Implementing an Executive-Function Syllabus: Operational Issues

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ABSTRACT

A recent approach to pedagogic challenge, contrastive to the *hierarchy* approach of Bloom, Anderson, Gagne, Van Hiele, Marzano, Webb and many others, identifies pedagogic challenge with executive function: Pedagogy is defined as challenging if it addresses executive function. Executive function, in turn, is defined by the presence of multiple modalities of topic approach and a multi-parameter development of the topic. This paper discusses operational issues in implementing a teaching methodology based on multi-parameter problems. This paper advocates teaching a multi-parameter topic using a step-by-step incremental approach, introducing one parameter at a time. Examples are presented from trigonometry, actuarial mathematics, statistics and (biblical) literary analysis. The paper also discusses the use of the incremental approach for problem creation and remediation.

Keywords: Executive Function, Cognitive Challenge, Educational Hierarchies, Incremental Approach, Remediation, Literary Analysis, Rashi, Biblical Commentary

1. APPROACHES TO PEDAGOGIC CHALLENGE

The first attempt at defining a pedagogic hierarchy was made by Bloom [3] in the fifties of the last century. Working with a team of psychologists, Bloom developed a six-stage educational hierarchy. At the low end of the hierarchy is *knowledge* referring to teaching or testing knowledge of raw facts. At the higher end of the hierarchy are *analysis*, *synthesis* and *evaluation*, referring to pedagogy in which the student must create (synthesize) something new or must identify critical components of a discipline (analysis) and evaluate them.

Since Bloom, several competing educational hierarchies have been proposed, including Anderson's modification of Bloom's initial hierarchy [1], as well as Van Hiele's hierarchy for geometry [30], and approaches of Gagne [7], Marzano [20] and Webb [31].

Hendel [12, 14] has recently proposed a unified approach to these disparate theories based on executive function. The simple idea is to identify pedagogic challenge with the brain function, executive function, associated with complex cognitive processing requiring the co-ordination of several subprocesses to achieve a particular goal. There are two main categories of psychological tests of executive function: open-ended and performance tests [22, 28]. This paper focuses on performance tests, the aspect of executive function that is involved in multi-parameter processing. This aspect of executive function is illustrated using the Wisconsin Card Sorting Test (WCST) [10].

Wisconsin Card Sorting Test
In the WCST, two rows of cards are presented to the examinee as illustrated in Figure 1. The first row contains three cards that differ in the dimensions of *number, formatting* and *letter*. The examinee must determine whether the card in the second row resembles more: the A card because of the dimension of *letter*, the B card because of the dimension of *number* or the C card because of the dimension of *formatting*. The administration of the test requires the examiner to present sequences of two rows of cards and to alternate between maintaining and changing the dimension of similarity between the two rows. Thus the WCST, besides measuring multi-parameter processing, also measures the capacity to make change and corrections. The WCST may be abstractly summarized by the equation *CorrectAnswer* = **F**(*letter, number, formatting*), where *F* is a Boolean function, a function based on the connectives *and, or* and *not*. Using this background, the fundamental thesis of this paper on pedagogy is now stated.

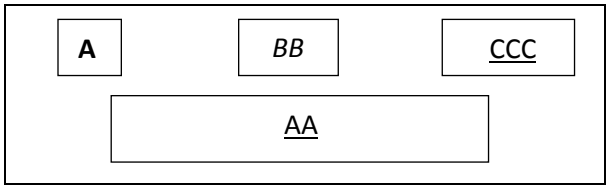

Figure 1: A Wisconsin Card Sorting Test, simplified for this paper. The actual test uses a 4-card first row with the dimensions of number, color, and shape.

A discipline is taught in a pedagogically challenging manner if multiple modalities of approach are continually used, each with multiple parameters.

