by upsampling along rows.

$$[1 \quad 3 \quad 3 \quad 1] // 8\,; \tag{10}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.164 & 0.000 & 1.596 \\ 1.164 & -0.392 & -0.813 \\ 1.164 & 2.017 & 0.000 \end{bmatrix} \begin{bmatrix} Y-16 \\ C_b-128 \\ C_r-128 \end{bmatrix} \tag{11}$$

4. RESULTS

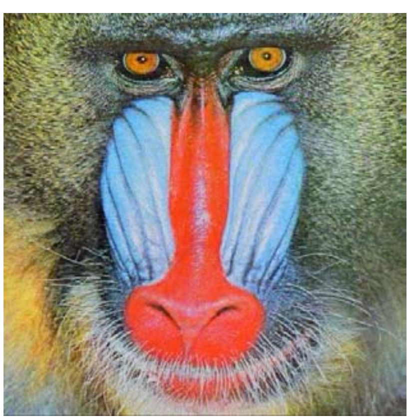
Table 1 shows the results after processing three images at low (<1 bpp) and high bitrates (> 1 bpp). These results show the bitrate of the areas compressed using DWT and using SVD. In this case, the improvement in PSNR at high bitrate is not much. This is because HC-RIOT [7] is intended for low bitrate high quality images. The images will have the best quality for low bitrate.

	Bitrate DWT (bpp)	Bitrate SVD (bpp)	Total bitrate (bpp)	PSNR (dB)
Mandrill	0.028	0.8361	0.653	26.03
Mandrill	0.201	0.8361	0.935	26.35
Mandrill	0.398	0.8361	1.440	26.38
Lena	0.039	0.647	0.220	32.46
Lena	0.163	0.647	0.560	32.92
Lena	0.413	0.647	1.230	32.96
Peppers	0.033	0.438	0.136	32.40
Peppers	0.127	0.438	0.412	33.18
Peppers	0.369	0.438	1.120	33.28

Table 1.Results for low and high bitrates.

Figure 4 shows the recovered images for low and high bitrates.

(a)



(b)

(c)

(d)

(e)

(f) 

Figure 4. Recovered images for (a) Mandrill at 0.653 bpp, 26.03 dB, (b) Mandrill at 1.216, 26.38 dB, (c) Lena at 0.22 bpp, 32.42 dB, (d) Lena at 1.23 bpp, 32.98 dB, (e) Peppers at 0.136 bpp, 32.40 dB, (f) Peppers at 1.12 bpp, 33.28 dB.

Table 1 shows that there is not much improvement of the recovered images (a) and (b) as the rate increases in tiles compressed by DWT because most of the tiles of the image are high activity pixels or low correlation tiles. In Peppers image there is a sensible reduction of distortion as the rate of tiles compressed by DWT increases because most of the high activity pixel tiles are compressed using SVD. Eq. (12) and (13) were used to calculate the MSE and the PSNR respectively.

$$MSE = \frac{1}{3xMxN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\Big(\big[r(m,n)-\hat{r}(m,n)\big]^2 + \big[g(m,n)-\hat{g}(m,n)\big]^2 + \big[b(m,n)-\hat{b}(m,n)\big]^2\Big) \quad (12)$$

$$\text{PSNR} = 10\log_{10}\left(\frac{255^2}{MSE}\right) \; dB \quad (13)$$

where

$r(m,n),\ g(m,n),\ b(m,n)$ = Original samples of the R, G, B components.

$\hat{r}(m,n),\ \hat{g}(m,n),\ \hat{b}(m,n)$ = Reconstructed samples of the R, G, B components.

M = Number of rows.

N = Number of columns.

5. CONCLUSIONS

We have presented a system for color image compression based on the HDWTSVD developed earlier for monochromatic images. Simulation results show that this system gives good results at both low and high bit rates. We can achieve even more compression at low bitrates by implementing an adaptive multistage VQ to encode the eigenvectors. An adaptive multistage VQ will help us to reduce the bitrate at low resolutions in the areas compressed by SVD while keeping the same bitrate and quality of tiles compressed by DWT. As we can see from images in figure 4, PSNR is not an indication of the recovered image quality.

