Pricing Options and Equity-Indexed Annuities in a Regime-switching Model by Trinomial Tree Method

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ABSTRACT

In this paper we summarize the main idea and results of Yuen and Yang (2009, 2010a, 2010b) and provide some results on pricing of Parisian options under the Markov regime-switching model (MRSM). The MRSM allows the parameters of the market model depending on a Markovian process, and the model can reflect the information of the market environment which cannot be modeled solely by linear Gaussian process. However, when the parameters of the stock price model are not constant but governed by a Markovian process, the pricing of the options becomes complex. We present a fast and simple trinomial tree model to price options in MRSM. In recent years, the pricing of modern insurance products, such as Equity-Indexed annuity (EIA) and variable annuities (VAs), has become a popular topic. We show here that our trinomial tree model can been used to price EIA with strong path dependent exotic options in the regime switching model.

Keywords: Option pricing, Regime switching model, Trinomial tree method, Equity-Indexed Annuity, Path dependent options.

1. INTRODUCTION

Since the seminal work of Black and Scholes (1973) and Merton (1973), option pricing has been a very popular topic. Due to its compact form and computational simplicity, the Black-Scholes formula enjoys great popularity in the finance industries. One important economic insight of the Black-Scholes option-pricing model is the concept of perfect hedging of options by continuously adjusting a self-financing portfolio under the no-arbitrage principle. Cox, Ross and Rubinstein (1979) introduce a discrete version of the Black-Scholes model, the binomial tree model,which provides further insights into the concept of perfect hedging in a transparent way.

The Black-Scholes' model has been extended in various ways. The Markov regime-switching model (MRSM) is one of the generalizations. The Markov regime-switching model (MRSM) has recently become a popular model. The MRSM allows the parameters of the market model depending on a Markov chain, and the model can reflect the information of the market environment which cannot be modeled solely by the Black-Scholes model. The Markovian process can ensure that the parameters change according to the market environment and at the same time preserves the simplicity of the model. It is also consistent with the efficient market hypothesis that all the effects of the information about the stock price would reflect on the stock price. However, under the MRSM, the pricing of the options becomes complex. There are many papers about option pricing under the regime-switching model. Naik (1993) provides an elegant treatment for the pricing of the European option under a regime-switching model. Buffington and Elliott (2002) tackle the pricing of the European option and the American option using the partial differential equation (PDE) method. Boyle and Draviam (2007) consider the the price of exotic options under regime switching using the PDE method. The PDE has become the focus of most researchers as it seems to be more flexible in pricing. However, if the number of regime states is large, and we need to solve a system of PDEs with the number of PDEs being the number of the states of the Markov chain, and there is no close form solution if the option is exotic, then the numerical method to solve a system of PDEs is complex and computational time could be long. In practice, we prefer a simple and fast method. For the European option, Naik (1993), Guo (2001) and Elliott, Chan and Siu (2005) provide an explicit price formula. Mamon and Rodrigo (2005) obtain the explicit solution to European options in regime-switching economy by considering the solution of a system of PDEs. All the close form solutions depend on the distribution of occupation time which is not easy to obtain.

The binomial tree model is one of the most popular methods to calculate the price of simple options like the European option and the American option. The Trinomial tree model of Boyle (1986) is a very flexible model. The extra branch of the trinomial model gives one more degree of freedom to the lattice and makes it very useful in the case of the regime switching model. Boyle and Tian (1998) use this property of the trinomial tree to price the double barrier option, and propose an interesting method to eliminate the error in pricing barrier options. Boyle (1988) uses a tree lattice to calculate the price of derivatives with two states. Kamrad and Ritchken (1991) suggest a $2^k + 1$ branches model for k sources of uncertainty. Bollen (1998) constructs a tree model which is excellent for solving the price of the European option and the American option in a two-regime situation. The Adaptive Mesh Model (AMM) invented by Figlewsho and Gao (1999) greatly improves the efficiency of lattice pricing. Aingworth, Das and Motwani (2006) use a lattice with a $2k$-branch to study the k-state regime switching model. However, when the number of states is large, the calculation of option price using the tree models mentioned above is complex.

