Logic as a Key to Interdisciplinary Integration for Students in the Mathematical Sciences

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Abstract. *We describe the creation and development of a course on mathematical logic and its extensions and limitations, in which coverage of technical material is interleaved with and related to discussion of relevant historical, linguistic, philosophical, and theological issues and of individuals of note.*

The new course, Logic, Limitations to Knowledge, and Christianity, presents an overview of topics in and related to logic, including development of formal logic and an axiomatic first-order logic. It explores the history of mathematics and logic in the Catholic Intellectual and wider Western Traditions, as well as the mutual interactions of mathematics, philosophy, language, and religion. It then considers extensions of first-order logic, and provable limits to knowledge: the three unsolvable problems of Euclidean geometry, and examples from Gödel, Turing, Arrow, quantum physics, and others. Epistemological issues will be emphasized throughout the course. The translation between natural language and expression in logical and reasoning formalisms is emphasized throughout.

As a Core Curriculum course at Seton Hall University, fundamental questions such as "What is logic?" and "What are its limits?" will be considered within the framework of Christianity's broader view of the human person and human intelligence.

Keywords. Logic, epistemology, history, reasoning, unsolvable problems, interdisciplinary, Catholic Intellectual Tradition

1. INTRODUCTION

Students in undergraduate programs in the mathematical sciences (mathematics, statistics, computer science) and allied fields (information science, data science, actuarial science, operations research) can expect to take multiple technical courses throughout the curriculum. At the same time, though depending somewhat on the university and its structure, they will be required to take a significant number of general education courses in the humanities, social sciences, science, and possibly the arts, business, or engineering.

Nonetheless, for most students, the two areas do not appear integrated, even with increasing emphasis on presentation and on ethics across the curriculum, and although courses late in the curriculum such as software engineering and data visualization (and some courses in mathematical modeling) deal with social and aesthetic issues.

Further, even for the students who enjoy technical material and who expect to pursue interesting and remunerative careers in these areas, many of their courses in the mathematical sciences appear dry, procedurally driven, and unconnected to either the rest of their education or the real world.

Logic is often seen as one of these courses. where students who do not take logic as a separate course in mathematics or philosophy will encounter it as a unit or thread in discrete mathematics, computer architecture, or databases or data science.

As Dr. Jeremy Horne has often observed [10], logic as presented as philosophy is often taught as a dry subject, taught in an introductory course for non-technical students as a formal exercise with examples that appear to be more riddles than results of any use, redeemed only by an amusing coverage of informal fallacies, or alternatively in a course in symbolic logic for technical students and philosophy majors, as an even more formal algebraic exercise, with applications left largely to the student's later experiences in philosophy, mathematics, computer science, or another discipline.

On the other hand, when a student is introduced to logic in courses in the formal/mathematical or natural sciences, it is most often presented as a tool for proof or for explaining the operation of computer programs. Regardless, logic is most often tied neither to practical issues, nor to the rest of philosophy, nor for that matter to its history and its importance in the Christian, Islamic, and broader Western intellectual traditions.

Dr. Horne continues:

> In many fields, especially technical ones, there are two worlds and corresponding forms of presentation: *episteme* and *techne*, theory and practice, science and technology, the "why" and the "what", the Greek Way and the Roman Way, etc. [… E]ach of these exists because of the other, the *episteme* induction with the deductive *techne*. My 'bone to pick' is that certain profound and critical philosophies have been lacking in foundational courses, thinking about origin, existence, process, and overall, what logic does – describe the foundations of our universe.

And later:

> Logic, then, stands to be a subject through which students can explore the nature of order, or structure, and about its meaning, as in whether reality is digital or analog. What is a closed versus open system? Actually, is there really something really closed, or does one perforce have to draw

on inductive methods to create a system? There is the nagging question about boundaries, ultimately leading one to consider that limits are what are imposed on a situation by humans.

The lack of integration of knowledge was of concern to Alexander von Humboldt, the 19th century scientist and polymath, even at the beginning of the differentiation and siloing of scientific disciplines [25].

