Analysis of the Largest Normalized Residual Test Robustness for Measurements Gross Errors Processing in the WLS State Estimator

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ABSTRACT

This paper purpose is to implement a computational program to estimate the states (complex nodal voltages) of a power system and showing that the largest normalized residual (LNR) test fails many times. The chosen solution method was the Weighted Least Squares (WLS). Once the states are estimated a gross error analysis is made with the purpose to detect and identify the measurements that may contain gross errors (GEs), which can interfere in the estimated states, leading the process to an erroneous state estimation. If a measure is identified as having error, it is discarded of the measurement set and the whole process is remade until all measures are within an acceptable error threshold. To validate the implemented software there have been done several computer simulations in the IEEE´s systems of 6 and 14 buses, where satisfactory results were obtained.

Another purpose is to show that even a widespread method as the LNR test is subjected to serious conceptual flaws, probably due to a lack of mathematical foundation attendance in the methodology. The paper highlights the need for continuous improvement of the employed techniques and a critical view, on the part of the researchers, to see those types of failures.

Keywords: Power Systems, State Estimation, Gross Errors, Normalized Residual.

1. INTRODUCTION

The real time power systems operation has the main objective to keep the electrical system operating. To achieve this goal it is necessary that the voltage, frequency, lines power flow and lines and equipment load levels be kept within safety thresholds.

The state estimation process plays an essential role for the monitoring and analysis of an electrical system, because it handles analog redundant information measurements contaminated by noise, in order to better estimate the complex voltages in the buses belonging to the supervised system [1].

The ability to detect and identify GEs is one of the important attributes of the state estimation process in power systems. Some GEs are obvious and can be initially identified and eliminated from the estimation process, through a simple verification of the measures input data. Such errors can be: absurd values of effective voltage, values far beyond those expected for measures of power and /or electrical current, etc. [2]. However, not all types of GEs are easily detectable and identifiable in this way, requiring the use of other methodologies.

The WLS state estimator works well when the noise in the measurements are Gaussian, but fails in the occurrence of one or more GEs [2]. To overcome this limitation, methods were developed for detection and identification of GEs, among which the most widely used, are based on the analysis of the measurement residual, because they provide information on possible violations of assumptions concerning the measurement model (the residue is the difference between the measured and estimated value of the measures).

The WLS estimator, associated with GEs processing techniques based on the analysis of the measures residues, give satisfactory performance in the occurrence of simple GE, or when there are multiple non-interactive GEs [3], but can fail in the following situations:

i) GEs associated with measures with low redundancy (critical measures or pertaining to critical sets of measures);

ii) Interactive multiple GEs;

iii) GEs that have the characteristic of being highly influential, i.e., to attract the convergence of state estimation process, called leverage point measures [3].

Because of the simplicity of its formulation, as well as the ease of its computer implementation, the WLS estimator associated with the largest normalized residual test is the most used in operation centers.

2. PROBLEM FORMULATION

This section will present a general formulation for the WLS estimator associated with the largest normalized residual test.

Power System Mathematical Modeling

In this work we adopted as a representation of an electrical system branch a generalization of the equivalent model of transmission lines, in phase and lagged transformers, from the model presented in [4]. From the application of Kirchhoff's laws, on the general model π, we obtain the following expressions for the active (P_{kl}) and reactive (Q_{kl}) power flow in the branch that connects the buses k e l [4]:

i) From bus k to bus l

$$P_{kl} = a_{kl}^2.V_k^2.g_{kl} - a_{kl}.V_k.V_l.g_{kl}.\cos(\theta_{kl}+\phi) + \\ - a_{kl}.V_k.V_l.b_{kl}.\sin(\theta_{kl}+\phi)$$

$$Q_{kl} = -a_{kl}^2.V_k^2.(b_{kl}+b_{kl}^{sh}) + a_{kl}.V_k.V_l.b_{kl}.\cos(\theta_{kl}+\phi) + \\ -a_{kl}.V_k.V_l.g_{kl}.\sin(\theta_{kl}+\phi) \quad (2.1)$$

ii) From bus l to bus k

$$P_{lk} = V_l^2.g_{kl} - a_{kl}.V_k.V_l.g_{kl}.\cos(\theta_{lk} - \phi) + - a_{kl}.V_k.V_l.b_{kl}.\sin(\theta_{lk} - \phi)$$
$$Q_{lk} = -V_l^2.(b_{kl} + b_{kl}^{sh}) + a_{kl}.V_k.V_l.b_{kl}.\cos(\theta_{lk} - \phi) + - a_{kl}.V_k.V_l.g_{kl}.\sin(\theta_{lk} - \phi) \quad (2.2)$$

Being:
b_{kl} - the component series susceptance;
b_{kl}^{sh} - the shunt susceptance of the transmission line;
$t_{kl} = a_{kl}.e^{j\phi}$ - the transformer turns ratio;
V_k and V_l - the voltage magnitudes at buses k and l;
θ_k and θ_l - the voltage phase angles at buses k and l.