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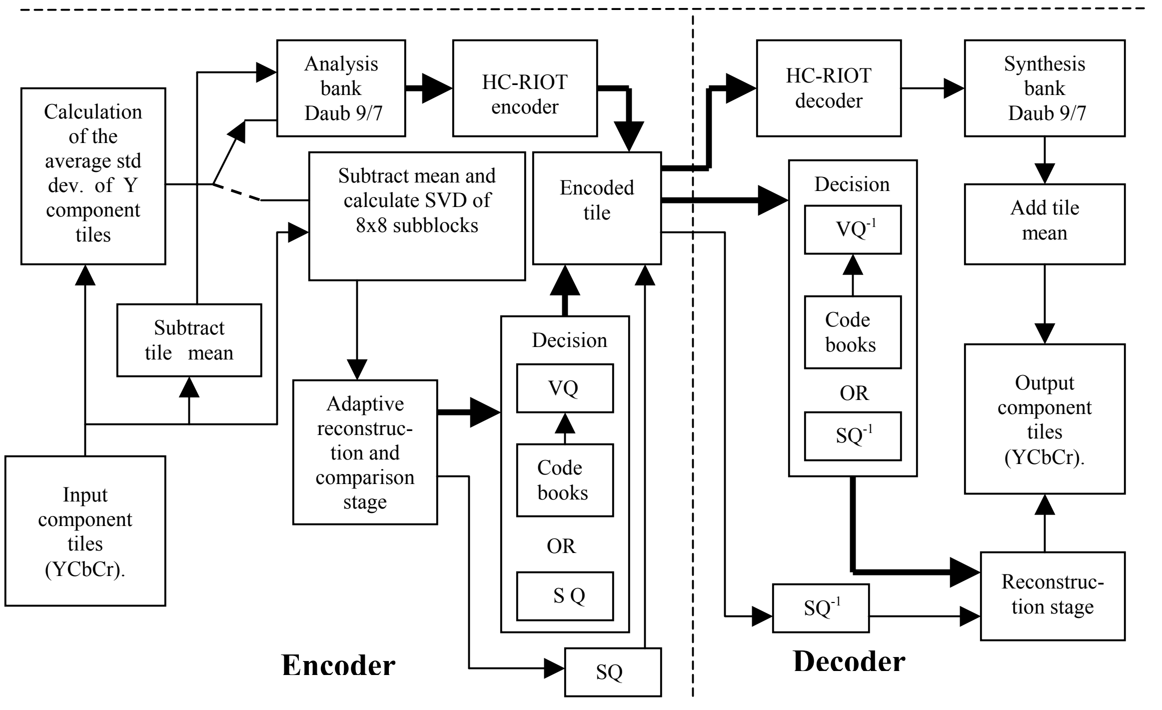

Figure 1. A Hybrid DWT-SVD Algorithm (HDWTSVD).

