The *Rule of Four,* Executive Function and Neural Exercises

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ABSTRACT

Deborah Hughes-Hallet has made many significant contributions to Calculus pedagogy. Among the tools she has introduced is the *rule of four*, which requires successful pedagogy to simultaneously address four approaches to each course concept, *verbal, graphical, algebraic and numeric.* We explore examples of this *rule of X approach* in other disciplines: i) Literary analysis is enhanced through the *rule of two,* a simultaneous approach of grammar and literary analysis; ii) Actuarial mathematics requires a *rule of six,* a simultaneous approach of *verbal, graphical, algebraic, calculator, modules, and English conventions;* (iii) Masters of Tic-Tac-Toe and Chess use a *rule of two*, simultaneously approaching the game *positionally and combinatorically.* We offer a unified and deep analysis of the *rule of X approach* by relating it to executive function, the area of the brain responsible for organizing and synthesizing multiple brain areas. We conclude the paper with an illustration of classroom activities that strengthen executive function and improve pedagogy. Our results are content independent, depending exclusively on paths of information flow, and consequently, our analysis is cybernetic in flavor [1].

Keywords: cybernetics, executive function, multi-dimensional processing, rule of four, Calculus, literary analysis, Chess, Tic-Tac-Toe, neural exercise.

1. OVERVIEW AND OUTLINE

Deborah Hughes-Hallet has made significant contributions to the current revamping of Calculus pedagogy. She has led educational projects with multiple institutions and authored several books. She has aggressively sought to identify critical components needed for eliminating the high failure rate in Calculus courses. One of the conceptual tools introduced by her is the *rule of four* [5,11,12,13].

Very roughly, the rule of four requires the instructor and student to approach each calculus concept, each illustrative problem, each homework exercise and problem, and each project using four methods: verbal, algebraic, graphical and numeric.

Hendel has adapted the *rule of four* to Actuarial mathematics where it has become the *rule of six.* The extra two rules, over and above the four used in, are actuarial English conventions and a rule of syllabus modules [9].

There are other disciplines that use a *rule of X* approach even though these disciplines do not explicitly formulate the multiple approaches using the phrase *rule of X*. In this paper, we will explore Chess, Tic-Tac-Toe and biblical literary analysis; historically, these disciplines have used a *rule of two.*

Sections 2-5 of this paper will present examples illustrating the *rule of X* approach in Calculus, Actuarial Mathematics, Chess/Tic Tac Toe, and biblical literary analysis, respectively. In Section 6, we explain *why* the *rule of X* approach has pedagogic appeal. The *rule of X* approach corresponds to executive function, the brain function that allows an individual to synthesize and integrate multiple disciplines. In other words, simplistically, this approach is successful as a pedagogic rule because it encourages executive function in the instructional arena. Section 7 presents simple classroom activities that are effective in strengthening executive function.

The idea of using multiple approaches is well established in education. For example, project based learning (PBL)[16] emphasizes an interdisciplinary approach. Deborah Hughes-Hallet’s contribution is a) to emphasize the consistent, continual, application of multiple approaches throughout the Calculus course and b) to apply the multiple approach method within one discipline. The contribution of this paper is i) to show that this *rule of X* approach is a useful concept in any course (not just Calculus), ii) to clarify the relationship between the *rule of X* approach and executive function and iii) to illustrate simple classroom activities that strengthen executive function.

2. CALCULUS

We present a simple Calculus modeling problem using a verbal, algebraic, graphical and numeric approach. The example could be presented in both Calculus and Pre-Calculus courses. The example addresses driving efficiency as a function of time.

<u>Verbal:</u> *Model the following: When you first start driving on a long trip you are not yet at peak driving efficiency. As you drive, your driving efficiency increases. After 3 hours of non-stop driving you are at peak efficiency. During the next 3 hours, your driving efficiency begins to decline. At 6 hours your driving efficiency is the same as it was at the beginning of the trip. After 6 hours, your driving efficiency continues to decline.*

<u>Algebraic:</u> In (Pre-) Calculus we learn how to model this data with the function $f(t) = 1- (3-t)^2/36$.

<u>Numeric:</u> Table 1 gives numeric values of driving efficiency as a function of time.

<u>Graphical:</u> Figure 1 graphically portrays both i) the algebraic function as well as ii) the table of numeric data.

