Modeling State Space Search Technique for a Real World Adversarial Problem Solving

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ABSTRACT

In problem solving, there is a search for the appropriate solution. A state space is a problem domain consisting of the start state, the goal state and the operations that will necessitate the various moves from the start state to the goal state. Each move operation takes one away from the start state and closer to the goal state. In this work we have attempted implementing this concept in adversarial problem solving, which is a more complex problem space. We noted that real world adversarial problems vary in their types and complexities, and therefore solving an adversarial problem would depend on the nature of the adversarial problem itself. Specifically, we examined a real world case, ***"the prisoner's dilemma"*** *which is a critical, mutually independent, decision making adversarial problem. We combined the idea of the Thagard's Theory of Explanatory Coherence (TEC) with Bayes' theorem of conditional probability to construct the model of an opponent that includes the opponent's model of the agent. A further conversion of the model into a series of state space structures led us into the use of breadth-first search strategy to arrive at our decision goal.*

Keywords: *State-Space-Search, Adversarial Problems, Evaluation Function, Explanatory Coherence, Heuristics*

1.0 INTRODUCTION

State Space Search is a process used in the field of Computer Science Artificial Intelligence (AI), in which successive configurations or states of an instance are considered, with the aim of finding a goal state with a desired property [1].

A state contains all of the information necessary to predict the effects of an action and to determine if it is a goal state [2]. State-Space searching assumes that:

- The agent has a perfect knowledge of the state space and can observe what it is in (i.e., there is full observability);
- The agent has a set of actions that have known deterministic effects;
- Some states are goal states, the agent wants to reach one of these goal states, and the agent can recognize a goal state; and
- A solution is a sequence of actions that will get the agent from the current state to a goal state.

The concept of State Space Search is widely used in Artificial Intelligence. The idea is that a problem can be solved by examining the steps that can lead to a goal state (solution).This may involve several searches from the initial state to the goal state within an optimal time [3].

A possible solution is to find a method to measure the "goodness" of a state, (that is, to determine how close a given state is to the goal state). If this can be evaluated correctly, then we look at the list of states, to determine which to use next to generate new state. We could pick the state closer to the goal, instead of picking at random. The advantage here is that it helps to determine the optimal path to the goal.

Most times, such measurement of a state's goodness is estimated. If the estimate is wrong, more time and effort could be spent on the search without obtaining an optimal solution. The better the ability to estimate goodness, the better the chance for optimality. For a single agent in a relatively non-hostile world, the search for the path from some single state to some goal state is not especially difficult. But the real world involves multiple agents each trying to achieve a goal of its own. This paper models this kind of competitive behaviour by defining possible paths that can lead to an optimal solution. Thus, the question in a competitive or adversarial situation is no longer "what is the optimal path to the goal?" But is instead "what is my path to the goal when someone else is trying to stop me?"

The fundamental change in the nature of the question results in a change state-space search in adversarial situations is conducted, thus giving rise to "adversarial or game search"(Since it is frequently used to build intelligent game-playing programs). This kind of search is frequently called "game search".

The principle of game search is to first generate the state space some levels deep, where each level correspond to one player's move (or more accurately, the set of all nodes that the player could possibly make at that point). After generating the state space for that number of level, the nodes at the bottom level are evaluated for goodness. In the context of game playing, those nodes are often called "boards" each one representing one possible legal arrangement of game pieces on the game board.

In adversarial search, the estimate of the goodness of a board is a little bit different from that of non-adversarial search. Since the opponent is a threat, an estimation function is set up so that it returns a spectrum of values, similar to non-adversarial search, but now the two extremes are boards. We apply our estimation function to those lowest level boards, and propagate the numerical value upward to help us determine which is the best move to make [3].

2.0 THE STATE SPACE CONCEPT AND CONFIGURATION

In the state space representation of a problem, nodes of a graph correspond to partial problem solution states and arcs represent steps in a problem- solving process.

An initial state, corresponding to the given information in a problem instance, forms the root of the graph. The graph also defines a goal condition, which is the solution to a problem instance. State space search characterizes problem solving as the process of finding a solution path from the start state to a goal state.

Arcs of the state space correspond to steps in a solution process and path through the space represent solutions in varying stages of completion. Paths are searched, beginning at the start state and continuing through the graph until either the goal description is satisfied or they are abandoned. The actual generation of new states along the path is done by applying operators, such as "legal moves" in a game or expert system, to existing states on a path. A state space is represented by a tuple [N, A, S, GD], where:

N is the set of nodes or states of the graph. These correspond to the states in a problem-solving process,

A is the set of Arcs (or links) between nodes. These correspond to steps in a problem-solving process.