In recent years, the equity linked insurance products, such as EIA and variable annuities (VAs), have become popular in the market. These products can be considered investment plans with associated life insurance benefits, a specified benchmark return, a guarantee of an annual minimum rate of return and a specified rule of the distribution of annual excess investment return above the guaranteed return. Earlier work on these kind products can be dated back to Boyle and Schwartz (1977) and Brennan and Schwartz (1976, 1979), etc. Tiong (2000) presents a comprehensive discussion on the equity-indexed annuities using assumptions of the geometric Brownian motion for asset price dynamics and constant interest rate and discusses the pricing of three common product designs. Lee (2002, 2003) provides an introduction to EIAs and studies several new designs of EIAs. Boyle and Hardy (2003) and Ballotta and Haberman (2003) study the guaranteed annuity options. Using the geometric Brownian motion asset price model, Milevsky and Posner (2001) and Milevsky and Salisbury (2006) investigate various variable annuities. Bauer at al. (2008) use the same model but take policyholders' behaviour into account. Coleman et al. (2007) examine the impact of the volatility of the underlying asset on the variable annuities' guarantees using Merton's jump-diffusion model. Lin and Tan (2003) and Kijima and Wong (2007) consider the EIA model with a stochastic interest rate and mortality risk. The equity linked insurance products have long maturity, the Black-Scholes model may not be a good choice in this case. Regime switching model is one way to deal this problem. We consider the valuation problem for EIA with embedded exotic option under a regime switching model. For the strong path dependent options, valuation becomes difficult using a regime switching model. Here, we introduce a method based on the work of Hull and White (1993) to solve the pricing problem of Asian options and EIA products in a Markov Regime Switching Model (MRSM).

The sections 2 and 3 of this paper summarize the main idea of Yuen and Yang (2009, 2010a, 2010b). Section 4 provides some new results, that is some results on pricing of Parisian options. Parisian options are barrier options with knock-in or knock-out feature. Pricing of Parisian options in the regime switching model is more difficult than the ordinary barrier options due to the knock-in or knock-out feature. In this paper we demonstrate that our trinomial model provides a fast and easy use method to solve the problem. We present some numerical results to illustrate the method and idea.

2. TRINOMIAL TREE FOR REGIME SWITCHING MODEL

We let $\mathcal{T}$ be the time interval $[0, T]$ where $T < \infty$. $\{X(t)\}_{t\in\mathcal{T}}$ is a continuous time Markov chain with a finite state space $\mathcal{X} := (x_1, x_2, \ldots, x_k)$, which represents different states of an economy.

Assume that there are two investment securities available to the investors in the market in our model, one is a bond and the other one is a stock. The risk free interest rate is denoted by $\{r_t = r(X(t))\}_{t\in\mathcal{T}}$ which depends only on the current state of economy.

The expected rate of return and the volatility of the stock price process are affected only by the state of economy and denoted by

$$\{\mu_t = \mu(X(t))\}_{t\in\mathcal{T}} \text{ and } \{\sigma_t = \sigma(X(t))\}_{t\in\mathcal{T}},$$

respectively.

In the CRR binomial tree model, the ratios of changes of the stock price are assumed to be $e^{\sigma\sqrt{\Delta t}}$ and $e^{-\sigma\sqrt{\Delta t}}$, respectively. The probabilities of getting up and down are specified so that the expected growth rate of the stock price matches the risk free interest rate. However, in the multi-state MRSM, the risk free interest rate and the volatility are not constant. They change according to the Markov chain. In this case, a natural way is to introduce more branches into the lattice so that extra information can be incorporated in the model. For example, Boyle and Tian (1998), Kamrad and Ritchken (1991) investigate prices of options in a model with multi-variables. Aingworth, Das and Motwani (2006) use $2k$-branch to study k-state model. However, the increasing number of branches makes the lattice model more complex, Bollen (1998) suggests an excellent recombining tree based model to solve the option price in the two-regime case, but for multi-regime states, the problem still cannot be solved effectively.

