A Discipline-Independent Approach to a Higher Cognitive Pedagogy

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ABSTRACT

We present a content-independent formulation of higher cognitive pedagogy, by identifying higher cognitive pedagogy with executive function which in turn we equate with continual multi-dimensional processing of drivers of outcomes. The key focus in this definition is on multiple dimensions. We apply our definition to four diverse disciplines: a) mathematical modeling of verbal problems is presented as an interaction between the dimensions of language and algebra; b) complex mathematical problems are presented as an interaction between multiple sub-problems participating in one solution; c) essay writing is presented as an interaction between specific atomic competency skills – creating meaningful sentence pairs – and hierarchical organization into greater wholes such as paragraphs and essays; d) foreign language translation is presented as a dimensional parsing of hypernyms and hyponyms; similarly, literary translation is presented as a dynamic interaction between multiple dimensions of a literary work. We show consistency and correlation between the executive-function pedagogical approach and the Bloom-Anderson approach.

Keywords: cybernetics, executive function, multi-dimensional processing, modeling, complex mathematical problems, organizational writing, Bloom-Anderson, hypernyms, hyponyms

1. OVERVIEW

Many independent sources have called for a reform of modern education with an emphasis on higher order thinking skills. Consequently, implementation of reform requires a precise definition of higher order thinking skills.

1.1 The Traditional Approach

The traditional approach of defining higher order thinking skills is to present hierarchies of learning attributes. This approach was initiated with the Bloom taxonomy [4]: – knowledge, comprehension, application, analysis, synthesis and evaluation. This taxonomy is typically applied as follows: If one is teaching a discipline, one evaluates each module and learning unit by ascertaining if the primary focus is exclusively on knowledge and comprehension versus whether applications of the learning unit are presented which require analysis, synthesis and evaluation on the part of the instructor and student.

Several other pedagogists, for example, Anderson [2], Webb [29-31], Marzano [20], present competing or complementary hierarchies. Some pedagogists – for example, Gagne [10], and Van Hiele [28] – characterize their hierarchies as representing stages of learning.

A recent development in this use of hierarchies is the demonstration of a correspondence and consistency between superficially different hierarchies. Thus Yazdani [33] showed the approaches of Gagne and Van Hiele equally effective in teaching geometry; Hess [13,14] studied the interaction of Bloom-Anderson and Webb by creating a matrix and classifying several thousand mathematics and English homework assignments in K-12 by their Webb and Bloom levels.

1.2 This Paper’s Approach

In Section 2, we approach the problem of defining higher order thinking skills by appealing to brain function. More specifically, we identify higher order thinking skills with activities requiring executive brain function. While executive function itself is an elusive concept, there being several aspects to executive function [22,26], it is generally agreed that executive function is a higher order brain function. Executive function refers to the capacity of the brain to deal with complex tasks. By explicitly relating higher order thinking skills to brain function we take one step towards objectifying and concretizing the definition.

We provide further concretization by identifying commonality in several executive-function performance tests. Several tests of executive function assess continual multi-dimensional processing of environmental information to accurately determine drivers of outcomes. This, the multi-dimensional processing of information to determine drivers, becomes our working definition of higher order thinking skills.

Such an approach is objective, operational and mechanistic. It enables an instructor to instantly evaluate learning material for the presence of higher order thinking skills.

1.3 Atomic Skill Competencies

We have identified higher order thinking skills with multi-dimensional processing. We further suggest identifying the atomic skill competencies of each educational domain as the multiple dimensions interplaying in determining outcomes. In Section 2.2 we rigorously define atomic skill competency.

1.4 Cybernetic Approach

Throughout our discussion, analysis is exclusively dependent on information flow and independent of content. The analysis of a complex domain in terms of information flow independent of content is the distinguishing attribute of cybernetics [8].

We are particularly indebted to the cybernetician Ashby [9] who focused on eliminating terms such as higher order from psychology and replacing them with more mechanistic and operational concepts. In so doing, Ashby was not trying to remove complexity from psychology but on the contrary, trying to preserve it in a more respectable manner.
1.5 Outline
Section 2 presents the definitions of executive function and atomic skill competency. Sections 3-6 apply our definitions to the domains of verbal-problem modeling, English writing, mathematics, literary criticism and foreign language translation. The application of our definition to such diverse disciplines supports the content-independent nature of our approach.