In the complex power injection equating, at any power system bus, you must consider the possible existence of shunt elements connected to it. Thus, the expressions for the active and reactive power injections, at a generic bus k, can be written as:

$$P_k = \sum_{l \in \Omega k} P_{kl}$$
$$Q_k = -V_k^2.b_k^{sh} + \sum_{l \in \Omega k} Q_{kl} \quad (2.3)$$

Being:
b_k^{sh} - the shunt susceptance of a capacitor connected to the bus k;
Ωk - the set of adjacent buses to the bus k.

For more details of the complete equating of flow and power injection expressions see [4]. The next section will address the solution method chosen for the WLS state estimator.

Normal Equation Method

The state estimation is the calculation of unknown state variables through a set of inaccurate measures; therefore, the estimation obtained will not be exact. Thus, the estimation problem is to find a way to achieve the best estimation and, for this, from many existing statistical criteria, the one which has been used in most of the power systems is the weighted least squares. In this paper it is assumed that there are no errors in the model parameters. With respect to this representation, the nonlinear equations for state estimation in power systems are represented as:

$$\underline{z} = h(\underline{x}) + \underline{w} \quad (2.4)$$

Being:
$\underline{z}$ - the measures vector (m x 1);
$h(.)$ - the vector of nonlinear functions, which lists the measures with the state variables (m x 1);
$\underline{x}$ - the vector of state variables to be estimated (n x 1);
$\underline{w}$ - the vector of measures errors (m x 1);
m - the measures number;
n - the state variables to be estimated number.

The measures errors are considered as independent random variables, with zero Gaussian mean [5]. Calling R the covariance matrix of the measures error vector, with size m x m, we have:

$$R = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_m^2 \end{bmatrix} \quad (2.5)$$

Where σ_i^2 is the variance of measurement error i. Therefore $\underline{w_i} \sim N(0,R_{ii})$ for every "i".

By applying the methodology of weighted least squares, the best estimation of the state variables vector $\underline{x}$, called $\hat{\underline{x}}$, can be obtained by calculating the value of $\underline{x}$ that makes minimum the $J(\underline{x})$ index [5], given by:

$$J(\underline{x}) = \frac{1}{2}.\underline{w}^t.R^{-1}.\underline{w} \quad (2.6)$$

or

$$J(\underline{x}) = \frac{1}{2}.[\underline{z} - h(\underline{x})]^t.R^{-1}.[\underline{z} - h(\underline{x})] \quad (2.7)$$

Being R^{-1} the inverse of the covariance matrix of the measures error vector, used here as a weight matrix for the measurements.

The $J(\underline{x})$ index becomes a minimum when:

$$\frac{\partial J(\underline{x})}{\partial \underline{x}} = 0 \quad (2.8)$$

or

$$H^t(\hat{\underline{x}}).R^{-1}.[\underline{z} - h(\hat{\underline{x}})] = 0 \quad (2.9)$$

being $H(\hat{\underline{x}})$ the matrix of first derivatives of the nonlinear functions of vector $h(\underline{x})$, known as the Jacobian, calculated at the point represented by the vector of estimated state variables $\hat{\underline{x}}$, and represented by:

$$H(\hat{\underline{x}}) = \left.\frac{\partial h(\underline{x})}{\partial \underline{x}}\right|_{x=\hat{x}} \quad (2.10)$$

Because the $J(\underline{x})$ index is a quadratic nonlinear function, to obtain $\hat{\underline{x}}$ we apply an iterative method to solve a linear equation at each iteration k, in order to calculate the current estimation of the state variables vector, through successive corrections [5], given by:

$$\underline{x}^{k+1} = \underline{x}^k + \Delta\underline{x}^k \quad (2.11)$$

However, to determine the correction $\Delta\underline{x}^k$, we perform the linearization of the equations $h(\underline{x})$ around the point $\underline{x}^k$, represented by the expression:

$$h(\underline{x}^{k+1}) \cong h(\underline{x}^k) + H(\underline{x}^k).\Delta\underline{x}^k \quad (2.12)$$

