# Causal Bayes Model of Mathematical Competence in Kindergarten

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## ABSTRACT

In this paper authors define mathematical competences in the kindergarten. The basic objective was to measure the mathematical competences or mathematical knowledge, skills and abilities in mathematical education. Mathematical competences were grouped in the following areas: Arithmetic and Geometry. Statistical set consisted of 59 children, 65 to 85 months of age, from the Kindergarten Milan Sachs from Zagreb. The authors describe 13 variables for measuring mathematical competences. Five measuring variables were described for the geometry, and eight measuring variables for the arithmetic. Measuring variables are tasks which children solved with the evaluated results. By measuring mathematical competences the authors make causal Bayes model using free software Tetrad 5.2.1-3. Software makes many causal Bayes models and authors as experts chose the model of the mathematical competences in the kindergarten. Causal Bayes model describes five levels for mathematical competences. At the end of the modeling authors use Bayes estimator. In the results, authors describe by causal Bayes model of mathematical competences, causal effect mathematical competences or how intervention on some competences cause other competences. Authors measure mathematical competences with their expectation as random variables. When expectation of competences was greater, competences improved. Mathematical competences can be improved with intervention on causal competences. Levels of mathematical competences and the result of intervention on mathematical competences can help mathematical teachers.
**Keywords:** mathematical competences, causal Bayes model, intervention, causal effect

## 1. INTRODUCTION

Mathematics is the foundation of the child's understanding of natural and social laws. Mathematical competence is developed by encouraging the child to develop and apply mathematical thinking in solving problems in different activities and every day situation [1]. Adoption of basic mathematical concepts influences the development of mental functions and the thinking process. When children learn math, they need to play with the real objects and study the real problems that interest them [2]. When acquiring mathematical skills, child masters the thinking skills that are prerequisites for understanding mathematics. Child acquires mathematical skills by moving the objects, comparing them and revealing their properties. A child must acquire that knowledge before school so he can master the mathematical content. For the successful studying of mathematical competence, it is necessary to examine the causal connection between the parts of the mathematical competence.

## 2. MATHEMATICAL COMPETENCES

When defining mathematical competences in the preschool education, one must start from the goals. Main goal is the introduction to basic mathematical concepts that will be used and studied by preschool children in the elementary and secondary school [3].
Fundamental mathematical competences can be divided into two areas of mathematics: Geometry and Arithmetic. Each of these areas has its own mathematical competences. Mathematical competences for geometry include: Space relations (more – less, before – after, above – under, left – right) and Geometric objects (lines and surfaces, triangle, rectangle, circle). Mathematical competences for arithmetic include: Numbers (recognize numbers, counting, writing numbers) and relation and operations with numbers (comparing numbers, adding and subtracting). The goal of our work is to measure mathematical competences in kindergarten. It is necessary to determine the statistical set and measuring variables that are being measured. This paper is the result of the author's previous research [4] [5] [6].

## 3. STATISTICAL SET AND VARIABLES

Elements of statistical sets were 59 children, 65 to 85 months of age, from Kindergarten Milan Sachs from Zagreb, Croatia [7]. Measuring variables are tasks which children solved, together with the evaluated results of the tasks. Testing was performed as a part of an ordinary testing of children's readiness for the elementary school and development monitoring parts of the preschool education curriculum [1].
Measuring variables for geometry are:
Left and right relations (LERI)
In the front and behind relationship (IFBE)
Above and below relationship (ABUN)
Recognizing triangle (TRIA)
Recognizing rectangle (RECT)

Measuring variables for arithmetic are:
Counting to 30 (CO30)
Understanding the numbers to 10 (UN10)
Knowing the number of fingers on both hands (NFBH)
Distinguishing numbers and letters (NULE)
Adding +1 (AD+1)
Adding +2 (AD+2)
Subtracting -1 (SU-1)
Subtracting -2 (SU-2)
Every measuring variable for competences was described separately through the tasks. Every task was evaluated with 0 (no competence) and 1 (competence). Tasks for measuring variables are:
Left and right relations (LERI). Competence of the left – right relationship was examined through orientation to one's own body. Child was asked to show his right leg, left ear, left eye and other body parts.
Front and behind relationship (IFBE). With the help of pictures, child was asked: Who is in front of the hen? Who is behind the girl? (Picture 1.)
Above and below relationship (ABUN). With the help of the table and a train toy, child was asked the following question: What is above the train toy? What is under the table? If a child knew the relationship he was evaluated with 1, and if a child did not know the relationship he was evaluated with 0. (Picture 2.)


Picture 1. LERI


Picture 2. ABUN

Recognizing rectangle (RECT). Child was shown pictures of various geometric shapes and asked: Where is a rectangle? (Picture 3.)
Recognizing triangle (TRIA). Child was shown pictures of various geometric shapes and asked: Where is a triangle? (Picture 4.)