S, is a nonempty subset of N, contains the start state(s) of the problem.

GD, a nonempty subset of N, contains the goal state(s) of the problem. The states in GD are described using either:

1. A measurable property of the state encountered in the search or
2. A property in the path through this graph from a node in S to a node in GD.

The task of a search algorithm is to find a solution path through such a problem space. Search algorithms must keep track of the paths from a start to a goal node, because these paths contain the series of operations that lead to the problem solution [4].

3.0 METHODOLOGY

We consider here an example of a real world adversarial problem and adopt the Theory of Explanatory Coherence (TEC) of Thagard [5].

The real-world problem in consideration, involves a typical day-to-day problem in our society. It is important to note here that there is no standard model applying to all social adversarial problem since the problems are varied in nature in terms of complexity and dimensions. The seven principles of TEC are very applicable in real-world social problems since they enable us build and run a mental model of an opponent. TEC shows how explanatory breadth, simplicity, explanations by higher level hypotheses, competing hypotheses, analogy, and negative evidence can all affect the acceptability of a hypothesis.

The theory consists of the following principles:

- Principle 1: **Symmetry.** Explanatory coherence is a symmetric relation, unlike, say, conditional probability.
- Principle 2: **Explanation.** (a) A hypothesis coheres with what it explains, which can either be evidence or another hypothesis; (b) hypotheses that together explain some other proposition cohere with each other; and (c) the more hypotheses it takes to explain something, the less the degree of coherence.
- Principle 3: **Analogy.** Similar hypotheses that explain similar pieces of evidence cohere.
- Principle 4: **Data Priority.** Propositions that describe the results of observations have a degree of acceptability on their own.
- Principle 5: **Contradiction.** Contradictory propositions are incoherent with each other.
- Principle 6: **Competition.** If P and Q both explain a proposition, and if P and Q are not explanatorily connected, then P and Q are incoherent with each other. (P and Q are explanatorily connected if one explains the other or if together they explain something.)
- Principle 7: **Acceptance.** The acceptability of a proposition in a system of propositions depends on its coherence with them.

3.1 A Real-World Case: The prisoners' dilemma.

On the 10th day of January, 2006, a case of burglary was reported to the police. Two alleged burglars, Peter and John were caught near the scene of the burglary and interrogated separately by the police. Both knew that if they confessed to the crime, they will each serve five years in prison for burglary, but if both refuse to confess, they will serve only 1 year each for the lesser charge of possessing stolen property.

Peter and John who were old time friends, have in recent time had severe conflict over some personal issue.

The interrogating police officer offered each a deal: If one testifies against the other as leader of the burglary ring, he will go free while the other will serve 10years in prisons if the other refuses to testify. Peter and John were faced with the prisoners' dilemma: Should they testify or refuse?

3.2 Analysis of the problem

We present as follows, the analysis of the problem, showing the evidence, hypotheses, explanations and possibly the contradictions.

<u>EVIDENCE</u>

Proposition E0:	Police investigates crime of burglary
Proposition E1:	Peter and John were caught near the scene of burglary with stolen properties
Proposition E2:	Peter and John were interrogated separately.
Proposition E3:	Peter and John were old time friends
Proposition E4:	Peter and John in recent times had severe conflicts over some personal issues
Proposition E5:	Peter and John earn 5years sentence each in prison
Proposition E6:	Peter and John earn 1year sentence each in prison.
Proposition E7:	Peter gains freedom, and John earns 10yrs sentence in prison.
Proposition E8:	John gains freedom, and Peter earns 10yrs sentence in prison.

HYPOTHESES

Proposition A1: Peter and John are suspects in burglary case.
Proposition A2: Peter would testify against John.
Proposition A3: John would testify against Peter
Proposition A4: Peter would not testify against John.
Proposition A5: John would not testify against Peter.
Proposition A6: Peter wants to be exonerated
Proposition A7: John wants to be exonerated.

The task is the most convenient decision to make. Clearly, both would desire to be exonerated, but only one person can be completely exonerated, while the other serves a greater punishment. Peter and John will not know what either of them would say to the policeman, since they are both interrogated separately and had no opportunity to confer with each other.

The above scenario is modeled using predicate calculus clauses in Fig. 3.0.