Rewriting equation (2.4), in relation to the approximations made in $h(\underline{x})$, we obtain the measurement model which has become linear:

$$\underline{z} = h(\underline{x}^k) + H(\underline{x}^k).\Delta\underline{x}^k + \underline{w} \quad (2.13)$$

Or:

$$\Delta\underline{z}(\underline{x}^k) = \underline{z} - h(\underline{x}^k) = H(\underline{x}^k).\Delta\underline{x}^k + \underline{w} \quad (2.14)$$

Being $\Delta\underline{z}(\underline{x}^k)$ defined as vector of the measurements residues.

From the model of linear measurement, the objective function $J(\Delta\underline{x})$ becomes:

$$J(\Delta\underline{x}) = \frac{1}{2}.[\Delta\underline{z}(\underline{x}^k) - H(\underline{x}^k).\Delta\underline{x}^k]^t.R^{-1}.[\Delta\underline{z}(\underline{x}^k) + -H(\underline{x}^k).\Delta\underline{x}^k] \quad (2.15)$$

Whose minimum is calculated from:

$$\frac{\partial J(\Delta\underline{x})}{\partial \Delta\underline{x}} = H(\underline{x}^k)^t.R^{-1}.[\Delta\underline{z}(\underline{x}^k) - H(\underline{x}^k).\Delta\underline{x}^k] = 0 \quad (2.16)$$

Therefore, the solution can be obtained by the following equation:

$$\Delta\underline{x}^k = [H(\underline{x}^k)^t.R^{-1}.H(\underline{x}^k)]^{-1}.H(\underline{x}^k)^t.R^{-1}.\Delta\underline{z}(\underline{x}^k) \quad (2.17)$$

Which is called the normal equation, where:

$$H(\underline{x}^k)^t.R^{-1}.H(\underline{x}^k) = G(\underline{x}^k) \quad (2.18)$$

is the gain matrix (G).

The iterative process starts from an initial value $\underline{x}^0$ and, at each iteration k; the corrections in the state variables $\Delta\underline{x}$ are obtained using equation (2.17). The vector of state variables update is obtained using equation (2.11) until a stopping criterion is satisfied, such as:

$$max|\Delta\underline{x}^k| \leq \varepsilon \quad (2.19)$$

where ε denotes a predetermined error tolerance.

Thus, this criterion indicates that the iterative process will be terminated when the magnitude of adjustments in state variables is negligible. The algorithm of the WLS state estimator can be summarized by the following steps:

Step 1: Set $k = 0$ and choose an initial solution $\underline{x}^k = \underline{x}^0$;
Step 2: Calculate the matrices $H(\underline{x}^k)$ and $G(\underline{x}^k)$ at the point $\underline{x} = \underline{x}^k$;
Step 3: Get the state variables correction through the normal equation and update the variables:

$$\Delta\underline{x}^k = G(\underline{x}^k)^{-1}.H(\underline{x}^k)^t.R^{-1}.\Delta\underline{z}(\underline{x}^k)$$
$$\underline{x}^{k+1} = \underline{x}^k + \Delta\underline{x}^k$$

Step 4: Test the stopping criterion: if $max\left|\Delta\underline{x}^k\right| \leq \varepsilon$, the process converged. Otherwise, make $k = k + 1$ and return to Step 2.

Largest Normalized Residual Test

The method used in this paper to detection and identification of measurement GEs is through the normalized residues vector ($\underline{r}^N$) analysis. The residues vector is defined by:

$$\underline{r}(\hat{\underline{x}}) = \underline{Z} - h(\hat{\underline{x}}) \quad (2.20)$$

To normalize the residue is necessary to calculate the residues covariance matrix, defined by the equation:

$$\Omega(\hat{\underline{x}}) = R - H(\hat{\underline{x}}).G^{-1}(\hat{\underline{x}}).H^t(\hat{\underline{x}}) \quad (2.21)$$

Thus, the normalized residue is calculated by:

$$\underline{r}_i^N(\hat{\underline{x}}) = \frac{r_i(\hat{\underline{x}})}{\sqrt{\Omega_{ii}(\hat{\underline{x}})}} \quad (2.22)$$

where Ω_{ii} is the i diagonal element of the residues covariance matrix.