> In September 1828, [Humboldt] invited hundreds of scientists from across Germany and Europe to attend a conference in Berlin. Unlike previous such meetings, at which scientists had endlessly presented papers about their own work, Humboldt put together a very different programme. Rather than being talked *at*, he wanted the scientists to talk *with* each other. ... He connected the visiting scientists on a more personal level, ensuring that they forged friendships that would foster close networks. He envisaged an interdisciplinary brotherhood of scientists who would exchange and share knowledge. “Without a diversity of opinion, the discovery of truth is impossible,” he reminded them in his opening speech. [punctuation slightly modified for clarity]

In this paper, we describe a second course in mathematical logic that addresses these limitations and attempts at least a partial integration, and describes how the course came to be developed. The course interweaves history and philosophy with logic and natural language understanding, and describes the development of logic in the Classical world and the Islamic Golden Age, and then interaction of logic with the Catholic Intellectual Tradition (CIT) from the medieval period to the present day. It also considers extensions of logic (temporal and other modal logics, non-monotonic logic, fuzzy logic, and probability-based reasoning), followed by consideration of its limitations, from the three classical unsolvable Euclidean problems through medieval *insolubilia* [4] to Gödel and Turing, with side excursions on Heisenberg [9] and Kenneth Arrow [18].

In the process, the course explores connections with the development of science and engineering, the history of Western philosophy and of mathematics, mathematical structures and systems, language and meaning, cognitive science, and data science, and frequently touches on the mathematical and logical foundations of computer science and artificial intelligence.

With such a large scope, in content, viewpoint and historical span, the intent is not to ask students to memorize or perfectly master the course material, but rather to involve students (and instructors!) in a learning experience that not only immerses them in logic, but emphasizes understanding of context, concept formation, disciplinary and viewpoint integration, intellectual exploration, and critical thinking.

This article is an expansion of [17]. The rest of the paper is organized as follows: In Section 2, we provide a context for Seton Hall and its University Core Curriculum, including the need for an advanced course in the Catholic Intellectual Tradition relevant to a student’s major, and in Section 3, describe the process of finding a course in mathematics that would satisfy the requirements of the program. Then in Section 4, we describe the structure of the course and in Section 5, its course paper requirement. Then, in Section 6, we describe our experience with the course, and in Section 7, look at future directions. Finally, in Section 8, we present a summary and conclusions.

2. THE CATHOLIC INTELLECTUAL TRADITION — SETON HALL’S UNIVERSITY CORE CURRICULUM

Founded in 1856, Seton Hall University is a diocesan university, a Catholic institution of higher education associated with the Archdiocese of Newark, rather than with a religious order such as the Jesuits. It sees itself as a liberal arts institution, offering a wide range of undergraduate programs (other than engineering) and master’s degrees, together with a smaller number of doctoral programs. The university comprises seven schools and colleges—Arts & Sciences, Business, Education, Nursing, Diplomacy, Communication and the Arts, and the Immaculate Conception Seminary School of Theology, all offering both undergraduate and graduate degree and certificate programs, plus a School of Continuing Education, offering only certificates [21].

In the period 1980-2005, a series of sets of general education requirements (“Core Curricula”) for the University’s undergraduate colleges gave rise to diverse standards in the different schools—sometimes multiple core curricula within a school. Some of these differences were inevitable—different mathematics and science requirements in the College of Arts & Sciences for BS versus BA degree students, or a business orientation in some of the courses required in the School of Business—while others might be harder to justify.

Among these differences, these core curricula have differed greatly in the emphasis placed on the traditional humanities, and in particular on theology, philosophy, and the Catholic (or more widely, Western and Christian) intellectual tradition. Further, pressures within disciplines and schools, a desire to allow students more flexibility in complementing their academic major, and a change in credits for the degree from 130 to 120, led specifically to decreases in required core curriculum credits in philosophy and religion, as well as implicit constraints on the size of an academic program.