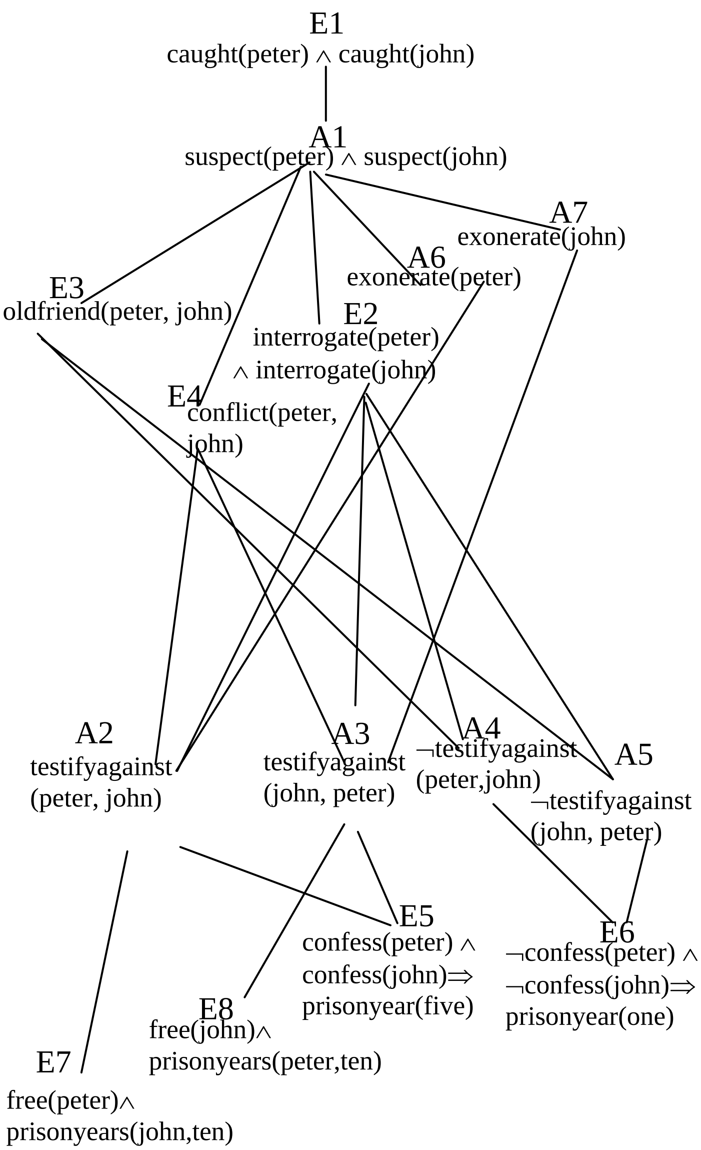

Fig. 3.0 Network model of the prisoner's dilemma using predicate calculus expressions

Applying the stipulated principles in Theory of Explanatory Coherence (TEC), let a statement (EXPLAIN (H1 H2) El) whose interpretation is that hypotheses H1 and H2 together explain evidence E1 be taken as input. Each proposition is represented by a network node called a unit, and constructs links between units in accord with TEC.

The Ai's in the model above show the "Hypotheses" that explain the 'Evidence' Ei's. E.g. Hypothesis A1 (Peter and John are suspects) explain the Evidence that Peter and John were caught near the scene of the burglary. The Straight lines indicate excitatory links produced by virtue of explanation.

4.0 STRATEGY AND SIMPLIFICATION TECHNIQUE FOR RESOLUTION

We consider the use of probabilistic approach, specifically the "Bayesian" approach of Thomas Bayes (1702-61) for the resolution of the problem. Recall that in State Space Search, every current state provides information for the next state. The "Bayesian" approach concerns the determination of probability of some event A (already known), given that another event B (not known) has taken place, i.e. the determination of the conditional probability $P(A|B)$. Bayes's results provide a way of computing the probability of a hypothesis following from a particular piece of evidence, given only the probabilities with which the evidence follows from actual causes (hypotheses).

Bayes' theorem states:

$$p(H_i|E) = \frac{p(E|H_i) * p(H_i)}{\sum_{k=1}^{n} p(E|H_k) * p(H_k)} \qquad \text{Eq(1)}$$

where:

$p(H_i|E)$ is the probability that H_i is true given evidence E.
$p(H_i)$ is the probability that H_i is true overall.
$p(E|H_i)$ is the probability of observing evidence E when H_i is true
n is the number of possible hypotheses.