Driving Time	0	1	2	3	4	5	6
Driving Efficiency	75%	89%	97%	100%	97%	89%	75%

Table 1: Numeric table for the algebraic function $f(t)=1-(3-t)^2/36$

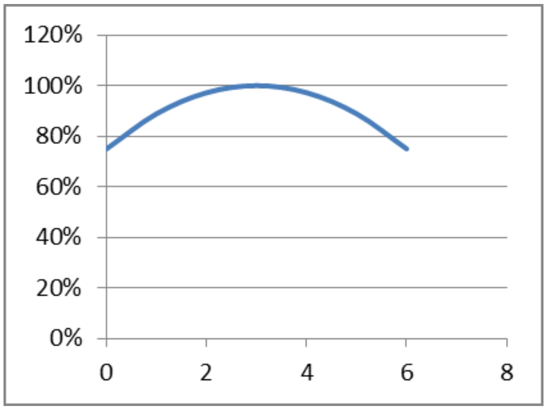

Figure 1: Graphical illustration of the function and numeric data presented in Table 1. The X-axis represents time in hours; the Y-axis represents driving efficiency.

Using this example, we can proceed further. The verbal problem and the graph show a maximum driving efficiency of 100% at time 3 hours. We can also infer this maximum (among integer values) by inspecting Table 1.

In Calculus, the following interplay of graphical and algebraic properties of the maximum are taught:

- Graphical: At a maxima, the *tangent line* of the curve is *flat*
- Formal: The geometric term *flat* numerically means 0 slope. The geometric term *tangent line* algebraically refers to the derivative function. We conclude that at a maxima, the derivative is 0.

We can in fact verify that $f'(t)=-2(3-t)/36 =0$ at $t=3$, as required.

Here again we see a convergence of verbal (peak efficiency), formal ($f'=0$), graphical (flat tangent line) and numerical (maximum row value) to describe the concept of maxima.

We now summarize this example and add the relationship with executive function that we will present in Section 6: *The rule of four requires both the instructor and student, in each concept, illustrative problem, homework exercise or problem and class project, to use four areas of the brain: i) verbal area ii) visual area (graphics), iii) numeric area and iv) abstract formal area. This simultaneous use of multiple areas of the brain requires executive function.*

3. ACTUARIAL MATHEMATICS

As remarked in Section 1, Hendel uses a *rule of six* when teaching Actuarial Mathematics. We use the following problem from the Society of Actuary examinations to illustrate.

Problem [17]: A i) ten year ii) $100 par value bond pays iii) 8% coupons *semiannually*. The bond is iv) priced at $118.20 to yield an annual nominal rate of v) 6% convertible semiannually. Calculate the vi) redemption value of the bond.

Module Identification: Actuarial Mathematics has distinct modules each with its own language, convention and formulae. The presented problem belongs to the *Bond* module. A bond is a technical term indicating a buyer who pays a *price* for the bond instrument at time 0, in exchange for which the buyer receives both a *redemption value, C,* at the maturity date, *n,* and possibly also, *coupons* at periodic intervals up to and including the maturity date. The *coupons* are expressed as a *coupon percentage, r,* of the *face value, F.*

English Conventions: In the statement of the problem, we underlined the English phrase nominal rate convertible semiannually. By *convention*, a 6% nominal rate convertible semiannually means a 3% rate every 6 months. Similarly, by convention, coupon rates are interpreted nominally. So the 8% coupon rate is interpreted as a nominal rate, that is, a 4% coupon rate every 6 months.

Verbal: In the verbal approach to a problem we seek to create a *correspondence* between keywords and algebraic variables. One exercise (that we will explore further in section 7) facilitating such a correspondence, requires identification of all keywords. Towards this end we have identified items i) – vi) in the problem statement (These indicators do not occur in the actual problem statement but are supplied by students (either mentally or explicitly) when they do the problem).

We can think of this correspondence as a sort of a verbal-algebraic dictionary. For this reason, in application of *the rule of six*, we treat separately English conventions and module identification.

- The module identification is not part of the verbal-algebraic dictionary. Rather, the module identification, *Bonds*, tells us which variables are needed in this type of problem. Bond problems require identification of 6 variables.
- Similarly, the English conventions are not part of the verbal-algebraic dictionary. Rather, the English conventions reinterpret certain English phrases (such as nominal convertible semi-annually) into equivalent English phrases.