Picture 3. TRIA


Picture 4. RECT

Counting to 30 (CO30). Child was asked to count. It was expected that he knew how to count to 30.
Understanding the numbers to 10 (UN10). More than 10 crayons were placed on the table. Child was asked to count them one by one, joining a number of crayons with the proper sequence of numbers. (Picture 5.)
Knowing the number of fingers on both hands (NFBH). Child was asked: How many fingers do you have on both hands? (Picture 6.)


Picture 5. UN10


Picture 6. NFBH

Distinguishing numbers and letters (NULE). Children had mixed group of numbers and letters on the table and had to make two groups, the group of numbers and the group of letters. (Picture 7.)
Adding +1 (AD+1). Understanding the operation of adding from 1 to 5 was examined by using the questions such as: You have 3 marbles and mother adds 1 marble. How many marbles have you got?
Adding +2 (AD+2). Understanding the operation of adding from 1 to 5 was examined by using the questions such as: You have 3 marbles and mother adds 2 marbles. How many marbles have you got?
Subtracting -1 (SU-1). Understanding the operation of subtracting from 1 to 5 was examined by using the questions such as: You have 3 marbles and your mother takes away one marble. How many marbles have you got?
Subtracting -2 (SU-2). Understanding the operation of subtracting from 1 to 5 was examined by using the questions such as: You have 3 marbles and your mother takes away two marbles. How many marbles have you got?


Picture 7. NULE


Picture 8. AD & SU

## 4. BAYES MODEL

Bayes model is a probabilistic directed acyclic graphic model. This model has directed acyclic graph and probabilities on the graph. Directed acyclic graph (DAG) consists of a set of vertices and set of edges. In our model the mathematical competences for kindergarten, the vertices are mathematical competences in kindergarten $x_1$, $x_2, \ldots, x_{13} \in$ {CO30, UN10, NFBH, NULE, AD+1, AD+2, SU-1, SU-2, LERI, IFBE, ABUN, TRIA, REC}. This competences are measured probabilistic variable with two values 0 and 1 with measure probability $p(0)$ and $p(1)$. It is well known that value equality $p(0) + p(1) = 1$ or probability of not having competences and having competences is 1 or 100%. The edges connect vertices. Every edge connects two vertices and present causal relation [8] between two causal competences. For example edge $x_i \rightarrow x_j$ or an arrow presented $x_i$ as the cause competence and $x_j$ is the effect competence of cause relation. Our graph is acyclic or has no cycles because causal model has no cycles on graph presentation. On the explained DAG we build Bayes model as Bayes network. Bayes network is DAG with probabilistic relations between causa related mathematical competences. Every edge has four probabilities. For example edge $x_i \rightarrow x_j$ has probabilistic relation or conditional probabilities $p(x_j = 0 \mid x_i = 0)$, $p(x_j = 1 \mid x_i = 0)$, $p(x_j = 0 \mid x_i = 1)$,

$p(x_j = 1 \mid x_i = 1)$.
The probability of effect competence $x_j$ is:

$$p(x_j = 0) = \sum_{k=0}^{1} p(x_j = 0 \mid x_i = k)$$

$$p(x_j = 1) = \sum_{k=0}^{1} p(x_j = 1 \mid x_i = k)$$

*or*

$$p(x_j = n) = \sum_{k=0}^{1} p(x_j = n \mid x_i = k) \qquad for \qquad n = 0,1$$

On our Bayes model we use Markov property or we calculate effect mathematical competence only with parents or causal competences. Parent competences are competences with direct edges from cause competences. For example in sub graph if we have $x_i \rightarrow x_j \leftarrow x_k \leftarrow x_m$ parent competences of competence $x_j$ are competences $x_i$ and $x_k$. Parent competences of competence $x_k$ are competences $x_m$, and competences $x_i$ and $x_m$ have no parent competences. We can write Markov property:

$$p(x_j) = \sum p(x_j \mid x_i \quad for \quad x_i \quad wich \quad is \quad parent \quad of \quad x_j)$$

In our paper we use free software Tetrad 5.2.1-3 [9]. Main part of this software is Linear Non-Gaussian Acyclic Model (LiNGAM) [10]. LiNGAM works with independent component analysis (ICA) [11 with estimation of coefficients with maximize log likelihood together with all the possible causal ordering. In the software Tetrad 5.2.1-3 we use program Linear Non-Gaussian Orientation Fixed Structure (LOFS). This program generates many different DAG.

## 5. RESULTS

The authors of this paper, director and psychologist from the kindergarten M. Sachs in Zagreb together with the professor of the Faculty of Teacher Education at the University of Zagreb, choose the most appropriate model in mathematic for kindergartens, as represented in this paper. Directed acyclic graph (DAG) is (Figure 1):