Bayes' decision rule uses the best available estimates of the probabilities of the respective states of nature (currently the prior probabilities), calculates the expected value of the payoff for each of the possible actions, and chooses the action with the maximum payoff [6].

The network model shown in Fig. 3.0 is transformed into its equivalent Bayesian network model Fig. 4.0 showing the direction lines based on the evidences and hypotheses, i.e, the probabilities of some proposed hypotheses given some evidences. We observe that there are two pairs of posterior probabilistic decisions which are equally likely, these form the **decision zone**. Furthermore, we notice a limitation here in the use of the Bayesian concept. Since the decisions of both parties (Peter and John) are mutually independent of each other we would not clearly know the prior conditions that determine the posterior decisions. This is because both parties are interrogated separately, so one cannot wait for the other's decision before he takes his own decision. We however suffice to say that prior probabilistic conditions are equally likely.

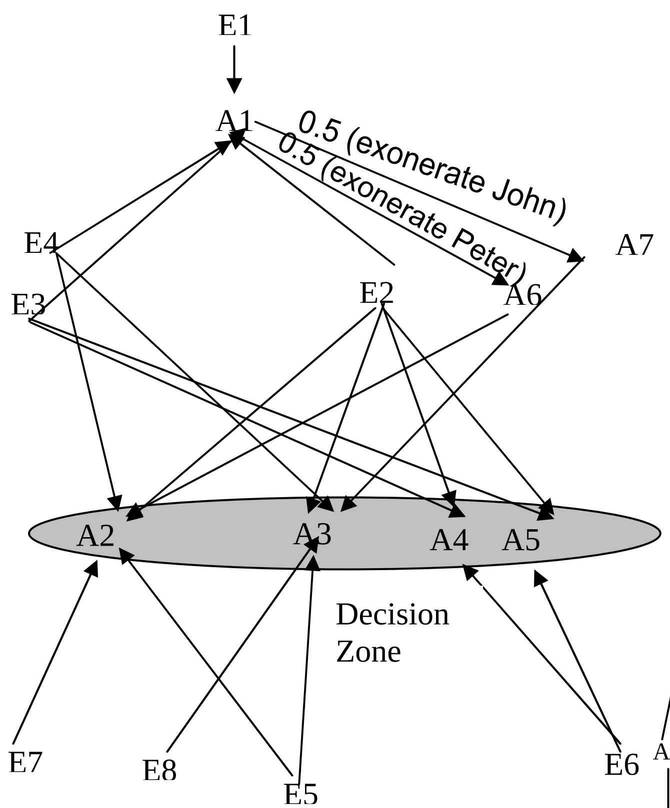

Fig. 4.0 Bayesian Network representation of the Prisoner's dilemma problem

The goal as we can see lies within the decision zone. The decision(s) that has/have the highest probabilistic value is/are the most probable decision. To arrive at that, we convert the model into a series of state space structures linked to an initial null state which together form a complete state space of the problem (see Fig.4.1). We used the breadth-first search strategy to locate the decisions. The probabilistic weight of each encounter of a decision is noted and incremented steadily as each of those decision encounters re-occur. Noting the frequency of occurrence, the mean of each of the probable decisions is taken. The highest value(s) is/are taken as the most probable decision(s).