To many in both the administration and faculty, the diversity in standards, the concomitant drift from a shared intellectual experience, and what has seemed to many to be a de-emphasis on parts of the University’s mission [20], was a concern. This gave rise in the period 2005-2008 to a move for a uniform University Core Curriculum [21], to be taken by all students, which was adopted in 2008 and initiated in Fall 2009 with that year’s entering class.

The University Core Curriculum [19] consists of courses in three groups, together with proficiency requirements in Numeracy, Reading and Writing, Oral Communication, Critical Thinking, and Information Fluency, and a requirement that all students have a culminating experience in their major program. The first group of courses comprises three courses already required of every freshman: University Life (a 1-credit orientation and skills course), and two College English writing courses, the second of which focused on writing a research paper. The second group consisted of two courses in the CIT, broadly understood: The Journey of Transformation, and Christianity & Culture in Dialogue, to be taken in the Freshman and Sophomore years, respectively. The first focuses on forging an identity in the world and as a moral being, through an invitation to explore "the transcendent mysteries of the human journey that are addressed by the world's religions, philosophies, art, music and literature" [21]. The second considers the relationship between Christianity and culture through an approach based on principles of dialogue, development, and community [21]," specifically including science as an aspect of culture, as well as a source of knowledge.

These courses are not organized or executed as traditional catechetical classes. They seek to engage students of any or no religious belief and encourage them to advance in their personal journey of self-transcendence and the quest for meaning very much in the spirit of the late Austrian neurologist and psychiatrist, Viktor Frankl, MD, PhD [6] Each includes readings, not only from Catholic authors over the centuries, but also from Classical authors including Aristotle and Plato, from other faith traditions, and from philosophers, social scientists, and scientists—together with excerpts from literature and illustrations from the arts—from the medieval period to the present day, as well as commentaries on these readings.

3. THE HUNT FOR A MATHEMATICS COURSE FOR THE UNIVERSITY CORE

The third component (CORE III) is a course intended to integrate concepts and questions from the two courses of the second component with a student's own major or other area of interest. According to University Core Curriculum guidelines, the course should satisfy several requirements:

- It should be taken by juniors (or seniors), requiring only The Journey of Transformation and Christianity & Culture in Dialogue as prerequisites (and transitively, College English I & II).
- It should have a substantial emphasis on the concepts and questions from those courses, both in course materials and in assessment, and require readings and a paper related to that tradition.
- It should have serious intellectual content in the discipline(s) offering it, and it count toward the major program in which it is offered.

It quickly proved straightforward to create multiple courses in each department in the humanities, the social sciences, diplomacy, education, and the arts, and to develop one course each in business and nursing. Further, the University's interdisciplinary Program in Catholic Studies created and offers several such courses, most cross-listed with one or more other departments.

However, it proved somewhat more difficult to create these courses in the sciences. In the natural and physical sciences, ecology, the history and development of Western science, the neuropsychology of religion, and integrating scientific and religious views of creation provide grist for appropriate courses. Finally, integrating robotics, artificial intelligence and cognitive science led to a course, Robotics and the Human Mind, co-taught in computer science and psychology, and including a programming component, that neatly fits the CORE III requirements. A more comprehensive overview of the Catholic Intellectual Tradition component and its rationale, of CORE III requirements and course offerings, and in particular, of CORE III courses in STEM disciplines can be found in [13]; a full list of courses approved as of Spring 2016 can be found in [21].

In mathematics, however, this task proved to be much more difficult, and finding an appropriate mathematics course faced three challenges. First, mathematics curricula are almost inherently hierarchical, and a junior-level course without mathematical prerequisites seemed hard to justify. Second, modern mathematics is largely a creation of the 17^{th} through 20^{th} centuries and does not directly dealt with issues of significant theological or religious concern. In fact, the Department of Mathematics and Computer Science offers its own History of Mathematics course, beginning with Egyptians and Babylonians, through the development of modern mathematics with Newton and Leibniz into the 19^{th} century, and ideally up to the middle of the last century. In this course, the emphasis is on the development of mathematical concepts and notation, and religious questions and disputes are hardly if ever mentioned. Thus, third, it would seem difficult to find an area of mathematics that could be naturally and easily related to the questions of the CIT.