Corresponding to the 6 labeled verbal items i) – vi) we have the following correspondence presented in Table 2.

Label	i)	ii)	iii)
Verbal	ten year … semi annually	100 par value bond	8% coupons … Semi annually
Algebraic	n=20	F=100	r=4%

Table 2a: Verbal-algebraic correspondence for the keywords in the problem presented in Section 3. The Module identification indicates the set of required variables. The English conventions require conversion of certain English phrases. For further details see the text. Note, Table 2a is continued in Table 2b.

Label	iv)	v)	vi)
Verbal	Priced at $118.20	Yield … 3%	Redemption value
Algebraic	P=118.20	i=3%	C

Table 2b: Continuation of Table 2a.

Timeline: The problem can be compactly summarized using the timeline presented in Table 3.

Time	0	1	2	…	20
Cash flow	-$118.20	$4	$4	…	$4+C

Table 3: Timeline for the problem. The buyer pays $118.20 at time 0. The buyer then receives 4%*F = 4% x 100 = $4,* at the end of each period for all 20 periods. Additionally, the buyer receives an unknown redemption value, *C,* at the maturity date of 20 periods (10 years).

Formal: Each module has an *equation of value* relating inflows and outflows. In this case, the formula relates a paid price amount (outflow) with the two inflows of a one-time redemption value and a periodic cash flow of $4 per period. The formula for bond price, given by the *bond* module, is

$$P = Fr\,(1-v^n)/i + C\,v^n$$

with $v = 1/(1+i)$ [20].

Calculator: Unlike Calculus, which uses a numeric component in the *rule of four,* Actuarial Mathematics uses a calculator component. It should be emphasized that the calculator component is useful in defining levels of pedagogic challenge. Roughly speaking, if a problem can be done by pressing keys it is *simple.* A more challenging problem might have several component simple problems. Without the calculator however, the multi-component problem might not be solvable quickly. In this way, the calculator facilitates defining challenging problems. Table 4 presents the calculator line for this problem (and enables arriving quickly at a numerical answer)

BA II Plus Key	**N**	**I**	**PV**	**PMT**	**FV**
Entry	20	3	-118.20	4	CPT

Table 4: Computation of the redemption value. The BA-II Plus is the name of a calculator. The *N* key corresponds to the number of periods, the *I* key corresponds to the yield (and is entered as a number not as a percent), *PV* corresponds to the price, $118.20, and *PMT* corresponds to the periodic coupon payment of $4. By hitting *CPT FV* (compute future value) the calculator gives the value in the *future,* at the end of 20 periods, which is the redemption value, *C.*

In summary, in teaching Actuarial Mathematics six issues are addressed. Each module is identified with a collection of variables, description, and formulae. Before creating a verbal-algebraic dictionary for any problem we must be aware of English conventions, some of which apply throughout the course, and some of which apply to particular modules. Once the algebraic-verbal dictionary is identified we can approach the problem through a formula, through a timeline and/or through a calculator line.

Omitting any one of these six approaches can cause difficulties to students. For example, the author frequently sees students who know how to create an algebraic-verbal dictionary, know how to use the calculator, and know how to manipulate formulae and create timelines. However, if these students are unaware of English conventions they will get the problem wrong. The remedy to this is not to urge more practice but rather to identify the English conventions needed.

With an eye to Section 6, we see that mastery of Actuarial Mathematics requires executive function integrating six distinct brain areas: verbal, formal, visual (timelines), mechanical (calculator timelines) as well as mastery of a module hierarchy for the subject along with a repertoire of English conventions.

4. CHESS AND TIC-TAC-TOE

It may appear strange to discuss chess in a paper on teaching college level content. But historically, Chess was one of the first disciplines to change from a person to a skill approach. Therefore, some historical background will first be given.

Historically, chess went through what historians call the *romantic* era. During this era, good players were considered to be geniuses. It was William Steinitz who helped change the direction of Chess theory and instruction from a person-centered approach to a skill- centered approach. Steinitz accomplished this by changing the emphasis of theory from *combinatorics* to *positional* play [10,14].