Yuen and Yang (2010a) proposes a different way to construct the tree. Instead of increasing the number of branches, we change the risk neutral probability if the regime state changes. In this manner, we can keep the trinomial tree a recombined one. The method relies on the flexibility of the trinomial tree model, and the core idea of the multi-state trinomial tree model is to change the probability being used in each regime rather than increasing the branches of the tree so that all regimes can be accommodated in the same recombining tree which greatly improves the efficiency in valuation of derivatives in MRSM.

We present the main idea of Yuen (2010a) here. Assuming that there are k states in the Markov regime switching model, the corresponding risk free interest rate and volatility of the price of the underlying asset are $r_1, r_2, \ldots, r_k$ and $\sigma_1, \sigma_2, \ldots, \sigma_k$, respectively. The up-jump ratio of the lattice is taken to be $e^{\sigma\sqrt{\Delta t}}$, for a lattice which can be used by all regimes, where

$$\sigma > \max_{1 \le i \le k} \sigma_i. \tag{1}$$

For the regime i, let $\pi_u^i, \pi_m^i, \pi_d^i$ be the risk neutral probabilities corresponding to that the stock price increases, remains the same and decreases, respectively. Then, similarly to the simple trinomial tree model, the following set of equations can be obtained for each $1 \le i \le k$:

$$\pi_u^i e^{\sigma\sqrt{\Delta t}} + \pi_m^i + \pi_d^i e^{-\sigma\sqrt{\Delta t}} = e^{r_i \Delta t} \quad \text{and} \tag{2}$$
$$(\pi_u^i + \pi_d^i)\sigma^2 \Delta t = \sigma_i^2 \Delta t. \tag{3}$$

If λ_i is defined as σ/σ_i for each i, then, $\lambda_i > 1$ and the values of $\pi_u^i, \pi_m^i, \pi_d^i$ can be calculated in terms of λ_i:

$$\pi_m^i = 1 - \frac{\sigma_i^2}{\sigma^2} = 1 - \frac{1}{\lambda_i^2} \tag{4}$$
$$\pi_u^i = \frac{e^{r_i\Delta t} - e^{-\sigma\sqrt{\Delta t}} - (1 - 1/\lambda_i^2)(1 - e^{-\sigma\sqrt{\Delta t}})}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}} \tag{5}$$
$$\pi_d^i = \frac{e^{\sigma\sqrt{\Delta t}} - e^{r_i\Delta t} - (1 - 1/\lambda_i^2)(e^{\sigma\sqrt{\Delta t}} - 1)}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}. \tag{6}$$

Therefore, the set of risk neutral probabilities depends on the value of σ. In order to ensure that σ is greater than all σ_i, one possible value we suggest is

$$\sigma = \max_{1 \le i \le k} \sigma_i + (\sqrt{1.5} - 1)\bar{\sigma} \tag{7}$$

where $\bar{\sigma}$ is the arithmetic mean of σ_i. Another possible suggestion is that $\bar{\sigma}$ is replaced by the geometric mean. These suggestions are based on the parameters used in the trinomial tree models in the literature.

After the whole lattice is constructed, the main idea of the pricing method is presented here. We assume T to be the expiration time of the option, N to be the number of time steps, then $\Delta t = T/N$. At time step t, there are $2t + 1$ nodes in the lattice, the node is counted from the lowest stock price level, and $S_{t,n}$ denotes the stock price of the n^{th} node at time step t. As all the regimes share the same lattice and the regime state cannot be reflected by the position of the nodes, each of the nodes has k possible derivative's price corresponding to the regime state at that node. Let $V_{t,n,j}$ be the value of the derivative at the n^{th} node at time step t under the j^{th} regime state.

Suppose the transition probability matrix is given by $P(\Delta t) = (p_{ij}(\Delta t))$, the price of the derivative at each node can be found by iteration. We start from the expiration time, for example, for a European call option with strike price K,

$$V_{N,n,i} = (S_{N,n} - K)^+ \qquad \text{for all states } i \tag{8}$$

where $S_{N,n} = S_0 \exp[(n - 1 - N)\sigma\sqrt{\Delta t}]$.