2. EXECUTIVE FUNCTION AND ATOMIC SKILLS

This section reviews executive-function performance tests, clarifies the meaning of multi-dimensional processing, rigorously defines skill competency, and uses these two concepts - skill and multi-dimensional processing - to define our approach to pedagogy which we show consistent with Bloom-Anderson.

2.1 Executive Function Performance Tests

As already indicated in Section 1, executive function refers to multiple brain functions. There are multiple executive function tests the two main categories being performance and rating tests [26]. We examine three well known performance tests.

The Wisconsin Card Sorting Test (WCST) [12]: During the administration of the WCST, the examiner flashes several dozen two-row items such as those found in Figure 1. The examinee is asked to match the card in the bottom row with the appropriate card in the top row. An illustrative example is presented in Figure 1.

![Figure 1: A sample item in the WCST.](image)

Abstractly, Figure 1 presents three dimensions: a) letter (A,B,C), b) formatting (bold, italic, underline), and c) number (1,2,3). The examinee must determine if the two A’s in the bottom row of Figure 1 resemble the A card because of the dimension of letter, resembles the B card because of the dimension of number or resembles the C card because of the dimension of formatting (both are underlined).

Typically, after a few attempts the examinee will discover the correct driver of resemblance. The examinee will then have a streak of correct answers. The examiner may then change the driving dimension. For example, if in the last 10 trials the correct answer was based on a match of number, the examinee may create new trials where the correct match is based on the dimension of letter resemblance.

A wealth of information is gathered during the test. For our purposes, we see that the examinee is being tested on his/her capacity to correctly identify the driving dimension from a set of competing multiple dimensions (formatting, number, letter). Furthermore, as time progresses the examinee must continuously reassess the correct driver of correctness.

We conclude that the WCST is measuring the capacity of the examinee to continuously process multiple-dimensional drivers of outcome. We argue that this is the essence of higher order thinking skills.

The Trailmaking test [6,7,11]: This deceptive but beautiful test has two parts: A and B. In both parts, the examinee is asked to make a trail: In part A, the trail is 1-2-3-..., while in part B, the trail is 1-A-2-B-3-C... An illustrative example is presented in Figure 2. Although these tasks are easy, remarkably, the part B test always takes longer. The increased length is due to the presence of two dimensions: number and letter. The multi-dimensionality requires executive function and hence the increased time length. Despite the test’s simplicity, it is useful in diagnosing brain damage and recovery possibility, for example after a stroke.

![Figure 2: A sample Trailmaking test.](image)

The simplicity of this test highlights the importance of our proposed definition that higher order thinking skills equate with multi-dimensional processing. The trailmaking test is making the powerful point that any multi-dimensional processing transforms a mundane exercise into executive-function quality. Indeed, just adding the dimensions of letter to the dimension of number in the simple task of making a trail raises the quality of the task to executive-function quality.

The Stroop Interference Test [16, 25]: In this test, the examinee is presented with two lists of words and asked to identify the color of each word in the list. The first list typically has only one dimension, color, and hence can be done quickly. The second list has two dimensions word meaning and letter color. For example the word “red” may be written in blue font, requiring the examinee to process two dimensions, word meaning and color, to arrive at a correct answer. This multi-dimensionality requires executive function and hence the second test typically requires more time (over several dozen trials), something measurable.

Summary: We identify higher order thinking skills with executive function. Executive-function performance tests measure the capacity to continuously process competing multiple dimensions to ascertain driving forces. We conclude that higher order thinking skills should be identified with multi-dimensional processing capability.

2.2 Atomic Skill Competencies

Although the pedagogic literature uses the word skill, it is infrequently (if ever) precisely defined. The psychological literature defines a skill as any task that under repeated performance a) increases in speed and b) decreases in error rate [19] For example, reciting the alphabet, plugging into a mathematical formula, developing a topic sentence by giving examples or consequences, are all examples of skills.

