Preservice Mathematics Teachers' Solutions to Problems: Conversions within the Metric System

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Abstract:

This paper reports on the results of a preliminary investigation to determine if preservice mathematics teachers solve conversion problems within the metric system in multiple ways. Here, four metric conversion problems of escalating difficulty were administered within a mathematics methods course at a comprehensive state university in the Northeastern United States. Results show that preservice teachers solved the problems in the same way that they were taught in high school, and that in high school they were only taught one way.

Introduction

The argument for enhanced STEM education is well known in both the K-12 arena and within higher education. Many current initiatives call for improved inclusion of Science, Technology, Engineering, and Mathematics (STEM) topics in the high school curriculum. The Presidents Council of Advisors on Science and Technology (PCAST) called for the creation of STEM-related experiences for today's students while transforming schools into "vibrant" scientific learning communities (PCAST, 2011). The National Research Council (NRC) advocated that in order to support the United States economic well-being and foster scientific innovation, today's schools need increased student participation in science related coursework (NRC, 2011). One of the fundamental topics of both the mathematics and science curriculum is the metric system. The NRC report calls for teachers, and by extension, preservice teachers to be trained in teaching the required material, and also to develop the teaching capabilities and knowledge to help the youth of today be successful in future STEM careers.

The NRC report documented how children best learn STEM topics and noted that when students see connections and relevance to their coursework in school, they become motivated and are more likely to pursue careers in the science world. In order to create vibrant STEM learning communities, teachers need to develop comfort and flexibility in effective teaching strategies, particularly while teaching the metric system (DeMeo, 2008). This topic is also of central issue to the Common Core State Standards for Mathematics (CCSS, 2011) initiative, which calls for students to understand the structure of mathematics and apply that structure to other areas of the curriculum such that skills and understanding are developed together. However, scant research exists on developing teachers' knowledge to effectively teach conversions within the metric system (DeMeo, 2008). Therefore, this study seeks to examine the results of a preliminary pretest instrument to determine how preservice mathematics teachers solve conversion problems within the metric system. Information learned from this preliminary study will be used to inform and guide a larger scale study of the development of preservice mathematics teachers' knowledge of teaching conversion problems within the metric system.

Theoretical Framework

The desirability of multiple solution methods in mathematics classes has been established for some time. Brenner et. al (1997) showed that students who receive representation training were more successful in representing and using different methods to solve a function word problem. The National Council of Teachers of Mathematics (NCTM) continues to advocate that "terminology, definitions, notation, concepts, and skills" (NCTM, 2000, p. 14) emanate from teaching with understanding, and supports the advantages of teaching with multiple methods. More current reviews of research demonstrate that in order for students to learn mathematics with understanding, teachers must be able to present the material within a framework that promotes conceptual and procedural understanding (Hiebert & Grouws, 2007; Rakes et al., 2010). Several studies within physics education (Feldman, 2002; Tao, 2001) established the value of multiple solution methods on classroom instruction. DeMeo's (2008) research on the role of multiple representations in science classes documented the need for specific instruction on multiple solution methods to solve conversion problems, namely the use of both proportional reasoning and dimensional analysis. DeMeo concluded that teachers of mathematics and science should work together to express multiple representations of data, including making the connections between proportionality and the use of linear plots because students who acquire only algorithmic knowledge of solving conversion problems may be unable to transfer their shaky understanding to more complex conversion problems used in science classes. If today's students are not prepared by their mathematics teachers to understand the structures inherent within the metric system and develop flexible ways of solving conversion problems, future work in science becomes increasingly difficult.

Methodology

The purpose of this study is to determine how preservice mathematics methods teachers solved four typical conversion problems. Fifteen undergraduate preservice mathematics teachers participated in the study as part of an education course in Mathematics Methods for grades 7-12. Participants included seven white females, and eight white males. All preservice teachers were also Mathematics majors at the same comprehensive public university (a dual-degree). Participants were asked to solve four different conversion problems within the metric system (as outlined in table 1) problem and explain their solution. Participants were also asked a corresponding set of qualitative questions:

1. Is the way you solved the problems the way that you were taught in high school?
2. Do you recall if your high school mathematics classes taught you multiple solution methods?