A combination is a sequence of moves with certain (more or less) forced outcomes [21]. The combinatoricist typically can see many moves in advance and hence the combinatoricist appears to be a genius. Very often, combinations involve giving up pieces and yet results in a win. During the romantic era, the method of play was combinatoric. The games that were won always had an element of surprise and aesthetic appeal, hence the name *romantic era.* The players that won these games were considered geniuses because of their ability to carefully think several moves in advance and to understand all possible responses.

Steinitz introduced the idea of *positional* play. In positional play, one does not necessarily look ahead. The idea of positional play is that certain squares on the chessboard are *worth more.* If a player has pieces on these squares, or, if a player has pieces that can move to these squares, that player has a positional advantage. Steinitz wrote extensively and showed that combinations happen in positions with advantage and typically do not happen in positions without positional advantage.

Since the attributes that make up good positions (control of certain squares) could be taught, it followed that winning chess was an attainable skill. One should not look for combinations unless one first has a positional advantage. If one does have a positional advantage, the combination will follow.

Today all chess theorists and instructors acknowledge the need for an approach to games using both combinatorics and positional analysis. We may consider this a *rule of two* even though chess instructors and theorists do not explicitly call it that. In fact, there are other *rules* for chess mastery. For example, chess instructors require special knowledge of the opening moves and special knowledge of endgames, situations in the game when most pieces are gone from the board. For purposes of this paper, and to show relationship with the other *rules of X,* we suffice with a *rule of two.*

To illustrate these ideas with an example would be difficult. Chess notation and chess positions are technical. Besides the typographical problems in showing many positions in a 6 page paper, the reader most likely is not familiar with many facets of chess (perhaps not at all). Therefore the exposition would be difficult to follow.

Consequently, for illustrative purposes, we use Tic-Tac-Toe. Tic-Tac-Toe is easily describable and most people are familiar with it. Winning Tic-Tac-Toe requires a *rule of two*. Thus the Tic-Tac-Toe example presented below is illustrative of Chess (which of course is more complicated).

Table 5 below is a sample Tic-Tac-Toe game. The following features are noted (the notation was developed for this paper):

- Tic-Tac-Toe is played on a 3 x 3 board
- There are two players. We will call them O and X
- The players alternate in placing their letter in empty squares
- The game is won when 3 identical letters form a line (horizontal, vertical or diagonal)
- We indicate moves in the game by using a notation of a letter followed by a number.
    - For example, in the game in Table 5, the game was started by O, who placed his letter in the center square. Hence there is an "O(1)" in the center square.
    - X made the second move; X placed his letter in the center top row. Hence this square has "X(2)."

O(3)	X (2)	
	O (1)	
O(5)		X(4)

Table 5: An illustrative Tic-Tac-Toe game

Using the above game we show positional and combinatoric play. Recall our definitions above: Positional play means making a move because of attributes of the position while combinatoric play means making a move because of an analysis several moves in advance and consideration that a win is forced.

Positional play: We can count how many possible lines there are through each type of square. We find

- 4 lines through the center square
- 3 lines through each corner square (horizontal, vertical and diagonal)
- 2 lines through the center square of the outer rows and columns.

Consequently, positionally, the center square is the *best* square. Here, classification of this square as *best*, does not predict a win but rather focuses on attributes of the square. For this reason, O made the first move in the center.

On the 2nd move, X played in the center top row. But that is an inferior square. X should have played in a corner square. So far we don't see a way for O to win; but positionally, X's position is inferior.

Combinatoric play: Steinitz taught that in a superior position, one looks for combinations. O has a combination starting with the 3rd move by playing in the upper left corner (O(3)). Let us analyze the combination.

- O threatens to win on move 5 by placing an O in the lower right corner. O would then have a line of Os (diagonal)
- X stops this by playing in the lower right corner on move 4 (X(4))
- O now plays in the lower left corner (O(5)).
  Notice how O threatens to win on move 7 by *either* making a column of Os on the left column or a diagonal of Os on the diagonal from the lower left to upper right. Since X can't stop two threatened wins, O must win.

With an eye towards Section 6, we note that mastery of Tic-Tac-Toe requires both visual analysis (positional play-attributes of squares) and complex logical analysis (give and take and analysis of all possible 3-move sequences).