Contrastively, writing an essay, doing a complex math problem, writing a complete computer program are all non-skill acts. For example, you cannot speak about the error rate in writing an essay since essays are not right or wrong. Similarly, increased practice does not increase essay writing speed. It is not a skill.
We now explain the word atomic. An atomic skill competency is a skill that cannot be decomposed further. For example, writing a paragraph basically consists of applying multiple skills, that is, multiple methods of developing topic sentences. A topic sentence may be developed by cause, consequence, example, analogy, contrast etc. Each particular method of development is an atomic skill competency: You can practice saying develop a topic sentence by consequence until you can do so quickly and without error. Contrastively, the entire paragraph writing, the development of the topic sentence, is a skill composed of multiple atomic skills. Writing a paragraph is a skill competency but not an atomic skill competency.

Of utmost importance is that atomic skill competencies need not be classified exclusively as memorization and performance, lower order skills. To fully understand this recall that our definition of higher order thinking skills exclusively requires the presence of executive function. A recent study [15] shows that memorization and performance when combined with executive function is higher order; in fact, multi-dimensional performance surprisingly improves intelligence.

In this study, number-letter pairs were flashed at three second intervals to an examinee who had to identify resemblances to pairs two trials earlier. For example, in the sequence #1) 2A, #2) 3B, #3) 4A, #4) 3A, #5) 4A, the examinee has to recall that trial #3 resembles trial #1 in the dimension of letter, trial #4) resembles trial #2 in the dimension of number, and trial #5 resembles trial #3 in the dimensions of number and letter. The examinees practiced these recalls, 20 trials at a time, each trial exposure being three seconds. Over a period of several sessions, performance increased and error rate decreased, the criteria for atomic skill competency. The surprising result was that the practice also increased general fluid intelligence. We attribute this to the multi-dimensionality of the recall, which required that two dimensions, number and letter be recalled. Such multi-dimensionality requires executive function and it is not surprising that executive function increases intelligence.

2.3 The Executive-Function Approach to Pedagogy
We are now in a position to give a full statement of our approach to pedagogy. The role of the instructor, or alternatively, the goal of instruction, is

a) To identify the atomic skill competencies of a domain of knowledge,

b) To provide exercises and other resources to enable mastery of these atomic skill competencies,

c) To present higher cognitive problems requiring choosing between, and combining of, multiple dimensions - each dimension consisting of a single atomic skill competency – to achieve desired solution outcomes.

Examples will be presented in the remaining sections of the paper. For the moment we note consistency and correlation of the executive-function approach with the Bloom-Anderson approach since performance of atomic skill competencies corresponds to the lower order Bloom-Anderson levels of knowledge and comprehension while the analysis of multi-dimensional problems corresponds to the higher order Bloom-Anderson levels of analysis (into component dimensions), synthesis and evaluation (determining which competing dimensions drive outcomes).

We have deepened the understanding of Bloom-Anderson by adding specificity, mechanistically identifying the terms synthesis, analysis and evaluation with multi-dimensional processing and by further identifying knowledge and comprehension with atomic skill competency.

3 EXAMPLES: MODELING

In the next four sections we illustrate application of the executive function approach of pedagogy to several disciplines. In this section, we apply the executive-function approach to modeling of verbal problems. Modeling is a key example of higher order thinking skills and is frequently mentioned in discussions of educational reform [24].

The key point to emphasize about modeling is that it requires a continual multi-dimensional processing of the two dimensions of algebra and language. This is illustrated in Figure 3.