Although both the standards of the National Council of Teachers of Mathematics (NCTM)[21] and the Common Core State Standards [5] advocate using multiple representations when teaching mathematical content, the emphasis in this paper on multiple approaches is threefold: i) *each* course concept should *always* have multiple approaches (in all problems, illustrative examples etc.), ii) each approach should have multiple parameters, iii) the presence of multiple approaches and multiple parameters *suffices* to ensure pedagogic challenge; no further considerations (such as pedagogical hierarchies) are needed. Illustrative of this emphasis, is the Hughes-Hallett approach to teaching calculus using the *rule of four*: *Each* calculus concept is taught with i) graphs, ii) algebraic formalism, iii) verbal problems and iv) manipulative calculations [13, 15].

The purpose of this paper is to present an incremental pedagogic approach, useful when implementing an executive-function based curriculum. Sections 2 – 5 explore examples from trigonometry, actuarial mathematics, statistics, and literary analysis. These sections apply the incremental approach to problem creation, remediation and open-ended skills such as literary analysis.

2. TRIGONOMETRY

The material in this section was first presented in a (job) interview at (what was then) Utah Valley Community College, in July 1991.

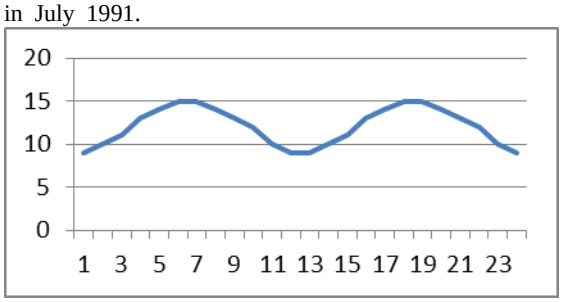

Figure 2: Duration of daylight in hours, over a 24 month period, starting at January, in Manhattan New York [6]. For example, the point X = 19, Y = 15 on the graph is interpreted to mean that in the 19th month (July of the 2nd year), daylight lasts 15 hours.

Trigonometry is the study of periodic phenomena. An illustrative example is presented in Figure 2, presenting the duration of daylight in Manhattan, New York over a 24 month period. Several features of this graph are apparent:

- Periodicity: The graph repeats (it is periodic).
- ***D***, Displacement: The low point of the graph is about 9 (the shortest day in Manhattan is 9 hours).
- ***A***, Amplitude: The difference between the shortest and longest day is about 15 - 9 = 6 hours.
- ***F***, Frequency: The graph repeats every 12 months.
- ***P,*** phase shift: The graph begins at its lowest point.

Because this paper is written for a wide audience, including non-technicians, the paper suffices with a *description* of the parameters without presenting any formal functions. Consequently, in the sequel, the letters, *D,A,F,P* refer to the parameters *displacement, amplitude, frequency* and *phase shift.*

This paper's basic thesis posits that to properly teach single variable periodic phenomena one should: i) use an incremental approach introducing one parameter at a time; one should then ii) provide sufficient illustrative examples and problems until students can master *simultaneous* use of all parameters; finally, one should climax the teaching unit with iii) presentation of verbal problems addressing the verbal and formal area of the brain and using all four parameters. Such pedagogy is rich and motivating creating student engagement. Note that the description of many natural real-world examples, such as the example in Figure 2, requires all four parameters.

There are many ways to assess if textbooks do this. For example, one might study the list of examples in expository sections. Figure 3 presents an alternative method of assessment: A review, in five textbooks, of the percentage of problems using one, two, three or four parameters.

Several salient features emerge. Notice how some textbooks ignore or diminish the number of problems with all four parameters. In addition, some textbooks are heaviest on two-parameter problems. The Swokowski textbook [26] is richest in number of problems, richest in percent of four-parameter problems and provides 25% verbal or graphical problems covering such diverse applications as brain waves, heart waves, temperature and daylight.

Summary
The executive function approach of this paper requires that single variable periodic phenomena be taught using four areas of the brain - verbal, graphic, algebraic and computational - and be taught using all four parameters *D*, *A*, *F*, *P*. Therefore, sufficient problems should be provided to achieve mastery. The new skills should then be applied to modeling real world phenomena. Such an approach is pedagogically challenging and engaging. Furthermore, such an approach can be developed without use of the educational hierarchies and is consistent with the NCTM and CCSS standards.

