

# Inspiring and Challenging Laboratory Exercise in Multivariable Control Theory – The Four-rotor Helicopter

Dag A. H. Samuelsen  
Department for technology, Buskerud University College  
3611 Kongsberg, Norway  
[Dag.Samuelsen@hibu.no](mailto:Dag.Samuelsen@hibu.no)

And

Olaf H. Graven  
Department for technology, Buskerud University College  
3611 Kongsberg, Norway  
[Olaf.Hallan.Graven@hibu.no](mailto:Olaf.Hallan.Graven@hibu.no)

## ABSTRACT

Engineering students in a module on multivariable control theory are given a laboratory exercise for developing their skills in practical implementation of control systems. This is done in an effort to create a more complete module that gives the students experiences in the practical sides of implementing control systems, while still being theoretically challenging and inspiring. Presenting students with this kind of real-life challenges like sub-optimal models, limited processing time and large degree of uncertainty, is a challenging task, partly due to the need of adapting the level of complexity to the student or group of students doing the exercise in order to keep them engaged throughout the exercise, and in part due to the university's need to reduce expenses related to the administration, supervision, and execution of laboratory exercises. The possibility of adapting the complexity of the exercise to each student's skill level is important, both through the design of the exercise and through the students choosing between different models. The eager student might be tempted by the better performing, but more complex models, while the struggling student can find satisfaction in stabilising the aircraft using the less complex models. The laboratory setup presented uses low-cost components, giving low investment and maintenance costs.

**Keywords:** DSP, helicopter, multivariable feedback control

## INTRODUCTION

Over last few decades the four-rotor or quadrotor helicopter [1], has been in existence as a full scale vehicle. The quadrotor helicopter as a small scale vehicle is also used in some universities as a training platform for students, in particular master and PhD students. The use of a quadrotor helicopter as a small vehicle is an attractive approach for several reasons: The low cost of the building materials, high reliability, and the use of a simple mechanical construction that is easy to continuously alter and adjust to fit the specific needs of the user. The low cost stems from the use of low cost components, such as simple standard available mechanical parts, standard power converters

used in hobby R/C devices, low cost processors, and simple motors without gears. The latter part also introduces high reliability in the sense that there are few moving parts which introduces wear and tear on the device. This is in stark contrast to the standard helicopter, with a main rotor and a tail rotor. A standard helicopter has a highly complex mechanical system to interconnect the rotors. A quadrotor helicopter uses highly complex control structures to control the speed of the four rotors independently in order to give the aircraft balance and controlled movement. This control structures are implemented in digital hardware giving added reliability and the reduced cost.

The implementation of the control structure and algorithms is even today an unresolved matter, making it a research topic of several institutions [2-5]. Sub-optimal solutions to this problem do exist, and today's implementations of the four-rotor helicopter use a simplified model [6] which is possible to balance out using the known control theory and the processing power available in the aircraft. The processing power is limited first and foremost by the power consumption, as the power consumed in the processor(s) will effectively limit the flying time of the aircraft. Low power DSPs can have a power consumption around 1W [7], but the processing power is then limited and only allows for simple models and low order controllers. Medium power DSPs have more processing power, but power consumption is then increased both in the DSP itself and in the external memory banks which needs to be introduced [8]. This may also require cooling fans. For very complex models and controllers, several DSPs are necessary to handle the amount of data processing and gives an unrealistic solution in terms of both power consumption and size. Cost is normally not a limiting factor in this context.

The known control algorithms for the four-rotor helicopter are simple enough to make a realistic implementation in an advanced control module of an engineering bachelor degree [9]. The laboratory installation is a low cost device, affordable for many institutions, and the maintenance cost (as students tend to break things they experiment with) will also be low due to low component cost and simple mechanical construction. It is with such a setup possible to give the students all the different challenges associated with this type of control problem: limited knowledge of the true models of the system that are to be

controlled, limited time and processing power to run the control loop algorithm, and the student's limited experience with problem solving of this type. When students starts working on problems for which they have limited experience with, it affects their ability to reach a working solution, and any solution is greatly affected by choices made in early stages of the controller synthesis. When success fail to come after completing an iteration, it is not always immediately obvious that an earlier bad choice might be the main reason for the control system failing. It is then important for the tutor to intervene and set the students on the right track. This may involve everything from simple steps like tuning parameters, to the need to change the control structure or even designing a completely new model. The students need to understand that all iteration of the controller design process must actually contain an evaluation and assessment of the model and a potential redesign. Knowing what to look for in order to assess what level of redesign is needed requires vast experience, and includes an analysis that tells whether the whole controller structure needs redesign, or just minor parts of the control. Parameter tuning is assumed to be done within the design process iteration. These types of challenges are typical for control system development for the not-so-many-years-of-experience engineer, and are therefore in the authors' opinion an important lesson to experience for the students.