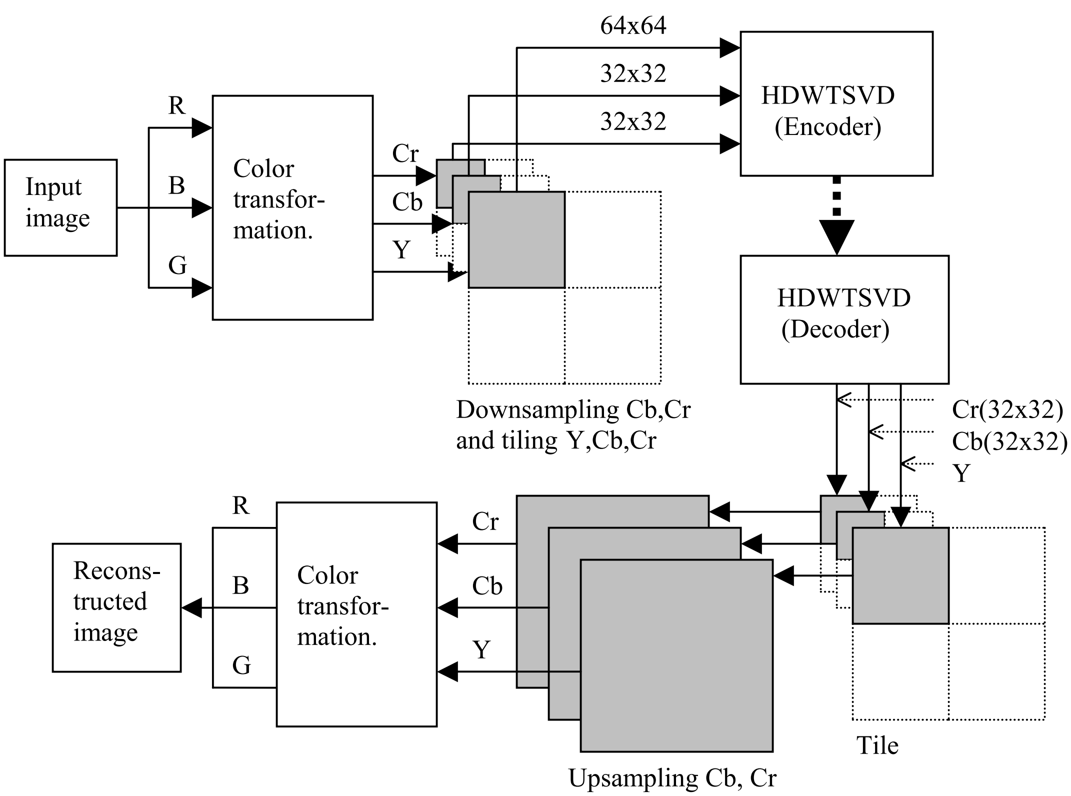

Figure 2. The HDWTSVD system for color images.

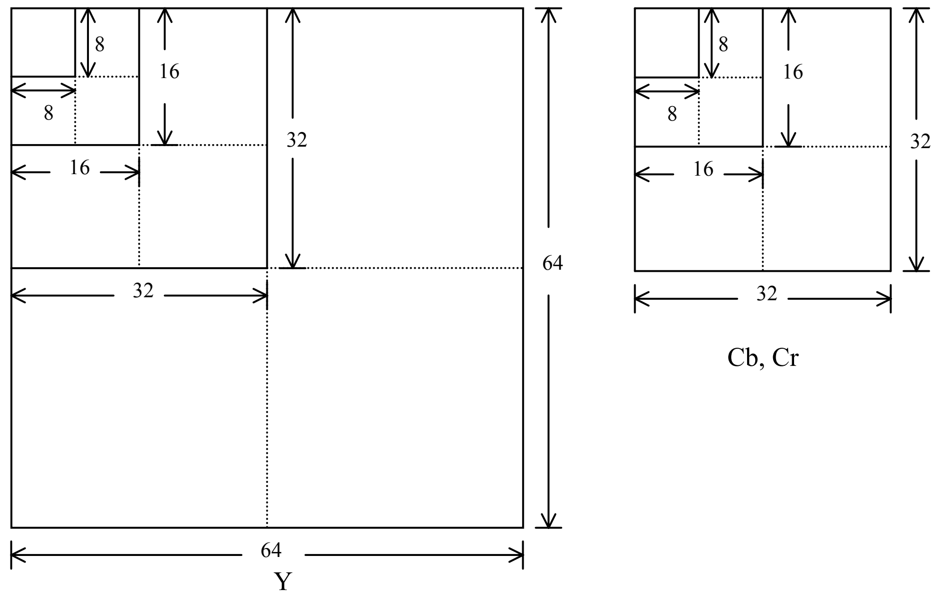

Figure 3. Subband decomposition of luminance (Y) and chrominance (Cb, Cr) components.