(There are rare exceptions, such as the Infinitesimal controversy [1]. These, however, seem to reduce either to Scholastic vs. Modernist disputes of little if any real theological import. Interestingly, in the Infinitesimal controversy, there was mathematical merit on both sides. The use of indivisibles and infinitesimals by the Modernists anticipated calculus and had applications to engineering and architectural applications. But the Scholastics were correct in that the approach did not have a firm foundation, which had to wait for Newton and Leibniz—or arguably Cauchy in the 19^{th} century or even Abraham Robinson in the 20^{th}.)

Years of discussion with faculty in the Department of Mathematics and Computer Science considered a number of possibilities, including modifying the History of Mathematics course, looking at mathematical models in the social and behavioral science, creating an interdisciplinary course in data science, or exploring the philosophy of mathematics. But each of these ran up against at least one of the above objections. In addition, either the course would be tangential to an undergraduate mathematics major or important and standard material would have to be omitted. (Interestingly, the resulting course would include content from each of these four areas.)

The solution to this dilemma arose organically when the first author began discussions with Dr. Horne, Dr. Nagib Callaos [5], and others about logic, reasoning, unsolvable problems, and the need for logic to be placed in a wider context. Simultaneously, the second author was teaching courses in the Department of Mathematics and Computer Science, the Department of Systematic Theology, and the Department of Catholic Studies.

Still one more impetus was a request to the first author from a colleague at William Paterson University for a presentation to his Interdisciplinary Honors Seminar; the presentation on limits to knowledge in science and mathematics, much of which informed the discussion and the course.

The first realization was that logic offered an opportunity to interweave history, philosophy and theology, since Aristotelean and later, other logics have had a long and substantial influence on Catholic and other Christian theologians and philosophers, since before St. Thomas Aquinas and through the present day, and conversely, Catholic thinkers, including clergy, have contributed to its development in Europe since the 11th century.

Second, we noted that, whereas all junior mathematics and computer science majors will have seen logic in discrete mathematics or computer architecture, and many other students will have taken a philosophy course in Logic or Symbolic Logic, there were no advanced courses in logic. This presented an opportunity for an advanced course in a major area in mathematics in which the mathematical prerequisites could remain implicit.

Finally, we realized that an advanced course in logic was completely suitable for an undergraduate major in the mathematical science, but would not have a specified content that precluded integration of other topics. It emerged there was an enormous body of intellectual content beyond simple propositional and predicate logic in the mathematical sciences: in mathematics, sets, relations and functions, plus axiom systems and proofs in linear and abstract algebra; in computer science, reasoning and natural language understanding in artificial intelligence, undecidability and incomputability in algorithms, and the use of logic in databases and in data science, as well as the use of temporal and modal logics and abstraction in the analysis of program specification and design, and of the correctness of computer programs. Logic also led to consideration of language, grammar, and meaning, impacting not only communication, but also relates to programming language and compilation, and once more to artificial intelligence and aspects data science.

Moreover, both directly and through mathematical modeling, computing, and more recently data science, logic and its extensions have had a major impact on the sciences, social sciences including economics, and even the humanities.

Further, setting the course in the context of logic and philosophy would allow discussion of the nature of mathematics, logic and science, and their relationship to language, philosophy and reality, and their interaction with theology. This provided not only the disciplinary content we needed, but also opportunities to relate the course material to the experience of the students, and to encourage the internalization and integration of concepts from the course.

4. THE COURSE: INGREDIENTS AND STRUCTURE

As a result of these inputs and discussions, the Department of Mathematics and Computer Science created a course, Logic, the Limits to Knowledge, and Christianity, cross-listed as a CORE III course, a Mathematics course, and a Computer Science course. As required, the course counts as a major elective in either major program, and will also count as an elective toward a major, minor or certificate in Catholic Studies.