We assume that the Markov chain is independent of the Brownian motion, thus the transition probabilities will not be affected by changing the probability measure from the physical probability to the risk neutral measure.

With the derivative price at expiration, using the following equation recursively:

$$V_{t,n,i} = e^{-r_i\Delta t}\left[\sum_{j=1}^{k} p_{ij}(\pi_u^i V_{t+1,n+2,j} + \pi_m^i V_{t+1,n+1,j} + \pi_d^i V_{t+1,n,j})\right], \tag{9}$$

the price of the option under all regimes can be obtained.

In Yuen and Yang (2009), a recombined trinomial tree method for pricing options under jump diffusion model with regime switching is studied. We show that the method is powerful for jump diffusion models with regime switching as well. For the problem of pricing strong path dependent options, such as Asian options, Yuen and Yang (2010b) uses Asia option as an example and proposes a method based on the paper of Hull and White (1993). The idea is to use representative sets of values, the price of the Asian option with the average stock price equal to representative sets of values is calculated; when the average stock price level is not the representative sets of values, linear approximation is used to obtain the option price. For detailed description of the method, we refer to Yuen and Yang (2010b).

3. PRICING EQUITY-INDEXED ANNUITIES

Equity-Indexed Annuities are popular insurance products. How to value these products and how to manage the risk of these products are practically important and theoretically interesting problems. Since most the embedded options in these products are exotic options and have path dependent features. Moreover the maturity of these products are long in most cases. The regime switching model can fit the market data better than Black-Scholes model for the equity price. However, the valuation of derivatives in such model is challenging when the number of states is large, especially for the strong path dependent options such as Asian options. Our trinomial tree model provides an efficient way to solve this problem. We present the main idea of how to value the liability of these products. The detailed discussion can be found in Yuen and Yang (2010b). Let $S(t)$ be the equity index price process and $A(t)$ is the average index level from 0 to t, that is

$$A(t) = \frac{1}{t}\int_0^t S(u)du. \tag{10}$$

Then, we consider a general expression of a point-to-point Asian EIA which is similar to the one used in Lin and Tan (2003) and has cumulative return equal to

$$C(t) = \max[\min[1 + \alpha R_t, (1+\zeta)^t], (1+g)^t], \qquad (11)$$

where $R_t = A(t)/S_0 - 1$. R_t is the average return of the equity index throughout the period from time 0 to t, α is the participation rate that shows how is the extra return received by the investors per unit of the average return of the equity index, ζ is the cap rate which is the maximum annual return that can be enjoyed by the investors and g is the guarantee rate which is the minimum annual return of the EIA contract.

In practice, the annual reset EIAs or ratchet EIAs are more popular. These kind of EIAs allow the investors to lock their guarantee return every year rather than just a guarantee return for the whole contract period. The cumulative return of these kind EIAs is given by:

$$C(t) = \prod_{k=1}^{t} \max[\min(1 + \alpha R'_k, 1+\zeta), 1+g], \qquad (12)$$

where $R'_k = \int_{k-1}^{k} S(u)du/S(k-1) - 1$, which is the average index return of the k^{th} year. If the equity index follows the simple Black Scholes model, the appreciation rate of the index in a time interval is independent of the return rate in the previous intervals due to the independent increment property of Brownian motion and the expected return of the whole contract period is equal to the product of expected return in each year. However, in our MRSM, the future return rate and volatility of the index are affected by the current data due to the presence of regime-switching. For example, if the return in this time point is low, there is a higher probability that we are now in a regime with lower expected return and thus the expected return in the next time period will be lower because it is likely that we are still in this low return regime state. Fortunately, the regime switching process is a Markov chain, we are able to determine the expected return in the year with the regime information at the very beginning of the year. Therefore, we are able to solve this problem by considering a conditional expectation.

In our model, we can include the mortality risk of the investors. We assume that the ratchet EIA is payable at the end of the year that the investor dies or the EIA contract expires, there is no selection effect and the future lifetime random variable is independent of the Brownian motion and regime switching process. In this case we are still able to value the product. In Yuen and Yang (2010b), some numerical results are presented. The numerical examples show that our recombined trinomial tree method is easy to use and flexible.