The importance of residues standardization can be understood if it is taken into account that different types of meters have generally different variances, so that a discrepant measurement's residue value can be perfectly acceptable to another. The residues standardization places them in a single reference, thus allowing a fair comparison of their absolute values.

Admitting the hypothesis that the measurements errors ($\underline{w}_i$) are independent random variables with normal distribution with zero mean and known variance it is proved, in [6], that the elements of the normalized residues vector presents standard normal distribution, i. e.:

$$\underline{r}_i^N \sim N(0,1)$$

Thus, the existence of GEs can be verified by the following test:

- If any $\left|\underline{r}_i^N\right| > \beta$, with i = 1,...,m, there is suspicion of GE;
- If all $\left|\underline{r}_i^N\right| \leq \beta$, with i = 1,...,m, supports the hypothesis that there is no GE.

Usually it is assumed $\beta = 3$ [2].

Considering the hypothesis of a single measurement containing a GE and all other measurements as perfect, in [7] and [2], it is shown that for a measurement system, free of critical measurements and critical sets, the measurement with GE attend the largest normalized residual ($\underline{r}_{max}^N$). Thus, we can perform both detection and identification of the measurement with a GE, at the same time, by testing:

$$\underline{r}_i^N > \beta \text{ (Threshold)} \quad (2.23)$$

In the presence of single GE, the method does not identify GE on critical measurements, or on measurements pertaining to critical sets of measures. This is due to the fact that the critical measurements have zero residues [8] and measurements of critical sets have normalized residues equal in magnitude [9].

After identifying the measurement with a GE, some special treatment should be given to this measurement, in order to minimize its effect. Traditionally, the effect of the measurement with GE can be suppressed in two ways [7]:

i) withdrawal of the measurement with a GE from the measurement set and re-estimates the states;

ii) recovery of the measurement with GE through the value of estimated error and performs again an estimation of the states.

In this paper the measurement identified as having a GE is removed from the measurement set.

OBS. 1: In all the gross error detection test procedure, it was used the residual as a measure of the measurement gross error, without any proof that this assumption is correct.

OBS. 2: No proof at all is presented that the measurement with gross error is the one with the largest normalized residual. Again they are mixing measurement error with measurement residual, and they are completely different quantities [10].

OBS. 3: The conventional methodology is not considering that the residual space is of dimension equal to the measurements´ number minus the system state variables, that is, a correlated space. Otherwise the measurement error is a not correlated space; that is the measurement errors are not correlated.

OBS. 4: The consequence of the correlated space for the measurements is that instead of using a hyper-sphere in order to identify the measurement with error one should use instead a hyper-ellipsoid.

3. PROPOSED METHODOLOGY

The implemented program has the following flowchart:

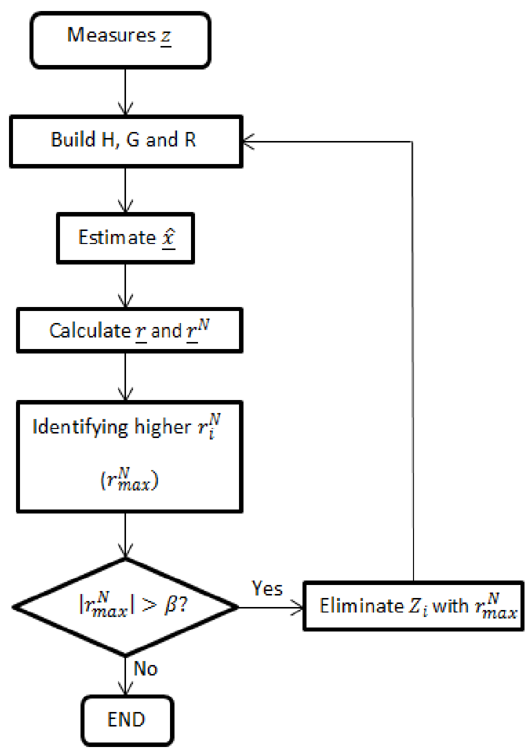

Fig.1: WLS estimator flowchart.

Obs.: β is chosen (how many standard deviations are accepted). In this paper it was considered $\beta = 3$.

The systems chosen for computer simulations are the IEEE's 6-bus and the IEEE-14 bus, where the program reads automatically the database in *.txt* format.