The course begins with a parallel consideration of syllogistic, propositional and predicate logic, together with translation between logic and natural language, enhanced with historical, philosophical, and Catholic theological aspects surrounding its development and elaboration, from Aristotle through the Islamic Golden Age and the Medieval Scholastics to the Enlightenment. Examples are drawn from mathematics, computer science and other areas, as well as from historical sources. Rather than using a specific set of axioms and rules of inference, the course considers the role of and options for axioms, rules of inference and proof techniques, including for example, arithmetic via the Peano Axioms and the resolution method for predicate logic. The issue of inductive reasoning versus deductive logic is also treated through consideration of al-Ghazali, ibn-Sina (Avicenna), Roger Bacon, René Descartes, David Hume and other historical figures.

Thereafter, the course examines extensions of logic, beginning with temporal and other modal logics, followed by consideration of various approaches to non-monotonic logic, and then probability-based reasoning and fuzzy logic. This is interleaved with consideration of later historical developments in the development of logic and its application to Catholic (or more generally Christian) theology, including, for example, Boole and Dodgson on the one hand, and the Neo-Thomists

[12] and Fr. Bernard Lonergan, SJ [14, 15] in particular on the other. This is followed by a very brief consideration of relevance logic [24], which attempts to address the "nonsense" accepted as true in strict, material logic, such as "If 2 + 2 = 5, then Hillary Clinton is President of the USA."

As its last major topic, the course considers inherent limits to knowledge and to mathematical reasoning, picking up from the medieval *insolubilia*. Consideration begins with the three classical unsolved (and unsolvable) problems of Euclidean geometry [2]: doubling the cube, trisecting the angle, and squaring the circle. After considering the view of mathematics culminating in Hilbert's Twenty-Three Problems [8], it considers the work of Gödel and Turing, and some of its implications, including Rice's Theorem [3]. Finally, it looks at some practical unsolvable problems in the wider world: the Heisenberg Uncertainty Principle, demonstrating the impossibility of knowing precisely both the position and the momentum of a particle, chaos, information theory, and Arrow's Impossibility Theorem, which shows that a fair voting method is impossible, or alternatively that it is impossible to find "the greatest good for the greatest number". This is complemented by a revisiting of Lonergan, and in particular his principle of reflection, a brief look at second-order cybernetics [7], a discussion of philosophies of mathematics, and consideration of the epistemological and ontological consequences.

These last two sections are accompanied by examples and motivation from artificial intelligence, including representation of context and implicit knowledge; parallels to certain mathematical constructs; semantics and analysis of computer programs and computer systems; cybernetic systems;; and use in formulating and discussing philosophical and theological issues, together with discussion of and readings from Fr. Stanley Jaki, OSB (and former Seton Hall Distinguished University Professor in Physics) and others [11]. Table 1 gives an approximate schedule of presentations.

Although the course is primarily intended as a theoretical and philosophical overview, it managed to touch on a surprising number of connections and applications.

Some of these connections are in STEM areas: in science and engineering, the structure of science and scientific experiment, process control systems, medical expert systems, and the idea of scientific progress and theory revision; and in mathematics, the concept of mathematics as the science of structure and pattern, including the concepts of generalization, specialization, and categorical differentiation, the concept of mathematical proof, and recurrence and recursion.

Others related to computer science: directly in logical models for programing and for verifying properties of programs, object-oriented and database design, logic in computer architecture, recursive axiom schemes as a basis for grammars, models for protocols in operating systems, networks, and security, and cellular automata, both as a model and as an application; much of the content of artificial intelligence and expert systems, especially including knowledge representation and natural language translation; and in data systems, many of the basic models for data mining and data analysis.