5. BIBLICAL LITERARY ANALYSIS

Like Chess, biblical literary analysis has a rich history with many periods. Also like Chess, most literary analysts would require a multi-disciplinary approach involving many rules [19]. For purposes of this paper, it was desirable to include a non-technical example (calculus, chess and actuarial mathematics are all technical disciplines) and illustrate a modest *rule of two*. We present a summary of a biblical passage below and then analyze it using a homiletic approach and a grammatical approach.

Like Chess, the *rule of two*, mirrors a historical situation. There was a period when the homiletic approach dominated. The homiletic approach seeks to use the biblical text to derive moral exhortations and lessons. This was followed by periods when the grammatical approach was used. The grammatical approach seeks to identify grammatical anomalies in the biblical text and possibly infer nuances of meaning.

The story [3]: M*oses had led the Jewish people out of Egypt. The Jewish people were promised Israel. Moses sent spies to explore the land for strengths and weaknesses and report back. The spies were each tribal governors. There were 12 spies. In the end, ten of them said that the land was unconquerable and that they should return to Egypt. Two of them said God could help the Jews conquer Israel.*

Let us look at some verses about the act of spying. *They spied from the Tzin desert to Chamath They went south, he came to Chevron where the giants lived…they came to the cluster river and they cut a cluster born by pole with two bearers and also from the figs and pomegranates.*

Homiletic approach: The homiletic approach searches for the classical four elements of literary analysis: *irony, paradox, ambiguity and tension*. The existence of irony, paradox, ambiguity and tension may be sufficient to justify the homily. The homiletic approach does not require further justification of the homily using textual or other sources.

In this case, we know that i) one of the spies was Calev, ii) Calev was one of the two spies with a minority opinion that the land could be conquered with God's help, and iii) the Bible indicates that Calev was rewarded with extra plots of land in Israel for standing up for himself.

Literary analysis asks us to explore the tension and ambiguity Calev must have been going through. After all, he was in a minority and was disagreeing with 10 other tribal governors. Didn't he have doubts in his thoughts? The homiletic literature offers the following homily proposing an answer to this question: *The biblical narrative mentions that the spies passed by Chevron, the burial place of the patriarchs. Undoubtedly, Calev stopped over to pray at the grave of the patriarchs to strengthen his resolve to disagree with his fellow governors.*

We again emphasize that the sole motivating factor for the homily is the fact of tension and ambiguity. Now let us examine the same homily from a grammatical point of view.

Grammatical approach: The grammatical approach notes the following: *In the biblical narrative cited above, all subjects are plural (they they they) except for the passage "He came to Chevron." We infer that Calev alone went to Chevron to pray for strength to keep his resolve.*

Summary: Using literary analysis, one can deeply appreciate how the rule of two operates. Already, literary analysis itself contains elements of *twoness*. Indeed, doubt, ambiguity and tension definitionally imply a doubt about which of *two* interpretations to use.

However, this *twoness* is exclusively in the literary sphere. Consequently, it lacks objectivity. By going to the formal grammatical sphere we acquire this objectivity: Why the contrast of *he* and *they?* Who is the *he* referring to? The grammatical sphere emphasizes and points but its emphasis is crystalized only with literary analysis.

It is precisely by using a *rule of two*, that we obtain a holistic analysis. We simultaneously benefit from the formal objectivity of grammar and the rich emotional empathy of literary analysis. In other words, the *rule of two* gives us a sense of completeness of approach with its consequent satisfaction.

Finally, with an eye to Section 6, we see that proper biblical literary analysis addresses two separate areas of the brain: One dealing with feeling and empathy and one dealing with technical grammatical form.

6. EXECUTIVE FUNCTION

In previous sections, we have suggested that the reason that the *rule of X* approach has pedagogical appeal is precisely because it integrates diverse areas of brain function. The capacity to integrate diverse areas of brain function is itself a brain function called executive function. In this section, we briefly review the definition and psychological tests associated with executive function [2,18].

There are multiple executive function tests the two main categories being performance and rating tests. We examine two well-known performance tests.

The Wisconsin Card Sorting Test (WCST) [8]: During the administration of the WCST, the examiner flashes several dozen two-row items such as those found in an illustrative example found in Figure 2. The examinee is asked to match the card in the bottom row with the appropriate card in the top row.