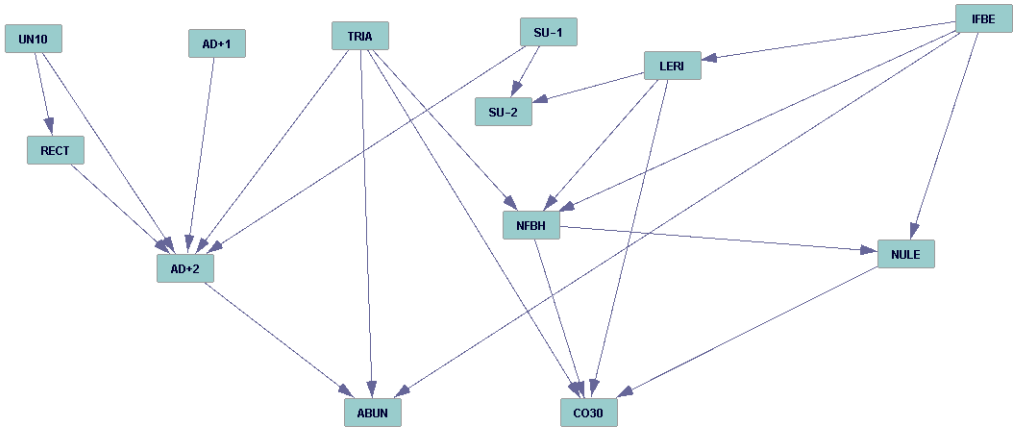


Figure 1. Directed acyclic graph (DAG)

From Figure 1 we can see five levels of causal structure. Causal mathematical competences UN10, AD+1, TRIA, SU-1 and IFBE are on the first level. These competences are fundamental and cause for all other mathematical competences. Mathematical competences RECT and LERI are on the second level. This competences are effect of competences from the first level. Competences effect causes from the first and the second level are on the third level. Third level competences are AD+2, SU-2 and NFBH. ABUN and NULE are on the fourth level. The last fifth level is competence CO30 or effect competences from all previous levels.
Using Bayes estimator from software Tetrad 5.2.1-3 we can describe causal structure of competences in our research or relation between causal competences and effect competences. Effect of causa is satisfactory when expected value is bigger than before modeling (measured competences):

First level (measured competences):

| TRIA=0 | TRIA=1 |
|---|---|
| 0.0533 | 0.9467 |

| IFBE=0 | IFBE=1 |
|---|---|
| 0.1467 | 0.8533 |

| SU-1=0 | SU-1=1 |
|---|---|
| 0.1467 | 0.8533 |

| UN10=0 | UN10=1 |
|---|---|
| 0.0933 | 0.9067 |

| AD+1=0 | AD+1=1 |
|---|---|
| 0.0533 | 0.9467 |

Second level

| IFBA= | LERI=0 | LERI=1 |
|---|---|---|
| measured | 0.3600 | 0.6400 |
| 1 | 0.2812 | 0.7188 |

| UN10= | RECT=0 | RECT=1 |
|---|---|---|
| measured | 0.1467 | 0.8533 |
| 1 | 0.1029 | 0.8971 |

Third level

| LERI= | IFBE= | TRIA= | NFBI=0 | NFBI=1 |
|---|---|---|---|---|
| | | measured | 0.0933 | 0.9067 |
| 0 | 1 | 1 | 0.0000 | 1.0000 |
| 1 | 0 | 1 | 0.0000 | 1.0000 |
| 1 | 1 | 0 | 0.0000 | 1.0000 |
| 1 | 1 | 1 | 0.0222 | 0.9778 |

| SU-1= | LERI= | SU-2=0 | SU-2=1 |
|---|---|---|---|
| | measured | 0.1333 | 0.8667 |
| 1 | 0 | 0.0000 | 1.0000 |
| 1 | 1 | 0.0000 | 1.0000 |

| UN10= | AD+2= | SU-1= | TRIA= | RECT= | AD+2=0 | AD+2=1 |
|---|---|---|---|---|---|---|
| | | | | measured | 0.1333 | 0.8667 |
| 0 | 1 | 1 | 1 | 1 | 0.0000 | 1.0000 |
| 1 | 1 | 0 | 0 | 1 | 0.0000 | 1.0000 |
| 1 | 1 | 1 | 1 | 1 | 0.0185 | 0.9815 |

Fourth level

| NFBH= | IFBE= | NULE=0 | NULE=1 |
|---|---|---|---|
| | measured | 0.1200 | 0.8800 |
| 1 | 1 | 0.0317 | 0.9683 |

| AD+2= | IFBE= | TRIA= | ABUN=0 | ABUN=1 |
|---|---|---|---|---|
| | | measured | 0.1467 | 0.8533 |
| 1 | 1 | 1 | 0.0175 | 0.9825 |

Fifth level

| IFBE= | NULE= | LERI= | TRIA= | CO30=0 | PO30=1 |
|---|---|---|---|---|---|
| | | | measured | 0.4000 | 0.6000 |
| 1 | 1 | 1 | 1 | 0.2273 | 0.7727 |

In the previous tables last column is expectation as random variables of mathematical competences. These expectations are the result of intervention on causal competences. The greater expectation of compete means that we can improve mathematical competences. Level of competences suggest new teaching in kindergarten in order to improve mathematical competences.

## 6. CONCLUSIONS

From the previous results we can conclude that our causal Bayes model is good for our discussion about the relation between mathematical competences in kindergarten. At the same time, the teachers understand causal relation between mathematical competences.

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