Table 1. Presentations and Discussions

- Overview of propositional and predicate logic
- Aristotle's logic
- Translating between natural language and logic
- The Islamic tradition: transmission and extension
- Medieval science and logic
- Renaissance studies and Descartes
- Modern mathematics & the infinitesimal controversy
- The scientific method: philosophy, structure, variations
- First-order logic and proofs
- Axiom, definition, theorem & proof
  - Proofs by resolution
  - Algebraic systems
  - Introduction to the Peano Axioms
- The Enlightenment, the transition to mathematical logic, and the Neo-Thomists
- Temporal and modal logics
- Temporal logic for modeling and analyzing computer programs
- Logic, mathematics & the philosophy of Lonergan
- Modal logics in philosophy & theology
- Non-monotonic reasoning and artificial intelligence
- Natural language, context and understanding, and representation of implicit knowledge
- Probabilistic reasoning and fuzzy logic
- Gödel, Turing, and the limits to knowledge
- Impossible problems: Euclid, Arrow, and more
- Stanley Jaki, Pope Benedict, and the integration of modern science with theology
- Heisenberg and the progress of science
- Chaos theory, cybernetics, and modern physics
- Cellular automata and self-organizing systems
- Philosophies of mathematics
- Reflection: Observations from Lonergan, software engineering, and second-order cybernetics.
- Epistemological and ontological consequences

The next most obvious are connections to philosophy and theology: not only the direct connections to logic, but questions related to ethics and morality; epistemology, cognitive science, and learning; essence, existence, and accidents (or, somewhat analogously, class, entity and attributes); the structure of language and communication; and the limits to knowledge. The existence and nature of God and the coherence of Catholic theology are also addressed in the readings.

Still others, however, lay in other areas. These included: aspects of legal philosophy, especially for Islamic law and equity, with applications to business ethics; social science and social justice issues related to modal logics, as well as the consequences of Arrow's Theorem and related results for politics and economics; the history of science and the idea of progress; and even the use of logic and mathematics in literature, specifically including the works of Lewis Carroll (Charles Lutwidge Dodgson).

Throughout, the goal is less to have students memorize or compute than to conceptualize and integrate, to understand the development of the mathematical sciences and their interrelation with history, philosophy and reasoning in general, and to have them think about deeper issues in language understanding and comprehension of the external world, as well as connections to the Catholic Intellectual Tradition.

5. COURSE EVALUATION AND THE COURSE PAPER/PROJECT

Other than a major course paper (with associated artifacts), the course emphasizes translation to and from logical notation (in various logical formalisms), connections of the material presented to material in the student's major program, reflections on readings of source materials, and a few proof exercises.

Table 2: Readings and Assignments

- Reflection on brief readings from Aristotle & Aquinas
- Translation between natural language and first-order propositional logic
- Reflection on Descartes and the Neo-Thomist integration of science and theology/philosophy
- Short proofs using resolution and (separately) the Peano axioms
- Translation between natural language and temporal and modal logics
- Reflection on Lonergan, the philosophy of modern science and its connections to theology
- Reflection on the implications of the limits of knowledge on philosophy, theology, and knowledge itself

The course project artifacts include an annotated bibliography, a major paper, and a presentation. The paper must arise from (but may extend) material covered or discussed in the course, and must relate to the CIT, on a topic approved by the instructors. The recommended areas for the paper are: (1) the interaction of the CIT and the development of logic and its extensions; (2) a philosophical topic or discipline (other than logic itself) considered from both the perspective of the course and the CIT, and (3) investigation of a major modern area of interest within the CIT, considered in light of the content and approaches in the course, but students may propose topics from other perspectives, as should be evident in Table 3. The paper may place the discussion in the context of the wider Western Intellectual Tradition, and/or extend to consideration of other cultures, traditions, and intellectual disciplines.

We have developed a list of sample topics and a fairly large bibliography, and are available for consultation with students in selecting a topic and resolving any problems during the process.

6. OUR EXPERIENCE

This past semester, the two authors team-taught a course to 21 students, roughly half mathematics and half computer science majors, with two mathematics minors, majoring in marketing and classical studies, respectively. Several were also pursuing a credential in data science.

Assignments have included reflections on readings and on the connections between the course emphasis and their own majors, natural language translation, and proofs. A sample of readings and assignments is presented in Table 2.

While having two very different faculty perspectives at (almost) every class has proved to be a real plus, there have proved to be a few cases in which presentations will have to be reordered, integrated, or revised, and in a few cases, extended, when the course is offered again. In addition, while we at first worried that we would have difficulty in holding to our schedule and covering all the material, we were able to keep to our schedule and, as mentioned above, even add additional material.