4. RESULTS

In this section we present the computer simulations results for IEEE's 6 bus and 14 bus systems.

IEEE´s 6 Buses System

The system has the following topology:

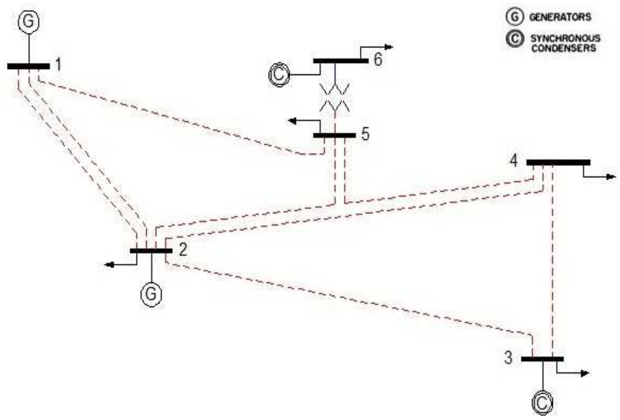

Fig. 2: IEEE's 6 buses system topology.

From the results of a load flow program, the measurement plan was built with a measuring overall redundancy index equal to three times the number of state variables to be estimated, therefore, consists of 33 measurements, without adding in the initial case, random noise and without the presence of critical measurements or critical sets of measurements. The measurements are shown in the following tables:

Table 1. Power injection measurement values

Active Measurements		Reactive Measurements	
Measurement	Value (MW)	Measurement	Value (MVAr)
AI 1	149.0	RI 1	-3.7
AI 2	18.3	RI 3	2.2
AI 4	-47.8	RI 4	3.9
AI 5	-7.6	RI 6	-14.0

Table 2. Power flow measurement values

Active Measurements		Reactive Measurements	
Measurement	Value (MW)	Measurement	Value (MVAr)
AF 1-2	103.1	RF 1-5	3.2
AF 1-5	45.9	RF 2-3	4.7
AF 2-3	62.5	RF 2-4	-0.6
AF 2-5	22.8	RF 3-4	4.4
AF 3-4	-33.4	RF 4-5	8.9
AF 4-5	-48.3	RF 5-6	14.7
AF 2-1	-101.3	RF 2-1	6.7
AF 3-2	-60.8	RF 5-1	-4.4
AF 4-2	-33.7	RF 4-2	-1.2
AF 5-2	-22.5	RF 5-2	-4.0
AF 5-4	48.6	RF 4-3	-3.8
AF 6-5	-11.2	RF 6-5	-14.0

Table 3. Voltage measurements

Measurement	Value (V)
V1	1.060

To weigh the measurements used by the WLS state estimator it was assumed that all meters have standard deviation calculated by the following equation:

$$\sigma_i = \frac{pr.\left|z_i^{lf}\right|}{3} \quad (4.1)$$

Where *pr* is the meter precision (considered 3% in this work by author´s choice) and z^{lf} is the measurement value obtained from a load flow simulation. After running the implemented software the following results were obtained for the estimated state variables:

Table 4. Estimated state variables

Bus	Magnitude (pu)	Angle (rad)
1	1.0602	0
2	1.0452	-0.0558
3	1.0102	-0.1700
4	1.0255	-0.1115
5	1.0284	-0.0911
6	1.0701	-0.1150

After the estimation process the largest normalized residual test is performed, to detect possible measurements containing GEs. In this case, the calculated largest normalized residual was: $r_{max}^N = 0.0150$ on the flow measure RF 1-5. As the $r_{max}^N \leq 3$, we accept the hypothesis that there is no measurement with a GE.

Now let's add a 5σ error on the measurement AF 1-2 (chosen randomly). Repeating the estimation process the following results are obtained:

Table 5. Estimated state variables

Bus	Magnitude (pu)	Angle (rad)
1	1.0587	0
2	1.0431	-0.0580
3	1.0074	-0.1743
4	1.0232	-0.1143
5	1.0261	-0.0936
6	1.0676	-0.1182

In this case: $r_{max}^N = 4.3806$ on the measurement AF 1-2. As expected, the test detected and identified the measurement carrying the GE. By eliminating this measurement from the measurement plan, the results were the same as the original case without the existence of measurement error. With the results obtained it can be observed the effect a measurement with GE can cause on the state estimation process, leading it to erroneous values for the estimated state variables.