The problems appear below:

Problem 1	Express 50cm as mm. Explain the solution. *500 mm*
Problem 2	Express 450,000,000mm as km. Explain the solution. *450 km*
Problem 3	Express 26Gm as nm. Explain the solution. *$26*10^{18}$ nm*
Problem 4	Given 450mg of a substance occupying a volume of 50 mL (450 milligrams occupying a volume of 50 milliliters) Calculate the density of the substance in grams per liter. Explain the solution. *9g/L*

Table 1. Four problems using conversions within the metric system

Problems were administered on the first day of class before any other instruction on multiple solutions methods was discussed or assigned. These problems are representative of typical conversion problems found in high school chemistry (Brown, LeMay, & Bursten, 2000) and mathematics (Bellman, et al, 2010; Gantert, 2007) textbooks.

Results

Results are tabulated by question as follows. In Problem 1 all preservice teachers solved it correctly as shown in Table 2. This problem represented the most basic conversion problem. Six students admitted in their explanations that they felt "rusty" or "it had been a long time since they had solved these problems." Three preservice teachers said that they pictured a ruler. Two preservice teachers wrote a staircase on their papers showing the often used mnemonic "King Henry Drank Milk During Class Monday," however one student wrote the mnemonic incorrectly as "King Henry's Mother Drinks Chocolate Milk." Four others stated they simply knew the conversion. This may indicate that that these six preservice teachers are not concerned with a solution for understanding but rather a procedural solution. Of the six students that used proportional reasoning the most often set of conversions were $50cm * \frac{1m}{100cm} * \frac{1000m}{1m}$. Of the three that used proportional reasoning, they set the problem up as $\frac{1\ cm}{10mm} = \frac{50cm}{x}$.

N=15 t	Express 50 cm as mm. Explain. 500 mm	
t	Number Correct	Number Incorrect
Mnemonic t	2 t	0
Dimensional Analysis	6 t	0
Proportional Reasoning	3 t	0
Other method	4 t	0

Table 2. Problem 1 Results

t In Table 3 below (problem 2), it is interesting to note that more participants turned to dimensional analysis. The one preservice teacher who used another method knew the meaning of these metric prefixes, and consistent with the first solution, the student simply counted the decimal places. This problem involved a larger magnitude of conversion and another preservice teacher lost track of their place value giving an incorrect solution that was off by one decimal place, i.e., the student moved the decimal five places to the right. This student used the mnemonic King Henrys Mother Drinks Chocolate milk (as described in Problem 1 above- omitting one of the words for "deca." The two preservice teachers who incorrectly used dimensional analysis had an incorrect conversion factors $\left(\frac{1km}{100m}\right)$ for a result of 4500; the other student used $\left(\frac{1km}{1000cm}\right)$ for a result of 45000. In summary incorrect solutions were based on an incorrect mnemonic or an incorrect understanding of the equivalent metric form of 1km.

N=15 t	Express 450,000,000 mm as km. Explain.	
t	Number Correct	% Number Incorrect
Mnemonic t	1 t	1
Dimensional Analysis	9 t	2
Proportional Reasoning	1 t	0
Other method	1 t	0

Table 3. Problem 2 Results

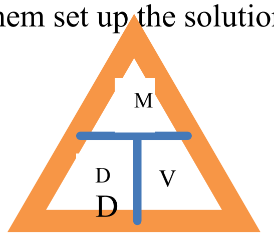
t In Table 4 below (problem 3), 14 of 15 preservice teachers could not solve the problem. Seven preservice students admitted that they did not know the meaning of the prefixes but they attempted a solution based upon dimensional analysis, and six simply stated I can not solve the problem. The prefixes were less commonly used in high school mathematics classes, however, the term giga is used often in today's computing realm. The seven who attempted the dimensional analysis solution used a potentially correct set-up of the problem had they known the mathematical meaning of the prefixes. These students reported a sense that the decimal should be moved to the right- but they were not sure how many places. It is possible that with a conversion chart for the meaning of the prefixes, more preservice teachers might have solved the problem correctly.

N=15 t	Express 26 Gm as nm. Explain.	
t	Number Correct	% Number Incorrect
Dimensional Analysis	1 t	8
Could not set up problem	0 t	6

Table 4. Problem 3 Results

In Table 5 (Problem 4), 13 of 15 students solved the problem correctly. All students applied the formula D = M*V. One student wrote that he remembered it as the "Department of Motor Vehicles." Two others wrote a triangle to help them remember how to set up the equation (see table 5). Another student interestingly used both the mnemonic "King Henry" above and then the equation D=M*V. Three students made an arithmetic mistake as follows: First, in the division of $\frac{450mg}{50mL}=\frac{90mg}{mL}$, second as $\frac{4.5}{0.05}=\frac{90mg}{mL}$ and finally as, $\frac{450mg}{50mL}=\frac{9mg}{mL}$.