Figure 2 presents three dimensions: a) letter (A,B,C), b) formatting (bold, italic, underline), and c) number (1,2,3). The examinee must determine if the text in the bottom row of Figure 2 i) resembles the A card because of the dimension of letter, ii) resembles the B card because of the dimension of number, or (iii) resembles the C card because of the dimension of formatting (both are underlined).

A	*BB*	CCC
AA		

Figure 2**:** A sample item in the WCST. Throughout this section, performance tests have been modified, from their standard format, for typographical reasons and reasons of space.

Typically, after a few attempts the examinee will discover the correct driver of resemblance. The examinee will then have a streak of correct answers. The examiner may then change the driving dimension. For example, if in the last 10 trials the correct answer was based on a match of number, the examinee may create new trials where the correct match is based on the dimension of letter resemblance.

A wealth of information is gathered during the test. For our purposes, we see that the examinee is being tested on *his/her capacity to correctly identify the driving dimension from a set of competing multiple brain areas (formatting, number, and letter). Furthermore, as time progresses the examinee must continuously reassess the correct driver of correctness.*

We conclude that the WCST is measuring the capacity of the examinee to *continuously process multiple-dimensional drivers of outcome in different brain areas.*

The Trailmaking test [4,6,7]: This deceptive but beautiful test has two parts: A and B. In both parts, the examinee is asked to use a pencil to connect items on a piece of paper into a trail. In part A, the trail is 1-2-3-…, while in part B, the trail is 1-A-2-B-3-C…. An illustrative example is presented in Figure 3. Although these tasks are easy, remarkably, the part B test always takes longer. The increased length is due to the presence of two dimensions or two brain areas: number and letter. The multi-dimensionality requires executive function and hence the increased time length. Despite the test's simplicity, it is useful in diagnosing brain damage and recovery possibility, for example after a stroke.

2	4	1	3	B	A	1	2

Figure 3**:** A sample Trailmaking test.

The simplicity of this test highlights the importance of our proposed explanation that the driver of good pedagogy is multi-dimensional processing. The trailmaking test is making the powerful point that *any* multi-dimensional processing transforms a mundane exercise into executive-function quality. Indeed, just adding the dimensions of letter to the dimension of number in the simple task of making a trail raises the quality of the task to executive-function quality.

Summary: We have identified the *rule of X approach* as an important pedagogical approach. The *rule of X approach* requires integrating skills from multiple brain areas. But this is exactly the definition of executive function. We conclude that the driver of appeal of the *rule of X approach* is its emphasis on executive function.

7. NEURAL ACTIVITIES

In the previous sections, we have explored the *rule of X* approach and its relation to executive function. It is natural to ask for activities that strengthen executive function.

One approach, adopted by Hughes-Hallet, is the skillful construction of projects with the requirement that project problems be addressed using the *rule of 4*, that is, that problems be solved on a graphical, numerical, algebraic and verbal level.

Another approach, adopted in my classrooms, is to *practice* creating verbal-algebraic dictionaries. This was illustrated in the problem presented in Section 3. There we showed how to focus on each phrase in a target text and identify its algebraic correlate. This can also be done in literary analysis, exploring each phrase and sentence for grammatical anomalies as well as for elements of irony, paradox, tension and ambiguity.

The following dialogue that occurred in a class of mine illustrates the utility of such exercises.

> Me: Explain the nuances of *100 par value bond.*
> Student 1: *Par Value* indicates *Face amount*, which is 100.
> Me: That is true.
> Me: But you have explained an English phrase *100 par value* with an English correlate, *Face value.* You are only using one area of your brain.
> Me: Can someone else explain the phrase using multiple domains.
> Student 2: F=100.
> Me: That is correct. You have translated the *English* phrase into an *Algebraic* phrase.

Note the resistance of Student 1 in using two brain areas. This justifies the use of such activities in class to strengthen executive function. In a typical class, the author might go around the room several times letting each student tackle one phrase.

8. CONCLUSION

The ideas presented in this paper have tremendous potential for application to pedagogy. The approach of this paper has applicability to other disciplines such as the teaching of Geometry. We have already pointed out that actuarial examination problems frequently emphasize problems that require multiple approaches. We would advocate that model curricula, such as found in the Common Core Standards, and placement examinations such as the Advanced Placement Tests, adopt the rule of 4 as a basis for suitable problems.

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