Now, let´s admit that the measurements are no more perfect, but having an associated random noise, so that they vary from $\pm 3\sigma$ of its original values, so, not characterizing a measure with GE. This was done to test the robustness of the largest normalized residual test, since, in real systems; the set of measurements is subject to noise. In this case: $r_{max}^N = 3.6882$ on the measurement AF 6-5. We conclude that the test failed, since the added noise in the measurements was less than 3σ (in magnitude), not characterizing GE, as the test found. Now, adding a GE of 5σ on AF 2-3, the test resulted in: $r_{max}^N = 4.5204$ on the measurement AI 1. It appears that the test detected the presence of GE, but not on the measurement that GE was inserted, failing again.

IEEE´s 14 Buses System

The system has the following topology:

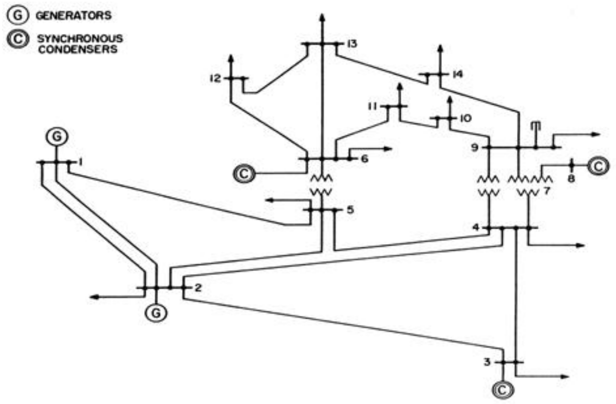

Fig. 3: IEEE’s 14 bus system topology.

For this system the measurement plan consists of 81 measurements, leading the overall redundancy level equal to 3.0.

Table 6. Power injection measurement values

Active Measurements		**Reactive Measurements**	
Measurement	**Value (MW)**	**Measurement**	**Value (MVAr)**
AI 1	232.4	RI 1	-16.5
AI 2	18.3	RI 3	6.1
AI 4	-47.8	RI 4	3.9
AI 5	-7.6	RI 6	5.2
AI 7	0	RI 7	0
AI 8	0	RI 9	-16.6
AI 10	-9.0	RI 10	-5.8
AI 11	-3.5	RI 12	-1.6
AI 13	-13.5	RI 13	-5.8
AI 14	-14.9	RI 14	-5.0

Table 7. Power flow measurement values

Active Measurements		**Reactive Measurements**	
Measurement	**Value (MW)**	**Measurement**	**Value (MVAr)**
AF 1-2	156.9	RF 1-2	-20.4
AF 2-3	73.2	RF 1-5	3.9
AF 2-4	56.1	RF 2-4	-1.55
AF 3-4	-23.3	RF 2-5	1.2
AF 4-5	-61.2	RF 4-5	15.8
AF 4-9	16.1	RF 4-7	-9.7
AF 5-6	44.1	RF 5-6	12.5
AF 6-12	7.8	RF 6-11	3.6
AF 6-13	17.7	RF 6-13	7.2
AF 7-9	28.1	RF 7-8	-17.2
AF 9-10	5.2	RF 9-10	4.2
AF 10-11	-3.8	RF 9-14	3.6
AF 12-13	1.6	RF 13-14	1.7
AF 2-1	-152.6	RF 2-1	27.7
AF 5-1	-72.7	RF 5-1	2.2
AF 3-2	-70.9	RF 3-2	1.6
AF 4-2	-54.5	RF 4-2	3.0
AF 4-3	23.7	RF 5-2	-2.1
AF 5-4	61.7	RF 4-3	-4.8
AF 7-4	-28.1	RF 5-4	-14.2
AF 9-4	-16.1	RF 7-4	11.4
AF 11-6	-7.3	RF 6-5	-8.1
AF 12-6	-7.7	RF 11-6	-3.4
AF 13-6	-17.5	RF 12-6	-2.4
AF 8-7	0	RF 13-6	-6.8
AF 9-7	-28.1	RF 9-7	-5.0
AF 14-9	-9.3	RF 10-9	-4.2
AF 11-10	3.8	RF 14-9	-3.4
AF 13-12	-1.6	RF 11-10	1.6
AF 14-13	-5.6	RF 14-13	-1.6

Table 8. Voltage Measurements

Measurement	**Value (V)**
V1	1.060

After running the implemented software the following results were obtained for the estimated state variables:

Table 9. Estimated state variables

Bus	**Magnitude (pu)**	**Angle (rad)**
1	1.0599	0
2	1.0449	-0.0870
3	1.0098	-0.2222
4	1.0175	-0.1800
5	1.0194	-0.1532
6	1.0697	-0.2483
7	1.0614	-0.2333
8	1.0899	-0.2333
9	1.0558	-0.2609
10	1.0508	-0.2636
11	1.0567	-0.2582
12	1.0549	-0.2631
13	1.0501	-0.2646
14	1.0353	-0.2799

After performing the state estimation the obtained largest normalized residual with perfect measurements was: $\underline{r}_{max}^{N} = 0.0177$ on the measurement RF 6-11. Note that $\underline{r}_{max}^{N} \leq 3$ then the hypothesis that there is no measurement containing a GE is accepted. Now let’s add a -6σ noise on the measurement FA 5-6 (chosen randomly) in the same perfect measurements set. Remaking the process we obtained the following results:

Table 10. Estimated state variables

Bus	**Magnitude (pu)**	**Angle (rad)**
1	1.0627	0
2	1.0477	-0.0863
3	1.0125	-0.2214
4	1.0210	-0.1769
5	1.0230	-0.1498
6	1.0743	-0.2265
7	1.0646	-0.2258
8	1.0932	-0.2254
9	1.0586	-0.2518
10	1.0539	-0.2528
11	1.0594	-0.2426
12	1.0592	-0.2410
13	1.0544	-0.2436
14	1.0390	-0.2656

For this case: $\underline{r}_{max}^{N} = 4.5849$ on the measure AF 5-6. As expected, the test detected and identified the measurement containing a GE. Eliminating it from the measurement plan the results were the same as the original case without GE, validating the test for this case.

Similarly to the case of the 6 buses system, we added random noise in the set of measures. For this case: $\underline{r}_{max}^{N} = 4.1846$ and, the test failed again when the measures have noise, even though they were lower than 3σ. Finally, adding a GE of

6σ on the measure AF 13-6 we obtained $r_{max}^N = 5.5143$ on this same measure, thus the test was effective in this case. However, simulating the system again, we obtained $r_{max}^N = 3.6290$ on the measure AI 10, thus the test detected the GE, but was not able to correctly identify the measurement containing a GE.

As stated initially, it is shown that the largest normalized residual test fails many times. Through the geometric interpretation, the author [11] proves mathematically that the measurement error is composed of components detectable and undetectable, also shows that the detectable component of the error is exactly the noise of the measurement error. The methods previously used for the processing of gross errors (GEs), consider only the detectable component of the error, then as a consequence, may fail.

Through orthogonal projections defined by the equation of the projection matrix, [11] also showed that errors in measurements that are very close to the range space of the Jacobian matrix, relative to other measures, are difficult to detect when using the largest normalized residual test, thus, depending on the amplitude of the components of the error, this method may fail. So is being studied and proposed a new methodology to process the measures with GE. This proposition is obtained by decomposing the measurement error in two components: the first is orthogonal to the range space of the Jacobian matrix, whose amplitude is equal to the residue of the measure; the other belongs to the range space of the Jacobian matrix and therefore does not contribute to the residue of the measure.

5. CONCLUSION

By the tests presented, it is verified the effectiveness of the proposed algorithm, applied to power systems state estimation, by using the WLS state estimator method.

The implemented software allows the operator to directly read the database from the solution of a load flow problem in a *.txt* format. After reading the database it solves the problem of state estimation by the weighted least squares method, taking into consideration the measurement quality, given by their respective variances.

To validate the results obtained by the WLS, the largest normalized residual test is performed in an attempt to detect and identify possible measurement containing GEs, which interfere negatively in the estimation process. If such measurement is detected, through a threshold test for r_{max}^N, it is discarded from the measurement set and the process of estimation is remade.

The results obtained using the software shows that the theoretically expected results of the classic state estimation analysis fails some times and the reasons for that is the lack of theoretical consistency used in the classical state estimation proposition. The simulations results showed that the largest normalized residual test fails and again the reason are the inconsistencies of the used theoretical background. For example even in the case of not having gross error in the measurement set, the test detected GE and in other cases, correctly detected the presence of GE, but erroneously identified the measurement containing a GE. These facts make clear the lack of robustness of the largest normalized residual test when using measurements sets with random noise.

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