Three also used this triangle mnemonic to help them set up the solution:



N=15 t	Given 450 mg of a substance occupying a volume of 50 mL (450 milligrams occupying a volume of 50 milliliters) Calculate the density of the substance in grams per liter. Explain.	
t	Number Correct	Number Incorrect
Dimensional Analysis	12 t	3

Table 5. Problem 4 Results

t Finally, in Table 6, the results of the two follow up questions are presented. 14 of the preservice teachers believed that they solved the problems the way in which they were taught in high school, and only two recall being taught multiple solution methods for problems like this in high school.

t	Number replied yes	Number replied no	Do Not Remember
Is the way you solved the problems the way that you were taught in high school?	14 t	0 t	1
Do you recall if your high school mathematics classes taught you multiple solution methods [for these types of problems]?	2 t	10 t	3

Table 6. Qualitative Questions Results

Resoundingly, this cohort of preservice teachers did not learn to solve these problems in multiple ways. This prohibited them from a meaningful check of their solution, and more importantly, they lack other teaching strategies or methods to teach the problems.

Discussion and Conclusion

The results of this preliminary study indicate that it is worthwhile to further study preservice teachers' solution methods of solving conversions within the metric system. The long standing problem of promoting the use of in-depth understanding through multiple methods is well known. Despite prior research that teaching based upon procedural *and* conceptual understanding is worthwhile, this particular cohort of preservice teachers seemed to gravitate towards the methods taught in high school. Many even used (incorrect) mnemonics. Without intervention, this pattern is likely to continue (DeMeo, 2008). The PCAST report suggested that the federal government should "ensure the recruitment, preparation, and induction support of at least 100,000 new STEM middle and high school teachers who have strong majors in STEM fields and strong content-specific pedagogical preparation, by providing vigorous support for programs designed to produce such teachers." (PCAST, P. 12). Therefore, future study is warranted on an intervention that might promote the development of teachers' knowledge to value and incorporate the use of problem solving, and in-depth understanding though the use of multiple methods for conversion problems. This future research should address the question: Can the in-depth study of unit conversion problems in a mathematics methods course encourage preservice teachers to create and refine models of teaching conversion problems within the metric system?

Furthermore, what is missing from these solutions are ways that focus on coherence and structure in the metric system and that align within the CCSS. At least two disparate methods are missing from these solutions- the use of scale factors to solve the problems (see Lappan, 1998), and the use of the meaning of metric prefixes to solve the problems such as applying the multiplicative identity:

> Express 450,000,000 mm as km:
> $10^{-3} \times 10^{3} = 1$
> insert the prefix you want to move to: kilo=10^{3}
> 450,000,000 mm =
> 450,000,000 x 10^{-3} m =
> 450,000,000 x 10^{-3} m x $\mathbf{10^{-3} \times 10^{3}}$ =
> 450,000,000 x 10^{-6} x 10^{3} m =
> 450 km

Using a mnemonic may indicate that preservice teachers do not know the meaning of the prefixes, so they were not able to use the structures of mathematics using exponents (as above) to devise a solution. When the preservice teachers were confronted with vastly different prefixes (like nano and giga), they could not rely on the structures of the prefixes to solve the problems. In short, they ran out of stairs to climb! Knowing a second or third way helps students to verify their solution and build confidence. The seminal work done by Shulman (1986) established that teachers can not teach what they do not know. Future research in the area of conversions within the metric system needs to promote the problem solving skills and use of multiple methods of preservice teachers so that they can, in turn, help their students.

Despite indications that conversions within the metric system are a worthwhile topic for further study, several limitations exist within this preliminary study. First, it is possible that if the preservice teachers were given information on the meaning of the prefixes in problem 3, (nano and giga), they might have been able to solve the problem. Second, preservice teachers were asked if the method they used was the same as the method taught in high school, but the question did not specify mathematics or science classes. This should be clarified. Third, the preservice teachers overall preferred the DA method, and this is the method most typically used in the sample textbooks consulted. A more exhaustive review of the methods used in high school textbooks should also be a part of further study. Fourth, although the second follow up question indicated that preservice mathematics teachers were only taught one way to solve the problems, the follow up questions did not specifically ask preservice teachers to solve the problem in a variety of ways, nor did the study include appropriate qualitative participant interviews. In future research, the use of an initial survey should be followed by individual student interviews, include an appropriate intervention to promote the use of multiple methods, and finally a post test.

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