Mathematics and blindness: the legacy of Abraham Nemeth

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ABSTRACT[1]

In this paper, we present the life and the achievements of American blind mathematician Abraham Nemeth (1918-2013), with the main focus on his extraordinary contribution to mathematical culture. His inventions opened the doors of mathematics to blind and heavily visually impaired people and are still the starting point for many advanced assistive technologies. The paper is structured as follows: after a brief introduction on mathematics and blindness in Section 1, Section 2 explores the biography of Abraham Nemeth, highlighting some points of interest for the development of his scientific career. In Sections 3 and 4 we present his two most remarkable achievements; the inventions of the Nemeth Braille Code and of MathSpeak respectively. Finally, we draw some conclusions in Section 5.

Keywords: Abraham Nemeth, accessibility, MathSpeak, Nemeth Code, spoken mathematics.

1. INTRODUCTION

One of the most hard to fall prejudice about blind and visually impaired people is the idea that scientific subjects, and especially mathematics among them, are not a good choice for their education and career. Science is still largely seen as a set of disciplines in which the visual component is essential; this is why blind and partially sighted people are often advised, in spite of their natural inclinations and talents, to choose alternative professional paths. Nevertheless, it is not difficult to disprove this stigma with a few examples of successful blind or partially sighted scientists.

The first blind scientist in history was indeed a mathematician. Nicholas Saunderson (1682-1739) was a British mathematician and physicist from Yorkshire; he lost his sight at the age of one due to smallpox, but his father encouraged him to study and taught him the letters of the alphabet guiding his son's hands on the engraved writings on the grave stones of the Penistone cemetery. Saunderson became very proficient in many subjects, especially in mathematics in which he rapidly surpassed his teachers, only relying on someone that used to read for him. As a self-taught scholar, he was chosen as Lucasian professor at the University of Cambridge, the most prestigious chair of mathematics in the world, and kept the role between 1712 and 1739. He was the inventor of the Saunderson Palpable Arithmetic Table, the first tactical support for blind people to perform mathematics, and member of both the Board of Longitude and the Royal Society of London ([1], [2], [3]).

Another, certainly the most famous, blind mathematician was Leonhard Euler (1707-1783), considered among the four greatest mathematicians of all times. He started losing his sight at the age of 27 and lost it completely later, but he published almost 900 scientific discoveries and inventions, many of which after his sight started declining. In particular, he produced almost a third of his results in the first seven years after completely losing his sight at the age of sixty ([4], [5], [6]). Other notable blind mathematicians include Joseph Plateau, Norberto Salinas, Lev Semenovich Pontryagin, Bernard Morin and Lawrence Baggett.

However, despite the evidence provided by many blind or partially sighted scientists throughout the centuries and the theories by Italian typhlologist Augusto Romagnoli, who encouraged blind and partially sighted people to study mathematics, a discipline he considered free from the often misleading need for a visual support, the hostile environment is still a reality in some cases, and of course it was even worse in the first decades of the twentieth century, when Abraham Nemeth was born.

2. BIOGRAPHY OF ABRAHAM NEMETH

Abraham Nemeth was born in New York's Lower East Side on 16 October 1918 from a family of Hungarian Jewish immigrants. Because of a combination of a macular degeneration and retinitis pigmentosa, he was

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never able to sense light, at least ophthalmologically speaking, but, as he states in [7], his family always encouraged him to challenge himself without being oppressive. His father was a very influential figure during his early years: he used to take Abraham around the streets of New York, describing every change of direction, indicating the cardinal directions they were moving to and guiding his son’s hands on domed letters on objects such as mailboxes and shop banners.

Nemeth’s family was far from wealthy and Abraham was sent to a little public school in the Lower East Side: in spite of the stereotypes, this institute proved to be inclusive and efficient and Nemeth rapidly gained good command of Braille and managed to catch up with the rest of his classmates. It was in his school years that he noticed a worrying lack of attention on mathematics for blind people, all his major scientific contributions were probably inspired by his own personal experience during those years and by the periods he spent at Brooklyn College ([7]).

He started to attend Columbia University but he was strongly advised not to choose the faculty of mathematics because of the predominant prejudice that wanted this subject not to be accessible to blind people. Thus, Nemeth studied for and obtained a degree in psychology, but he didn’t find a job in that field and was forced to employ at the American Foundation for the Blind with a marginal role and a modest salary. It was on this workplace that a colleague, who later became his first wife, told him: "Tell me the truth. Wouldn't you rather be an unemployed mathematician than an unemployed psychologist? ([7])”.