Table 3: Student Paper Topics

- Developments in AI: Moral Considerations
- Consciousness of AI: Logical and CIT Perspectives
- AI and the CIT: Some Moral Questions
- AI, Human Intelligence, and the CIT
- Philosophy of Mathematics
- Compatibility of Religion and Science
- The Link between Logic & Religious Discourse
- Mathematics and Theology
- The Islamic Tradition: Consistency with the CIT
- Catholic vs Islamic Culture: Approaches to Logic
- The Jesuit mission to China: Science & Theology
- Comparison of Arguments for God's Existence
- Limits of Logic and Reason for God's Existence
- Does the Universe Exist? Is it Comprehensible?
- Theodicy: The Logical Problem of Evil
- Fuzzy Logic & the Resolution of Moral Questions
- Capitalism, the CIT, and Deontic Logic
- Robotics, Medicine, and Ethics
- Catholic Education, Logic, and STEM
- Stop-word Analysis for Biblical Understanding

Students have been delighted with the class, although providing several helpful suggestions for the future. They would appreciate more reading assignments, more frequent evaluation, and discussions based on those readings, together with even earlier deadlines on initial drafts of the annotated bibliography and the paper—one of the few occasions in our experience when students have actually asked for more work.

The students especially appreciated the course as a capstone, giving perspective and making sense of essentially their entire general education requirements and the University Core, and tying these together with their studies in the mathematical sciences. Students learned a lot about the theory and applications of logic, and the difficulties of precise representation of language and reasoning. For both Catholic and non-Catholic students, the course deepened their understanding and appreciation of the Catholic Intellectual Tradition. Notably, students very much appreciated the differing perspectives of the instructors and the dialogue between them—both instructors and perspectives.

For all the unanimity in evaluation of the course, students found different aspects intriguing or provocative. In some cases, this was due to familiarity, in others to surprise, and in still others, to an opportunity to explore a tangential interest after being afforded a context. The range of students' interests can be seen in the summary of student paper topics presented in Table 3.

The course has been demanding of the instructors, especially in this first offering, as we race to elaborate topics, and then work to place them in a wider context. But it has also been extremely satisfying. The interaction with a fellow instructor, the enthusiasm of the students, and the fascination of the material, made both of us look forward to every class, and to make additional connections as the semester wore on. We expect such activities to continue, if at a reduced pace, in future semesters.

7. FUTURE DIRECTIONS

The course has gone even better than we dared hope. Prior exposure—every student had some prior exposure to logic in mathematics, computer science, data science, or philosophy courses—undoubtedly helped, but somewhat to our surprise, not only through familiarity with notation and application. Based on feedback from more than one student, the course offered a way to tie that notation to both an historical and philosophical context and expression in natural language, and to see the use of logic in the mathematical sciences as part of a much broader effort to obtain intellectual certainty as a basis for understanding the two worlds of ideas and of reality. Both instructors and students agree that this course is a valuable and important enhancement to their education in the mathematical sciences. Moreover, a number of department alumni and others have expressed a wish that they had had the opportunity to take such a course.

Nonetheless, any course can always be improved. In researching medieval logic, we realized the need to cover the medieval Islamic tradition and the transmission of both classical and Islamic learning to the West [23]. This has been provided as a supplement this time, but was used in several student papers, and will be integrated into the course the next time the course is offered.

We also will modify presentations, integrating some material now in separate presentations, moving some material from one presentation to another, and in a few cases changing the order of presentations. Likewise, we will modify some assignments and add others. In at least one case, we expect to reorder two assignments, so students can use lessons learned in what is currently the latter to aid in completing the former. We will also determine what changes have to be made to the research paper assignment timing, guidelines, and required artifacts.

Finally, we already have a lengthy bibliography, but will be adding to it both works we consulted in developing course materials and assignments, and selected works based on student annotations.