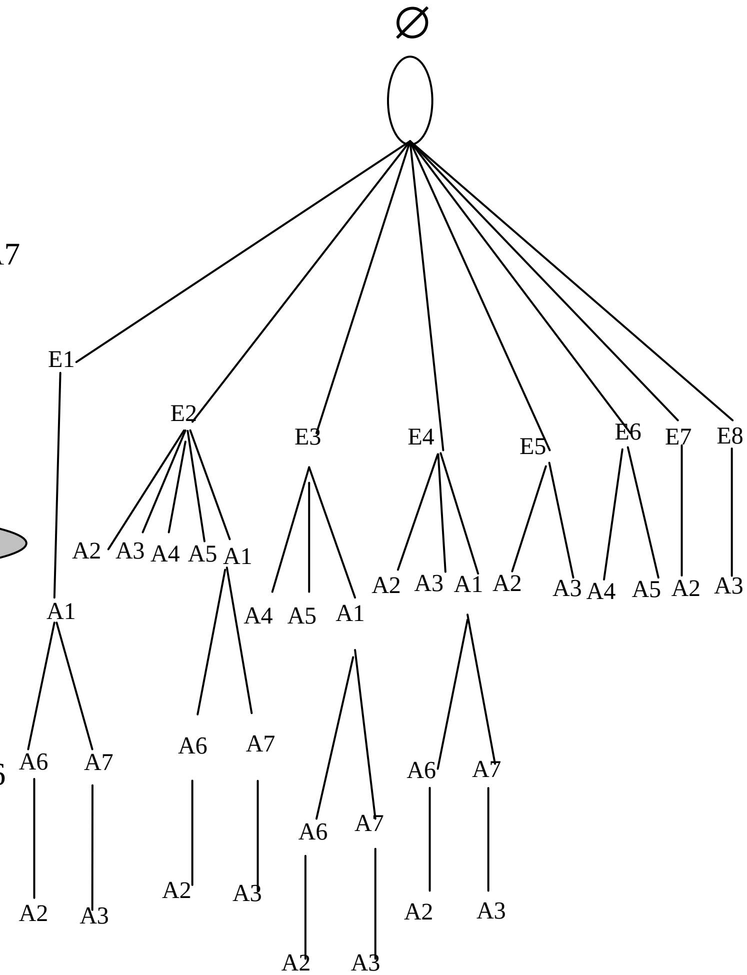

Fig. 4.1 Complete State Space for the "**Prisoner's Dilemma**" problem

5.0 SOLUTION APPROACH

The problem poses a triangular picture which shows three major probable actions Viz:

- Must occur actions
- May occur actions
- May not occur actions

Fig. 5.0 Triangular picture of the major probable actions

The probable actions that ***may*** occur are:

- That either Peter or John would be exonerated
- That either Peter or John would testify against each other
- Both Peter and John would testify to the crime

The probable action that ***may not*** occur is:

- That both Peter and John will not testify against each other.

The probable actions that ***must*** occur are:

- That either Peter or John will serve 10yrs jail term
- That both Peter and John will serve 5yrs jail term each
- That both Peter and John will serve 1yr jail term each.

5.1 SOLUTION

- Given that Peter and John were caught at the scene of the crime, clearly, the probability that both are suspects is 1.
- Since Peter and John may have likely desire to be exonerated, the chance that Peter wants to be exonerated and the chance that John wants to be exonerated will be 0.5 each.
- Probability of A2 (Peter testifies against John) is a joint probability of the probabilities A1 (Peter and John are suspects) and A6 (Peter will be exonerated)

i.e. $$P(A2) = P(A1 \text{ and } A6) = P(A1) * P(A6) = 1 * 0.5 = 0.5 \qquad \text{Eq (2)}$$

- Probability of A3 (John testifies against Peter) is a joint probability of the probabilities A1 (Peter and John are suspects) and A7 (John will be exonerated)

i.e. $$P(A3) = P(A1 \text{ and } A7) = P(A1) * P(A7) = 1 * 0.5 = 0.5 \qquad \text{Eq (3)}$$

- Probability of A2 (Peter testifies against John) given E7 (John will be sentenced to 10yrs imprisonment while Peter is set free), clearly is 1.

i.e. $$P(A2 \mid E7) = 1 \qquad \text{Eq(4)}$$

- Probability of A3 (John testifies against Peter) given E8 (John will be sentenced to 10yrs imprisonment while Peter is set free), clearly is 1.

i.e. $$P(A3 \mid E8) = 1 \qquad \text{Eq(5)}$$

- Probability of A1 (Peter and John are suspects) given E3 (Peter and John are old friends), clearly is 1.

i.e. $$P(A1 \mid E3) = 1 \qquad \text{Eq (6)}$$

- Probability of A1 (Peter and John are suspects) given E4 (Peter and John had conflicts in recent times), clearly is 1.

i.e. $$P(A1 \mid E4) = 1 \qquad \text{Eq(7)}$$

- Probability of A1 (Peter and John are suspects) given E2 (Peter and John were interrogated separately), clearly is 1.

i.e. $$P(A1 \mid E2) = 1 \qquad \text{Eq(8)}$$

- Probabilities of A4 (Peter would not testify against John) and A5 (John would not testify against Peter) given E3 (Peter and John were old time friends) are equally likely, therefore each has a probability of 0.5

i.e. $$P(A4 \mid E3) = 0.5 \text{ and}$$
$$P(A5 \mid E3) = 0.5 \qquad \text{Eq(9)}$$

- Probabilities of A4 (Peter would not testify against John) and A5 (John would not testify against Peter) given E6 (Peter and John earn 1year sentence each in prison if both refuse to testify) are equally likely, therefore each has a probability of 0.5

i.e. $$P(A4 \mid E6) = 0.5 \text{ and}$$
$$P(A5 \mid E6) = 0.5 \qquad \text{Eq(10)}$$