Textbook	# Problems	1	2	3	4
[25]	56	18%	71%	11%	0%
[2]	70	24%	54%	20%	2%
[9]	28	57%	28%	11%	4%
[26]	73	30%	30%	23%	17%
[16]	52	31%	38%	27%	4%

Figure 3: For each textbook, the number of problems on one-variable, periodic functions is indicated along with the percentage of those problems with 1,2,3 or 4 of the parameters, *D*, *F*, *P*, and *A*. For example, Sullivan's Precalculus [25] presents 56 exercises on periodic functions; 71% of these exercises involve only two of the possible four parameters.

3. ACTUARIAL MATHEMATICS

This section studies the application of an executive function approach to facilitate tutoring. The material in this section is based on the author's experience in teaching and tutoring actuarial mathematics at Towson University, which has recently been designated by the Society of Actuaries as a Center of Actuarial Excellence.

Roughly, speaking, actuarial mathematics is the study of risk [24]. The Society of Actuaries and Casualty Actuarial Society grant actuarial degrees to candidates who pass a series of very difficult actuarial examinations. Actuarial courses at a variety of institutions teach the material covered on actuarial exams. The preliminary actuarial examinations involve multiple-choice examinations, which require that students must get perfect answers without partial credit.

Although certain students know the syllabus content, they nevertheless, are unable to obtain perfect answers on very difficult problems. Problem types that students find difficult typically involve executive function; they involve either multiple brain areas or multiple subproblems.

One possible tutoring philosophy is to focus on problem templates and needed formulae; student failure is then attributed to insufficient practice. Contrastively, the approach of the author in tutoring is to explicitly identify executive function, a non-mathematical organizational skill, as something that must be learned to achieve mastery in problem solving. The sample problem below is illustrative.

Sample Problem
Abe deposits \$100 in bank **A** for 3 years at 4%. The accumulated value is withdrawn and deposited in Bank **B** for 4 years at 3%. Barbara deposits \$100 in Bank **C** for 7 years at *k*%.

Both Abe and Barbara have the same accumulated value at the end of 7 years. Compute *k*.

Interest Background

Prior to presenting our approach to tutoring, we review basic interest theory. The theory of interest relates the four variables: i) *P*, the principle amount deposited in a bank, ii) *i*, the annual effective interest rate paid by the bank, iii) *n*, the number of years the money remains in the bank and iv) *A*, the accumulated value of the principle which remains in the bank for *n* years earning interest at rate *i*.

These four variables give rise to four functional relationships. For example, $A(P,n,i)$ indicates the accumulated value of principle, *P*, deposited for *n* years in an account yielding *i*. Similarly, $i(A,P,n)$ indicates the interest rate, *i*, needed to accumulate amount *A* from an amount *P* deposited in a bank account for *n* years. The functional notation $P(n, i, A)$ and $n(i, A, P)$ have similar interpretations.

For purposes of this section, the *particular* functional form of these four functions need not concern us. For expositional purposes, it suffices to have an awareness that *some* functional relationship exists and that given numerical values for any three of the four quantities, the remaining quantity can be computed.

Solution Overview

The solution of the sample problem presented above requires combining the solutions to three subproblems, each an instance of the basic interest formula. More specifically,

(1) $A_1 = A(100, 4, 3\%)$,
(2) $A_2 = A(A_1, 3, 4\%)$,
(3) $k = i(A_2, 100, 7)$.

Notice that all 3 subproblems are each an application of the basic interest formulae. Also, notice how the three subproblems are intertwined: The principle in subproblem (2) is the accumulated value from subproblem (1). Similarly, the accumulated value in subproblem (3) is the accumulated value from subproblem (2). While the unknown in subproblems (1) and (2) is the accumulated value, the unknown in subproblem (3) is the interest rate, *k*.

Overview of Student Difficulties

Some typical difficulties of students during tutoring sessions are as follows:

a. Student solves subproblem (1) but does not go further.
b. Student sets up 3 subproblems but cannot evaluate them numerically.
c. Student solves subproblems (1)-(2) but gets stuck on (3).
d. Student does not know how to begin solving.
e. Student confuses "3" and "4" in subproblems (1) and (2).