<table>
<thead>
<tr>
<th>English</th>
<th>Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amy purchases</td>
<td>Math</td>
</tr>
<tr>
<td>4 Peanut bags</td>
<td>4P</td>
</tr>
<tr>
<td>And</td>
<td>+</td>
</tr>
<tr>
<td>1 orange juice quart</td>
<td>1O</td>
</tr>
<tr>
<td>For a total of</td>
<td>=</td>
</tr>
<tr>
<td>6 dollars</td>
<td>6</td>
</tr>
<tr>
<td>Bonnie purchases</td>
<td>Math</td>
</tr>
<tr>
<td>1 peanut bag</td>
<td>1P</td>
</tr>
<tr>
<td>And</td>
<td>+</td>
</tr>
<tr>
<td>4 orange juice quarts</td>
<td>4O</td>
</tr>
<tr>
<td>For a total of</td>
<td>=</td>
</tr>
<tr>
<td>9 dollars</td>
<td>9</td>
</tr>
<tr>
<td>How much does</td>
<td>Solve for</td>
</tr>
<tr>
<td>1 peanut bag</td>
<td>P</td>
</tr>
<tr>
<td>And 1 orange juice quart</td>
<td>Q</td>
</tr>
</tbody>
</table>

Cost?

Figure 3: Verbal modeling of a purchase problem with the two equations 4P+1Q=6, 1P+4Q=9. The table should be read both vertically (English, Math) and horizontally (English-Math correspondence).

<table>
<thead>
<tr>
<th>English Phrase</th>
<th>Mathematical Correspondent</th>
</tr>
</thead>
<tbody>
<tr>
<td>And</td>
<td>+</td>
</tr>
<tr>
<td>For a total of</td>
<td>=</td>
</tr>
<tr>
<td>Number followed by noun</td>
<td>Number x Noun symbol</td>
</tr>
</tbody>
</table>

Figure 4: Small list of verbal-algebraic skill competencies.

Figure 4 illustrates a small list of atomic skill competencies. I have found this approach – creating verbal-algebraic tables – extremely useful when teaching modeling to remedial students. Some would argue that I am replacing thinking with memorization of lists. But a deeper analysis shows this untrue. We have already cited results [15] that memorization can increase intelligence provided executive function is involved. Figure 4 illustrates such a memorization since it requires the two dimensions of language and mathematics. Figure 4 should be perceived as an exercise of executive function. I have seen, that after students successfully memorize Figure 4, they are more adept at new English-Math translation situations.

4 EXAMPLES: COLLEGE WRITING
There are a variety of approaches to textbooks on college writing. The Jones-Faulkner [17] textbook uses executive-function pedagogy. The book leaves grammar to an appendix!
The body of the book is organized into 3 major parts: sentences, paragraphs and essays. Each part is highly skill driven.

For example, there are 4 categories of sentence-pair types, each category having several subtypes. Typical examples of sentence-pair types are sentences connected by cause, consequence, contrast, analogy, illustrative lists, supporting data, etc. Consequently, the first part of the book is devoted to developing skill competencies on sentence pairs. A typical exercise might present several sentences and request creating a second sentence that is a consequence of it. The second part of the book develops skills in the five types of paragraphs. After mastering these skill competencies students are adequately prepared to write complex essays.

Figure 5 illustrates the approach of the book. It presents a paragraph and analyzes the sentence-pair relationships.

<table>
<thead>
<tr>
<th>Paragraph</th>
<th>Sentence Pair Relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Jim passed several actuarial exams. (2) He was immediately hired. (3a) His success was due to (3a) taking good prep courses and (3b) spending a lot of time studying and practicing. (4) Indeed, during his period of study he did not go to any parties.</td>
<td>#1 – Topic sentence. #1,#2 – Consequence #3,#1 – Cause #4,#3 – Supporting data #3a,3b – Parallel sentences #3b – List of items/examples</td>
</tr>
</tbody>
</table>

Figure 5: Illustrative paragraph and corresponding atomic skill competencies. For example, sentence #2, *Jim being hired*, is the consequence of sentence #1, *his passing actuarial exams*. Similarly, sentence #3a - *his taking good prep courses* – is the cause of sentence #1 – *his passing actuarial exams*.

5 EXAMPLE: ACTUARIAL MATHEMATICS

To illustrate the issues in teaching actuarial mathematics, consider the following problem:

5.1 An Illustrative Example

**Price, (that is ascertain how much money is needed in the bank)** an annuity that pays $5,000 at the end of year 1, $5,000 at the end of year 2, $5,000 at the end of year 3, $6,000 at the end of year 4, $7,000 at the end of year 5 and $8,000 for year 7.