- Probabilities of A2 (Peter testifies against John) and A3 (John testifies against Peter) given E4 (Peter and John in recent times had severe conflicts over some personal issues) are equally likely, therefore each has a probability of 0.5

i.e. $$P(A2 \mid E4) = 0.5 \text{ and}$$
$$P(A3 \mid E4) = 0.5 \qquad \text{Eq(11)}$$

- Probabilities of A2 (Peter testifies against John) and A3 (John testifies against Peter) given E5 (Peter and John earn 5years sentence each in prison) are equally likely, therefore each has a probability of 0.5

i.e. $$P(A2 \mid E5) = 0.5 \text{ and}$$
$$P(A3 \mid E5) = 0.5 \qquad \text{Eq(12)}$$

- Probabilities of A2 (Peter testifies against John), A3 (John testifies against Peter), A4 (Peter would not testify against John), and A5 (John would not testify against Peter) given E2 (Peter and John were interrogated separately) are equally likely, therefore each has a probability of 0.5

i.e.
$$P(A2 \mid E2) = 0.25$$
$$P(A3 \mid E2) = 0.25$$
$$P(A4 \mid E2) = 0.25$$
$$P(A5 \mid E2) = 0.25 \qquad \text{Eq(13)}$$

We now find the average probabilities of A2, A3, A4, A5 respectively.

i. $$\text{Ave. } P(A2) = \frac{\text{Sum of all Probabilities of A2}}{\text{Number of Cases of A2}}$$
$$= \frac{0.5+1+0.5+0.5+0.25}{5}$$
$$= \frac{2.75}{5}$$
$$= 0.55 \qquad \text{Eq(14)}$$

ii $$\text{Ave. } P(A3) = \frac{\text{Sum of all Probabilities of A3}}{\text{Number of Cases of A3}}$$
$$= \frac{0.5+1+0.5+0.5+0.25}{5}$$
$$= \frac{2.75}{5}$$
$$= 0.55 \qquad \text{Eq(15)}$$

iii $$\text{Ave. } P(A4) = \frac{\text{Sum of all Probabilities of A4}}{\text{Number of Cases of A4}}$$
$$= \frac{0.5+0.5+0.25}{3}$$
$$= \frac{1.25}{3}$$
$$= 0.42 \qquad \text{Eq(16)}$$

iv $$\text{Ave. } P(A5) = \frac{\text{Sum of all Probabilities of A5}}{\text{Number of Cases of A5}}$$
$$= \frac{0.5+0.5+0.25}{3}$$
$$= \frac{1.25}{3}$$
$$= 0.42 \qquad \text{Eq(17)}$$

We see that the maximum payoff probabilities here are P(A2) and P(A3) which have equal probabilistic values of 0.55. Therefore,

- Peter would testify against John and
- John would testify against Peter

5.0 CONCLUSION

In this study, we have attempted to bring the State Space Search concept from mere board games and simple puzzle problems to a real life situation. Using a complex adversarial problem scenario where information of individual opponent's actions are hidden from each other, a model was derived. A problem space of this nature forms a state space, and the drive towards a more realistic decision forms the state space search.

The partial combination of Thagard's Theory of Explanatory Coherence (TEC), and Baye's decision rule, together with breadth-first search were applied to find the optimal solutions for the state space search. Results from the study led to the conclusion that some real world adversarial problems are peculiar in nature and so should be treated with regards to their peculiarities.

REFERENCES

1. Wikipidia, **State Space Search,** Wikipedia Free Encyclopedia, 2013 Available Online at: www.en.wikipedia.org/../state_space_search

2. P. David & M. Alan, **Foundations of Computational Agents**, 2010 Available online at: www.artint.info/html/ArtInt_48.html

3. K. Eiselt, **CS2360 Lecture note on State Space Search,** 1997 Available Online at: www.static.cc.gatech.edu/computing/classes/cs2360

4. G.F. Lugar & W.A. Stubblefield, **Artificial Intelligence and the Design of Expert Systems,** The Benjamin/Cummings Publishing Company Inc., California, 1989

5. P. Thagard: **Adversarial Problem Solving: Modeling an Opponent Using Explanatory Coherence,** 1992 Available online at: www.wiwiss-fu berlin.de/dblp/page/record/journal/cogs

6. F.S. Hillier & G.D. Lieberman, **Introduction to Operations Research,** McGraw-Hill Companies Inc. New York., 2001