Executive Function Response to Typical Student Difficulties

The author's executive-function responses to the typical student problems (a) through (e) are presented below. Prior to this presentation, general characteristics of the tutoring approach are re-summarized. The author's tutoring approach emphasizes *organization vs. knowledge*, executive function vs. course content. None of the student typical issues, (a) – (e), has anything to do with course content, that is, with the topic formula $A=A(P, n, i)$. All students know this formula and they also know how to apply it in a single setting. Remediation focuses on how to integrate problem components together. Here are the typical responses to student difficulties (a)-(e):

a. The sample problem is not a single plug-in problem. It consists of three plug-in problems whose solutions have to be integrated together. Try first doing each subproblem by itself.
b. You set up the formula correctly and substituted the numerical values you knew. In the Bank **B** subproblem, you don't know the principle, *P*. What does the verbal text of the sample problem tell you about the principle invested in Bank **B**? (Answer: It is a derived number and is equal to the accumulated value, *A*, of the Bank **A**, subproblem.) Similarly, you set up the Bank **C** problem correctly. What does the verbal text of the sample problem tell you about the accumulated value in Bank **C**? (Answer: It is numerically equal to the accumulated value in Bank **B**.)
c. You are using the *plug-in* version of the formula, $A=A(P,n,i)$ for the Bank **A** and **B** subproblems. For Bank **C** you must use a different version of the formula. What is the unknown and what is the functional form to be used. (Answer: Use $i(A,P,n)$ vs. $A(P,n,i)$.)
d. You are thinking holistically. In class, you learned about the interest theory problem template, $A=A(P, n, i)$. If you are given a problem in that template form, you can solve it. Contrastively, the sample problem consists of *three* applications of the problem templates. To solve this problem you must first identify and solve the three subproblems, the Bank **A**, Bank **B** and Bank **C** problems.
e. You are not making a silly error. You are not confusing the "3" and "4". After all, you know the difference between "3" and "4". If I gave you just one subproblem, you would not confuse the "3" and "4". Rather, you are confusing the parameters of the Bank **A** and Bank **B** subproblems. You must learn while doing each subproblem to forget about the parameters of the other subproblems. (Note: Focusing on specific information and forgetting other information is one aspect of executive function. It is called *working memory* and consists of the ability to focus on the information needed at the moment and to forget other information.)

Syllabus Construction

The author has changed the construction of his course syllabus based on the above typical scenarios. The course syllabus now explicitly mentions, "besides the mathematical content of the course, students are expected to master applying mathematical content to multi-component verbal problems." In other words, the learning of executive function is explicitly made a separate course learning-objective.

4. STATISTICS

The material in this section is based on work done by the author in 1995 while teaching the topic binomial probability in a college-level statistics course. A typical binomial problem is the following: *Calculate the probability of a student achieving (by chance) a grade of at least B on a five-question multiple-choice test with 5 choices per question, two of which are correct* [19].

This problem has four parameters: i) *n* = the number of questions, *g* = the grade achieved, *c* = the number of choices per question, *t* = the number of true (correct) choices per question.

Other binomial problems are similar, e.g. *Find the probability of having a majority of boys in a family of five children.* Here, *n* = the number of births, *g* = the percentage of boys in the family, *c* =the number of possible genders per birth, *t* = the

number of genders that are boys (only one, of course).

Using the multiple-choice test question, one can easily see the confusion that might arise from teaching binomial problems using a formula approach and presenting all parameters in one sitting. Weaker students confuse the probability of a correct answer with the probability of a grade; they also confuse the identification of the number of questions and the number of correct answers. Finally, some students are simply overwhelmed by the large number of parameters and don't know *where to begin*.

The author managed to successfully teach binomial probability using an incremental approach. Since the incremental approach does one parameter at a time, the author felt this method could be used to teach people of any age. The author communicated his methodology to a 4th grade teacher, Amy Bassan, who taught talented students in a private school. Amy had considerable leeway in designing her curriculum and tried these methods on a class of 11 students. The author gave her basic principles and Amy created 10 lessons. Additionally, the students kept diaries. Thus, the study was qualitative. Amy's instructions to her 4th graders are presented below, as well as their diary descriptions of what they learned and how they felt.