Positively stimulated by this observation, Nemeth went back to Brooklyn College, this time to obtain a degree in mathematics. He undertook this adventure while he was still working at the American Foundation for the Blind, studying after work and developing his own way of representing mathematical formulas: this is how the Nemeth Code started. He had previously noticed from his personal experience that the Braille notation for advanced mathematical objects was poor or non-existent, so he developed a series of surprisingly simple Braille expressions to be correspondent to higher mathematical concepts and notations. Initially he kept the secret of this system, until Clifford Witcher, a blind physicist that used to collaborate with the American Foundation for the Blind, asked him for a Braille table of integrals and Nemeth invited him to learn his coding technique. Witcher was extremely happy with the code and suggested Nemeth to introduce it at a meeting of the Joint Uniform Braille Committee in 1952; the so-called Nemeth Braille Code for Mathematics and Science Notation, often simply indicated as Nemeth Code, was approved on that same day ([7], [8]).

Meanwhile, Nemeth obtained a PhD in mathematics at Wayne State University and progressed to became researcher, LECTURER, associate professor and, finally, full professor at the University of Detroit. Later in his career, he decided to address another problem that undermines the chances of a scientific career for many blind or partially sighted students, especially in the earliest stages of their education. The way mathematics is usually taught, and in particular spoken, tends to generate a surprisingly high level of ambiguity, analyzed in [9] and [10], that produces the effect of leaving visually impaired students, who mostly rely on the way the subject is orally presented, behind their classmates. That is why Nemeth conceived MathSpeak, a groundbreaking method to verbalize mathematics in an accessible way, a system that was incorporated in the first screen readers and adopted by ministerial teachers training programs in almost all English-speaking countries. Abraham Nemeth died in Livonia, Michigan, on 2 October 2013.

3. THE NEMETH CODE

The author in [11] describes a key episode for the creation of the Nemeth Code. During the break of a calculus seminar at Wayne State University, Nemeth was approached by his professor who asked him a few questions about Braille coding. After receiving the answers, the professor commented that it seemed to him that blind people were taught to write at the mirror with invisible ink. In fact, Braille users have to punch the back of the paper, moving from right to left and reproducing the signs in a way that allows them to find understandable messages after rotating the paper. Moreover, it appears clear that they cannot read what they write while they fix it on the paper. Things were even more difficult for the blind or heavily visually impaired students that wanted to approach mathematical contents: in fact, Braille was initially designed to reproduce words; so, while humanistic subjects have always been accessible to Braille users, a representation of mathematics was very limited as adequate and coherent codes for trigonometry, calculus and other branches were incomplete or non-existent.

Since there were no screen readers during the ‘40s, Braille appears as the most effective tool to convey mathematical contents at that time: it appears that, after a period of practice to get good command of the technique, the blind or heavily visual impaired student gains such an immediateness in interpreting Braille coding as to rapidly catch up in speed with seeing people. Nemeth started to develop the principles of his code, which was eventually published in 1952. The need for such a system was perceptible by every blind or heavily visually impaired individual interested in mathematics; however, there were many possible ways to provide a Braille coding of mathematical symbols. Nemeth’s research was focused on three aspects:

- Completeness: his code needed to include the representations of all possible symbols, letters and formulas.
- Coherence: the coding of all symbols, letters and formulas needed to be as close as possible to the

standard ink equivalent.

• Accessibility: the code needed to avoid potential ambiguity that could have occurred when the same Braille coding represented more formulas or concepts.

Braille dots can be arranged in 63 different positions to compose letters and, therefore, words, thus it appears evident that it is impossible to represent all different mathematical symbols plus all the letters of both the Latin and the Greek alphabets with this very limited number of dots combinations. Therefore, Nemeth created arrays of consecutive Braille symbols to represent even the most advanced concepts such as double integrals and matrix determinants. This was the only accessible method to provide such coding in a compact and non-ambiguous way. From letters of the Latin alphabet and numbers, whose coding already existed and that was effortlessly incorporated with no sensible modification, the learner that approaches the Nemeth Code goes through capital letters of the Latin alphabet, coded as their small equivalents preceded by a specific sign, and up to more complex notation, including indexes, subscripts, relation signs, logarithms and functions, as stated in [8]. As far as subscripts and exponents are concerned, the author in [8] underlines that locating them on a different line than the main one, as it is usually done with standard ink writing, could result in inevitable ambiguities for the blind or partially sighted reader; so the Nemeth Code provides univocal representations for them.

The Nemeth Code was explicitly designed to meet the need for a Braille representation of all the concepts that previous methods used to ignore. The effort was successful as it is now possible to represent all symbols, letters and formulas in a number of branches including linear algebra, combinatorics, statistics, analysis, algebra, geometry and logics.

However, if the completeness and the accessibility requirements were met adopting the most economical strategy, finding a number of all different strings to associate with symbols, letters and concepts in a non-ambiguous way, the hardest task was to keep the code coherent. Since its first version, the real strength point of the Nemeth Code was the very close similarity between the coding of mathematical symbols and their ink equivalent. This plays a key role in increasing the shared ground between the blind or partially sighted student and the seeing teacher, reducing the time they have to spend discussing the mechanics of coding and allowing them to focus solely on the mathematical content.