4. PRICING PARISIAN OPTIONS

In this section, we apply the trinomial tree method to price Parisian options in MRSM. Parisian options are barrier options with knock-in or knock-out feature which activates when the underlying asset remains above or below certain levels for a continuous period. The advantages of Parisian options over barrier option are that the options are more robust against short-term stock price movement and it is harder to manipulate the underlying asset price (Haber and Schoñbucher (1999)). However, the pricing of Parisian options is also complicated due to the presence of this continuous-time knock-in (or knock-out) features. The payoff and price of the simple barrier options depends on the event whether the barrier level is touched or not. There are only two situations. The price and payoff of Parisian options depends on the time that the price of the underlying asset stays above (or below) the barrier. The time can take over a range of values from 0 to the length of the time-window, denoted as D. An additional variable referring to the amount of time the underlying asset price staying within the window is required for calculating the price of Parisian options.

Different techniques can be used to calculate the price of Parisian option. Chesney, Cornwall, and Jeanblanc-Picqué (1997) find the price of Parisian options by Laplace transform. Haber and Schoñbucher (1999) use partial differential equation to calculate the price of Parisian options. Avellaneda and Wu (1999) price Parisian options using lattice method and distribution of first passage time of the barrier level. Here, we consider up-and-out Parisian put options and illustrate how to price them with our trinomial tree approach in MRSM. We let $V_{t,n,d,i}$ be the price of the up-and-out Parisian put option at the n^{th} node at time step t under the i^{th} regime with d units of time that the asset stays across the barrier currently. The strike and barrier are assumed to be K and B, respectively. Then, we have the following equations,

$$\begin{aligned} V_{t,n,d,i} &= e^{-r_i \Delta t}\Bigg[\sum_{j=1}^{k} p_{ij}(\pi_u^i V_{t+1,n+2,\phi(t+1,n+2,d),j} \\ &\qquad + \pi_m^i V_{t+1,n+1,\phi(t+1,n+1,d),j} \\ &\qquad + \pi_d^i V_{t+1,n,\phi(t+1,n,d),j})\Bigg], \qquad (13) \end{aligned}$$

and,

$$\phi(t+1,n+1,d) = \begin{cases} D & d = D \\ d+1 & S_0 u^{n-t} \geq B, d < D \\ 0 & S_0 u^{n-t} < B, d < D \end{cases}. \qquad (14)$$

For the up-and-out Parisian option, we know that

$$\begin{aligned} V_{T,n,d,i} &= (K - S_0 u^{n-T})^+ \quad \forall n, i \text{ and } d < D \\ V_{t,n,D,i} &= \quad 0 \qquad\qquad \forall t, n, i. \end{aligned}$$

With these two simple recursive equations and the boundary conditions, we can price the Parisian options effectively using the trinomial tree.

We consider a two-regime market. In Regime 1, the volatility of the underlying asset is 25% and risk-free interest rate is 4%; in Regime 2, the volatility of the underlying asset

Table 1: Prices of the Parisian options

	Strike = 110		Strike = 100	
N	Regime 1	Regime 2	Regime 1	Regime 2
100	13.4657	14.7585	8.3487	9.9382
200	13.5499	14.8824	8.3659	9.9831
300	13.5698	14.9233	8.3782	10.0055
400	13.5843	14.9489	8.3890	10.0213
500	13.5963	14.9677	8.3883	10.0260
600	13.6060	14.9822	8.3953	10.0350
700	13.6128	14.9927	8.3971	10.0392
800	13.6168	15.0001	8.3978	10.0421
900	13.6177	15.0045	8.3986	10.0446
1000	13.6233	15.0118	8.3997	10.0470
MC	13.6769	15.0794	8.4150	10.0945

is 35% and risk-free interest rate is 6%. The barrier level and the length of the time-window are set to be 130 and 0.04, respectively. The transition rates of the two regimes are both assumed to be 0.5. In Table 1, we summarize the prices of the one-year Parisian up-and-out put option with strikes 110 and 100 found by tree method. The prices obatined by simulations are also shown.

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