8. CONCLUSIONS

The University Core and its Core III requirement has created an opportunity in STEM education that allows a student to build on and integrate their disciplinary knowledge, general education, and a focus developed in specified previous courses. After some discussion and effort, a course created for mathematics and computer science majors, Logic, the Limits to Knowledge, and Christianity, aimed at Mathematics and Computer Science students, has been a spectacular instance of such a course.

The result, from the authors' experience, is a deeper student comprehension of their own disciplines and the role of their general education, better understanding of the role and purpose of technical language and its meanings, improved ability to learn, think, integrate, and present, and more awareness of some of the great philosophical issues and questions, and to deepen students' understanding and appreciation of the Catholic Intellectual Tradition.

While the program originated as a way of including a required exposure to the Catholic Intellectual Tradition for all Seton Hall University undergraduate students, it has also offered, in STEM, at least, an opportunity to move instruction and understanding from STEM to STEAM. One can expect, based on student feedback and instructor observation, this integration of disciplines to result in better modeling, design, and solution of problems in the formal, natural, and health sciences, embracing as appropriate philosophical and social concerns, such as ethics and issues of privacy.

In addition to its impact on students, collaboration in the development or teaching of these courses have created or enriched faculty and department connections, and have led to collaborative research and publications. We will consider use in the future of additional faculty guest speakers or discussants to increase and enhance these cross-disciplinary connections.

C. P. Snow, concerned about the siloing of academia, and the division between scientists and literary intellectuals, wrote:

> I believe the intellectual life of the whole of western society is increasingly being split into two polar groups. When I say the intellectual life, I mean to include also a large part of our practical life, because I should be the last person to suggest the two can at the deepest level be distinguished. I shall come back to the practical life a little later. Two polar groups: at one pole we have the literary intellectuals, who incidentally while no one was looking took to referring to themselves as 'intellectuals' as though there were no others. I remember G. H. Hardy once remarking to me in mild puzzlement, sometime in the 1930's: 'Have you noticed how the word "intellectual" is used nowadays? There seems to be a new definition which certainly doesn't include Rutherford or Eddington or Dirac or Adrian or me. It does seem rather odd, don't y' know.' Literary intellectuals at one pole—at the other scientists, and as the most representative, the physical scientists. Between the two a gulf of mutual incomprehension-sometimes (particularly among the young) hostility and dislike, but most of all lack of understanding. They have a curious distorted image of each other. Their attitudes are so different that, even on the level of emotion, they can't find much common ground. [22]

Through its influence on both students and faculty, this course, and other Core III courses, can begin to address Snow's concerns of an increasing gap, between (modernizing the discussion) the humanities and social sciences on the one hand, and STEM fields on the other, and to bridge "the gap of mutual incomprehension", at least by sensitizing STEM students to connections with philosophy and theology, the other humanities, and the social sciences including economics. In the process, a possible new intellectual and academic gap, between researchers in and practitioners of the mathematical sciences [16], may be in part ameliorated or avoided through broader understanding by the mathematical scientists.

As with many other institutions, Seton Hall will also seek to bridge the gap in the other direction. Efforts and plans across the nation and the world are based on developing STEAM-based instruction for primary and secondary grades, encouraging students to develop coding and information expertise, and creating courses and programs in the Digital Humanities and in Data Science/Big Data that involve students from the humanities, the arts, the social sciences, and the professional schools. However, at Seton Hall, these efforts are strengthened by the University Core program, and in particular its Core III classes, and by offerings such as the minor in the Faith and Science offered by the Catholic Studies program [21].

While this program was developed at a Catholic university, and relies on the Catholic Intellectual Tradition as its underlying matrix, the structure provides a template that can be adapted to many other situations, deepening the education of STEM students, and their preparation for career and life. See [13] for further discussion of this point.

9. ACKNOWLEDGMENTS

The authors wish to thank J. T. Saccoman, Eric Johnston, and Msgr. Richard Liddy, all of Seton Hall, whose input was valuable in creating the course described here, to Jeremy Horne and Nagib Callaos, for valuable comments on an earlier draft of this paper, and to Fr. Gerard Buonopane, creator of another STEM Core III course, who contributed to refining our understanding of the general context of such courses.

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