At the authors' institution, the final year bachelor students in electrical engineering complete a module in multivariable control theory. The learning objectives of the exercise is that the students are be able to set up models of basic multivariable processes, set up different controller structures and find the controller parameters for the same processes, and find the properties of the combined systems consisting of process, perturbation block and controller regarding both nominal and robust stability and performance. Doing this on a theoretical basis by simulations will give the students only parts of the real-life physical challenges multivariable control in the presence of uncertainties poses to engineers. Thus, there is a definite need for a hands-on physical laboratory for the students to work on. It is also desired that this laboratory also should give the students experience in the previously mentioned challenges:

- How limited knowledge of the true models of the system poses a challenge as to how to model the uncertainty covering for both the neglected dynamics and the unknown dynamics, which are both represented in some way by perturbations of the nominal system. Failure to cover all possible perturbations from the nominal model might cause the system to be unstable for a set of states. And the understanding that this is clearly not desirable and why.
- How limited time and processing power will effectively limit the number of calculations or processor cycles that can be used to calculate the next command signal. How a control loop is typically performed by using a timed signal to do measurements of the process output at fixed periodic intervals. Then how to do calculations based on measurements and previous controller states and then sending the command signal to the actuators to "push"



**Figure 1: Basic design of the helicopter**

the process in the desired direction. In order for the controller to be able to stabilise the process, the control loop has to be run at a specific rate, giving the sampling rate of the system. When the speed of the processor is limited (as it always will be), this gives the limit on the combination of sampling rate and number of calculations done in the control loop. The students should be able to maximise the utilisation of this limit but still keep within the limit.

- The knowledge of the students varies, and each student have to decide at what level the controller design should be laid, so that the control problem can be solved within the given time period set aside for completing the laboratory work within the module.

When the students start on this module, they have theoretical as well as practical skills in a number of areas from previous completed modules. The prerequisite knowledge needed for mastering this module is:

- Mathematics with linear system theory where they are supposed to develop basic mathematical models for physical systems and processes, mainly limited to amplification, time delay, 1<sup>st</sup>, and 2<sup>nd</sup> order linear, time invariant systems. This part also includes mathematical tools such as complex numbers.
- Control theory for single input - single output (SISO) systems. These modules cover the standard feedback and feed forward control loop with both continuous and discrete controllers, as well as stability and performance analysis.
- Basic programming of microcontrollers in C, with emphasis on the control of peripheral units and other basic tasks.

The purpose of the module for which the exercise program is described in this article, is to further extend this knowledge and skills to master the control of multivariable processes. The learning objectives of the module are [10]:

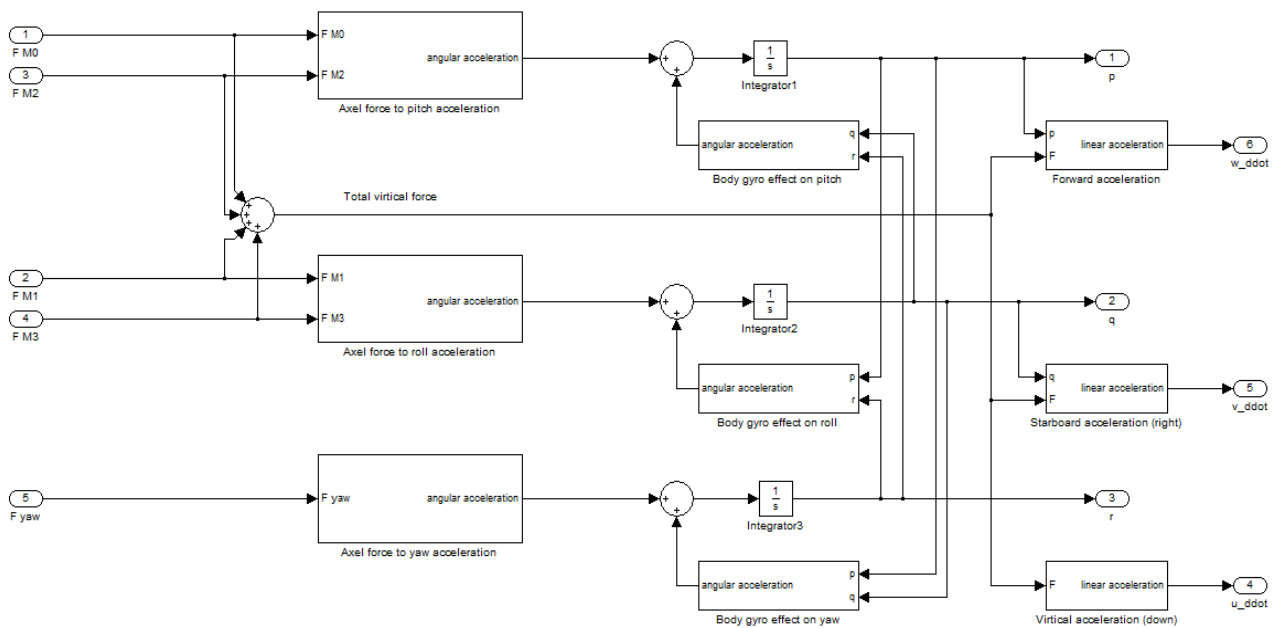


Figure 2: Flight dynamics for the model developed by the student group

- Weighted sensitivity for SISO systems.
- Linear system theory: Coprime factorization, State controllability and observability, Stability, Zeros, Internal stability, Nyquist stability, Norms. H2-norm, H-infinity-norm, Hankel norm.
- Limitations on performance in SISO and multiple input multiple output (MIMO) systems.
- Uncertainty and robustness for SISO systems.
- Robust stability and performance analysis
- Controller design and controller structure design, including LQG and H-infinity methods

### LABORATORY EXERCISE

The purpose of the laboratory exercise is to aid theoretical understanding by creating a link between the theoretical part and the physical world that is to be controlled, and to develop practical skills needed to perform the task of creating a control system. Thus, the laboratory exercise includes the creation of mathematical models for the process to be controlled, synthesis of a controller, simulations and uncertainty analysis with regard to stability and performance to verify the controller operations on the model and all of its perturbations, discretisation and application of the controller on the physical system for a final verification that the controller actually is apt for controlling the physical system. This is the point where the first iteration finishes, and the performance of the practically implemented controller is assessed. Normally, a number of weak points is found at this stage, and the controller performance and stability

is thoroughly investigated in cooperation between the students and the tutor. Specific measures can now be taken to alleviate the weaknesses of the controller, and this often includes redesign of the model, the controller or the synthesis of the controller, as the design process goes into its second iteration. Based on an estimate of the time required to perform the selected redesign, an important decision for the tutor to make at this point is whether the students should actually alter the system, or select a simpler, less time consuming redesign. Students doing a module find themselves in the (un)fortunate situation that the deadline is fixed, and any achievements gained after the deadline is of no value for their grades (although their self esteem might be affected). After the final iteration, it is usually necessary to tune the parameters of the controller in order to optimise the performance and stability properties of the physical system, and time has to be set aside for this as well.

The aircraft used in the laboratory exercise consists of a stiff frame with four arms perpendicular to each other for mounting of the motors driving the propellers. The power source is a high-capacity, lightweight battery pack, with low loss voltage converters to supply the microcontrollers/DSP processors, and power converters for controlling the speed of the motors. There is an option for several processors in the aircraft, each with a specific list of tasks. The main processor has significantly more processing power than the others, and is used for running the control loop algorithms. The other processors can be set aside to do other tasks not running in the main processor, such as communication, sensor interfacing, simple signal processing, and housekeeping in general. Some of these are mandatory, while the presence of others is up to the students to decide on.

The helicopter is given ready assembled to the students in order for them to have their full focus on development of the control algorithms. Which choices are then presented for the

## LABORATORY TRIALS

students? In the aircraft given to the students, the main processor will be a digital signal processor (DSP) [11], as the processing power needed to stabilise the aircraft is considered too much for a normal microcontroller. Installed is also a battery pack with fixed capacity, normal housekeeping circuits for power supervision, security, charging, and receiver for the remote control. The sensors available to the students are a solid state gyro and an accelerometer. The readings from these sensors are not very accurate and filtering through a Kalman filter/observer is highly recommended. This filter will have to be integrated within the control loop, and it is therefore natural to run the filter algorithm in the main processor. The helicopter can receive radio signals from a normal remote control for model aircrafts. In order for the helicopter to be controlled by a user via the remote control, whenever the receiver mounted in the aircraft receives control signal this must be read by the main processor or possibly buffered by any other microcontroller the students decide to put into the aircraft.

The helicopter and the motors must also be modelled by the students, and the choice of model will highly influence how well the control system will perform, or even if it is possible to synthesise a controller for the process model. The full model itself is not challenging to achieve, the main challenge lies in the highly non-linear characteristics of the model [12]. The mathematical tools for verification of controllers for non-linear models are limited, and hence the ability to develop stable, robust controllers is likewise limited. In this context, the full helicopter model has to be reduced [13] to a sub-model for which the mathematical tools exist. This model reduction is one of the main issues of the laboratory exercise, and is left for the students to handle.