The Nemeth Code has gone through four revisions and a number of minor updates over the years and still represents one of the most useful tools to convey mathematical knowledge and culture to blind and partially sighted people all over the world. It improved the cultural standards of Braille users, allowing them to gain access to an entirely new range of scientific topics and closing the gap with the humanistic subjects. Moreover, it upgraded previous methodology for conveying mathematical contents to blind and heavily visually impaired people and increased their autonomy.

4. MATHSPEAK

If Nemeth's first and most famous invention was designed to open the doors of higher mathematics to blind and partially sighted students, his second effort was to eliminate the gap that a wrong presentation of mathematics can produce, especially in the earliest stages of children's education. The level of ambiguity in spoken mathematics has been investigated only after Nemeth brought the topic under the spotlight: it had no sense to provide a tool for coding higher mathematics in Braille whilst deliberately ignoring the difficulties that can occur in the first school years and that could easily erase every potential interest in the subject.

In fact if the use of a well thought coding allows the expressions to be accessible and non-ambiguous for the blind student, the problem of spoken interaction with the teacher and the colleagues/classmates is still relevant. If not addressed with the right amount of attention and correct methodologies, this issue will always be profoundly subject to the risk of being confusing and ambiguous.

We now present a few examples to show how even simple details in presenting mathematics to blind or partially sighted children can produce ambiguity. In spoken mathematics we often use the word 'over' to describe a fraction, like in 'a over b '; but this can cause confusion in the students that only rely on oral presentation of the object as no Braille coding of fractions, neither the older methods nor the Nemeth Code, represents the numerator over the denominator. The same happens with the spoken expression 'a plus b over c': no one that relies only on hearing can understand if this spoken expression refers to a fraction with denominator c and numerator a+b or if it indicates the sum of a quantity a and a fraction with numerator b and denominator c.

Another notable example is given by the expression \sqrt{a+b} it is usually read as 'square root of a plus b' but if it is the square root of a sum or the sum of two numbers, one being the square root of a quantity and the other being another number outside of the square root sign, will be not clear to a blind listener. A similar example of ambiguity comes with the so-called cross multiply; in fact, no Braille coding puts the four quantities at the corners of a cross as they are represented on the same line.

Authors in [9] and [12] show that, together with fractions, the vast majority of ambiguities arises when the speaker fails to clearly signal the beginning and the end of a construct such as a square root, a grouping or an absolute value. They proved the extent of ambiguity of spoken mathematics with statistical evidence. First of all, they have found that 74.9% of mathematical contents in school books is presented ambiguously. They also surveyed the spoken presentation of this material by school teachers, finding that 86% of ambiguously mathematical expressions were spoken ambiguously. Another observation that arises from their research is the very low level of awareness of this problem among school teachers.

Again, the problem of grasping the concepts conveyed by mathematical objects moves to a lower level which is strictly related, in the case of spoken interaction, to the disambiguation of the expression. This aspect is peculiarly relevant in the case of visually impaired students which have not acquired a sufficient command of reading via the sense of touch; this is the case of partially sighted students who experience a sudden decline of their sight. The problem is particularly complex because only a few people are aware of the potential ambiguities as they automatically suppose that everyone can disambiguate every expression in a glance.

The solution to this problem came when Nemeth developed the first prototype of a series of rules to optimize the accessibility of spoken mathematics in [13], he called it MathSpeak and expanded it during the last decades of the twentieth century. This protocol, to be applied at every level of the education system, prescribes best practice guidelines for correctly reading mathematics without generating ambiguity: for instance, in the previous case related to the square root, MathSpeak indicates to signal when the square root sign starts and ends or, in the case of the fraction, to introduce numerator and denominator separately after explicating the presence of the fraction sign. MathSpeak covers the whole range of mathematical expressions, providing consistent indications on how to pronounce them for the blind or partially sighted student, without renouncing to use, when necessary, shortcuts to save time: this is the case of grouping signs for which the protocol provides efficient abbreviations.

MathSpeak was at the heart of some of the earliest screen readers prototypes and it is still a benchmark to assess the improvements of new methodologies. It has been developed even after Nemeth retired with encouraging results. Authors in [9], [10], and [12] prove the efficiency of MathSpeak both in terms of quality and of time with an extensive statistical study in collaboration with school teachers: they observed that, even after a short period of practice, teachers became able to fully follow MathSpeak protocol and to almost totally eliminate ambiguity from their oral exposition of mathematical formulas. They also provide an algorithm to insert pauses and improve the synthetic speech rendering of mathematical formulas.

5. CONCLUSIONS

Abraham Nemeth has pioneered the research on how to get mathematics accessible to everyone: his inventions accompanied blind and partially sighted students and professionals, including a number of scientists, from the early school years to their workplace. If his life was a stunning example of resilience, his achievements have unvaluable importance to the lives of many people around the world. However, his legacy does not only consists of his inventions: he concretely proved that minority groups can play their role in our society and that their achievements improve the whole environment.

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