Executive-function pedagogy requires identification of the atomic skill competencies. There are 3 basic annuities which a student must learn to price: level, increasing, decreasing.

- **Level**: A level annuity makes a level payment of $x at the end of every year for n consecutive years.
- **Increasing**: An annuity which pays $x in the first year, $2x in the second year, $3x in the third year etc.
- **Decreasing**: An annuity which pays $nx in the first year, $6(n-1)x in the 2nd year, etc. until paying $x in the n-th year.

Students are taught to recognize these basic three annuities, to calculate their purchase price, as well as the symbols and formula associated with them. These are atomic skill competencies since with sufficient practice students can do any of the three basic annuities i) quickly and ii) without error.

The major part of my teaching is devoted to multi-dimensional analysis of verbal problems, such as the problem introduced at the beginning of this section. One possible multi-dimensional analysis of the problem at the beginning of this section is presented in Figure 6. As can be seen, the sum of two of the three basic annuities – level and increasing – results in the desired payment.

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired payment</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>6000</td>
<td>7000</td>
<td>8000</td>
<td></td>
</tr>
<tr>
<td>Level annuity</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>Increasing annuity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1000</td>
<td>2000</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>Sum of level and increasing annuity</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>6000</td>
<td>7000</td>
<td>8000</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6: Analysis of the desired payments (row (2)) into 2 basic components, a level annuity(row(3)) and an increasing annuity (row(4)), whose sum (row(5)) is the desired payment.

In a certain sense, my classroom is flipped: I don’t spend time on formulae but rather spend time doing analysis and ask the students, after a brief introduction, to learn and practice formulae on their own.

5.2 Comparisons with other textbooks

**Kellison edition 2** [18]: a highly respected book that had no competitors for over 10 years, teaches the three basic annuities in the body of the text but only presents multi-dimensional problems such as the example presented in section 5.1, in the exercise section. This approach is used by many textbooks: minimal skills are presented in the text body but analysis and synthesis in the exercises. We advocate the reverse: the majority of textbook and class time should be spent on analysis.

Daniel-Valeer [27]: This textbook presents in the text body both multi-dimensional and atomic skill examples. However, this textbook ingeniously creates a new formula – a new atomic skill competency – that can “solve” two-dimensional problems. Thus the multi-dimensional problems are solved with a formula and hence they lose their multi-dimensional challenge.

In summary, it is possible to base an actuarial course on executive function. However, many textbooks prefer to opt the easy way out by leaving students to think on their own or by substituting formulae for thinking.

6 LITERARY CRITICISM AND FOREIGN LANGUAGE

6.1 Overview on Rashi

The material in this section is based on the biblical commentary of Rashi. Rashi is an acronym for Rabbi Isaac Solomon. Although many scholars in several civilizations had commented on the Bible [3], Rashi, a French medieval commentator of the 11th century, was the first commentator to comment both on general literary issues as well as individual words and phrases [5,21].

The derivation of each Rashi comment from the Biblical text has been the subject of much research by many people over several centuries. In this paper, we focus on the fact that rules governing Rashi derivations can be organized into ten categories each of which is composed of atomic skill competencies involving executive function [23].
6.2 Rashi and Executive Function

We have already identified executive function with multidimensionality. We can organize the Rashi rules according to their multi-dimensionality as presented in Figure 7.

- Two Dimensionality in
  - Words – meaning, grammar
  - Phrases – reference, parallelism, contradiction
  - Sentences (Broad/literal interpretation)
  - Paragraph structure (e.g. climax)
  - Languages
- Multi (more than 2)-Dimensionality
  - Database methods
  - Symbolism

Figure 7: The 10 categories of Rashi rules (italicized) organized by dimensionality and essay units (words, phrases, sentences).

6.3 Examples

Hyponymy [32]: Translation is typically thought of as a non-executive function activity. But in Section 2.1 we showed that even a mundane exercise like alphabet recitation can be transformed into executive function quality if it becomes multidimensional. And indeed, Rashi typically comments on word-pairs involving several dimensions.