When a suitable simplified model is found, the controller has to be synthesised and discretised so that it can be programmed as part of the software running in the main processor of the aircraft. As part of the controller discretisation, a specific sampling rate for the control loop is chosen. The first limiting factor is that a low sampling rate means that the command signal is updated too slowly to counteract unstable, low or mid frequency dynamics of the aircraft. So the sampling rate has to be fast enough. As a consequence of the limits set on the sampling frequency the limited processing power effectively dictates how many processor cycles that are available to calculate the next command signal to the motors. Due to the requirement for low power consumption, the DSP [11] in the aircraft has limited operating frequency. However the processor included has floating point capabilities. The processing limitations has the impact that the students will have to write efficient code, based on models that are optimal in the crossing point between model complexity and processing time of the controller for that specific model. This challenge is further complicated by the other limiting factors: limited knowledge of creating controllers for non-linear models, and the limited knowledge and skills the student or student group exhibit.

An average student group is expected to implement a controller which is able to stabilise the aircraft in the presence of small, possibly ramped changes in reference. For better performing groups, large, rapid steps are necessary to handle while keeping the aircraft stable, and also requirements on performance in the presence of model deviations may be set for the control system. For student groups with achievements below average, the aircraft must be possible to stabilise using the remote control for the aircraft.

The four-rotor helicopter model has been tried by a student group of 6 students in their final year project where the task given was to develop control algorithms to stabilize the helicopter so that is possible for a person with just basic training to fly the helicopter. The assignment included design and construction of the hardware, which is not a part of the assignment that will be given to future students within the multivariable control theory module. The main reason is that the hardware construction would take up too much time and that hardware construction is not a part of the learning objectives in the module. The project group managed to synthesise and tune a fairly good controller for the aircraft, meaning that the aircraft as difficult to control, but manageable. The students were considered to be above average in theoretical understanding and skills. When extracting the time resources spent on modelling the aircraft and the design of the control system, the students in the project group used on average less than 100 hours each, but stated that more hours spent on the design of the controller structure and tuning of parameters would likely result in a better performing aircraft. The students were not specifically prepared for this task, and were distracted in the project period by other side-activities such as group administration; meaning that 100 hours per person for a 3 person group is deemed sufficient for the laboratory exercise is more focused on the specific tasks.

In a 10 ECTS module, a student is expected to put a total of about 300 hours effort, including lectures, exercises, self-study, laboratory work preparing for exam and exam. It is the authors' plan that in this module in multivariable control theory a student would typically spend 60-70 hours on lectures and exercises, and equally much on self-study. Adding the exam preparation and exam, this leaves between 130 and 150 hours for the laboratory exercise project. An average student would then typically need more hours to complete the project at an acceptable level than did the student group in the trial. It is the authors' opinion that the laboratory is within the student's ability if groups are formed with about 3 students in each. In order to give all students a feeling of satisfaction the assignment will be divided into different levels, where the first assignment would be to stabilise the aircraft around one axis at a time, before moving on to the multivariable problem. This will allow the students to gradually approach the multivariable problem, while acquiring an intuitive understanding of the behaviour of the aircraft in relation to the models developed.

Figure 2 shows the Simulink diagram of the flight dynamics as developed by the student group doing the trial case with the four rotor helicopter. For controlling the process, the student group decided mainly to use a set of PI/PID controllers with decoupling, or decentralised control.

## CONCLUSION

In this paper a laboratory exercise setup allowing for hands-on training for students in multivariable feedback control has been presented. Using the four-rotor helicopter, the exercise can be adapted to be challenging enough for any student by letting the students develop models on their own, in search of a better performing control loop, while at the same time the less skilled students might fall back on simpler and well known models in order to stabilise the aircraft. The exercise can be done

evolutionary, in the sense that the students are given basic tasks at the start, like parameter tuning, before engaging in more complex elements of the control system design. In this way the students can be given tasks that are manageable at their own level, and the exercise setup is therefore adaptable to each student's needs and level of skills.

The laboratory has also been shown to give students challenges similar to what they might experience in their career as control engineers, like model errors, parameter uncertainty and limited processing time and power for the control loop. Another important aspect is how students handle their own limited experience with regard to the handling of unsuccessful control loop implementation and the strategies for solving these issues, e.g. the decision between of more parameter tuning or redesign of the controller or model.

The laboratory setup follows the syllabus of a control-theory module, and the steps necessary to complete the exercise are described with an estimate of 130 and 150 hours needed to complete the assignment. The test of the laboratory setup with a group of students seems to confirm the estimates given.

In addition to the shown advantages in learning outcome and engagement for the students of the described setup, the complete system is created with the use of low cost components making this setup an attractive alternative for institutions in need of training students in multivariable control theory.

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