Rashi, in his commentary on the grain offerings described in Leviticus 2, explains two Hebrew words describing the cooking utensils, pot and frying pan: “A frying pan is flattened while a pot has height.” Frying pans and pots are hyponyms of the hypernym cooking utensil. By translating hyponym pairs, Rashi introduces multi-dimensionality since meaning is based on the two dimensions of function (cooking) and form (height).

Hononymity: On the verse (Gen. 42:23), They were unaware that Joseph heard since he had used a translator. Rashi states that “In this sentence, heard means understand. In other words hear hononymically can refer to listening or understanding. Here again Rashi introduces multi-dimensionality since meaning is based on the two dimensions of function (understanding) and form (listening).

Grammar: Rashi also used word pairs to illustrate grammar. On the verse (Ex. 19:18) And mount Sinai was fully smoked Rashi comments Smoke is a noun and verb while smoked is an adjective. Here, Rashi introduces multi-dimensionality since meaning is based on the two dimensions of grammatical function (noun-verb) and spelling (smoke vs. smoked).

Grammar: The text of Num. 12:1 when translated properly reads, Miryam and Aaron speak against Moses. Note the grammatical anomaly indicated by the underlined s; proper English is either Miryam speaks or Miryam and Aaron speak but not Miryam and Aaron speaks.

Rashi resolves this contradiction, between a plural subject and singular predicate, by stating Miryam initiated the conversation while Aaron only echoed and participated Miryam’s requests. Here Rashi introduces multi-dimensionality by modifying the dimension of plural dialogue with a dimension of intensity of plurality (mutual dialogue, or initiated-echo dialogue).

Reference: On the innocuous looking verse (Deut. 26:5) Jacob went down to Egypt with a few people Rashi references Gen. 46 which lists the 70 people that went down to Egypt. Hence, the terse but executive-function comment: Few means 70. Here, Rashi introduces multi-dimensionality by merging the implications of two separate texts.

Parallelism: Unlike the reference method where the two texts have few words in common (the reference is based on meaning), in the parallelism method the two texts are almost identical; minor differences between the two texts illuminates meaning. Here multi-dimensionality is achieved through simultaneous awareness of two phrases. Figure 8 is illustrative.

(In the Messianic era) One washes clothing in wine and Suth in blood of grapes.

Figure 8: Parallel structure of Gen. 49:11.

The parallelism shows wine parallel with blood of grapes and clothing with suth. Hence the terse Rashi comment: Suth refers to a type of clothing.

Contradiction: Num..8:24 and Num. 4:2 discuss the age when Levites start working in the Temple: One verse says they start at age 25, while the other states they start at 30. Rashi resolves the contradiction: They start a 5 year training program at 25 and upon completion at 30, start actual Temple service. Again, Rashi introduces multi-dimensionality by focusing on multiple texts and multiple dimensions of Temple service.

Two languages: Rashi explains the peculiar biblical word totafoth: It means 4 since path in African means 2 and Tot in Caspian means 2. [The word totafoth refers to the 4-chambered Tefillin ornament worn by religious people]. Here Rashi introduces multi-dimensionality since multiple languages are being used to explain meaning.

Database: Databases definitionally study relations across multiple dimensions. A rather beautiful example is found in the Rashi commentary on the multi-colored Leviticus 10, a biblical chapter describing i) the death (by God) of Aaron’s two sons who impetuously offered an improper sacrifice, ii) Aaron, the High Priest’s silence and acceptance of his sons’ death, and iii) the Divine command to Aaron prohibiting drunk priests serving.

Rashi performs a simple database inquiry: how are biblical paragraphs introduced: Several dozen biblical paragraphs of God’s laws begin with the introductory sentence “God spoke to Moses to say over”, while only two paragraphs begin with the introductory sentence “God spoke to Aaron, to say over.” This anomaly suggests the following reasonable sequence of events: Aaron’s two sons were drunk; they therefore thought they were as good as their father and could offer anything he could offer. When they did so, they died at the hand of God. As a reward for Aaron’s acceptance of God’s will, evidenced by his silence, he was rewarded with teaching the commandment that priests should not serve drunk.

7. REFERENCES


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