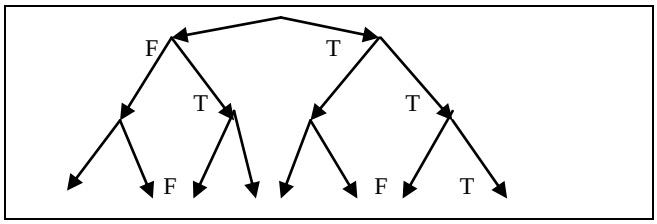

Figure 4: Tree construction of sample space for a 3-question true-false test. The test corresponding to the right most branching, TTT, achieves a score of 100%. The adjacent branch on the left,TTF, achieves a score of 67%.

To teach the material to elementary school students without introducing complicated formulae, some adjustments were needed. Formulas were replaced by trees. A sample tree for scoring a three-question true-false test is shown in Figure 4.

Besides the instructor's identification of skills, the students themselves in their diaries identified what they had learned; the instructor and student versions concurred. About a dozen skills were identified (some students had more; some had less). They include i) calculation of probabilities for one multiple choice question, ii) tree construction, iii) identification of parameters (*n, g, c, t*), iv) associating questions with trees, one question per level, v) calculating scores on tests from trees, vi) calculating probabilities of scores based on trees, vii) test creation, viii) experimental vs. theoretical probability, ix) conversion of trees with test scores to probability of a grade.

An additional student activity was the creation of multiple-choice arithmetic tests, which the students administered to the class. This enabled teaching the contrast of concepts, experimental vs. theoretical probabilities. A typical easy, intermediate and hard question created by the 4th grade students is presented in Figure 5.

In terms of the actual incremental instruction, students first i) learned to calculate probabilities on single multiple-choice questions with *c* choices, *t* of which were true. They then ii) advanced to constructing *n*-level trees corresponding to *n*-question tests. iii) As just mentioned, students created actual tests and administered them to classmates. They then learned iv) to score each test on the tree (theoretical), v) score each actual test, vi) convert a test score to a grade and vii) perform a separate probability assessment of reaching such a score.

EASY: *8 x 9 equals which number?*
INTERMEDIATE: *6 x (1+5) is one away from which number?*
HARD: *What number is 2, 3 or 4 away from 4 x 3 / 2?*

Figure 5: Sample problems from Amy's 4th grade students. Each question was followed by 5 choices. For example the choices for the hard question might be 9,10,11,12,13 with two of them (9 and 10) being correct (within 2 to 4 of 4 x 3/2). The students would create five-question tests and administer them to the class. This facilitated seeing the difference between experimental and theoretical probability.

The overall impression was that most students did master the material (e.g. they could answer questions they had not seen before) and most students enjoyed 50%-70% of the 10 lessons. Some observations and emotional reactions of the elementary school and college students are presented in Figure 6.

Issue	**Elementary School**	**College**
Time spent	10 lessons	2-3 lessons
Teaching entire tree method all at once	Confusion	Confusion
Sequential teaching.	50%-70% of lessons were fun. Material was understood.	Material understood better. Grades improved
Trees	Most enjoyed	Enjoyable
Multilevel trees (more than 4 levels) with lots of leaves	Boring and tedious	Boring and tedious
Making up tests and administering them to fellow classmates	Fun	NA

Figure 6: Observations and emotional reactions of elementary vs. college students to teaching of binomial probability.

Some final observations are worth noting:

- It is quite remarkable that, when presented in an incremental manner, even young elementary school children were able to grasp difficult concepts like binomial probability
- Especially of note is the fact that the use of tree diagrams meets the executive function requirement of multiple approaches in multiple brain areas (geometric, algebraic and computational).

5. LITERARY ANALYSIS

Advanced literary analysis focuses on how the forces of *paradox, irony, ambiguity* and *tension* work in the text [29]. Some instructors reject the idea that this is a learnable skill; rather, they argue it is an innate ability.

There are other approaches to literary analysis, for example the symbolist approach [27]. Furthermore, many enlarge the list of paradox, irony, ambiguity, and tension to include *nuance* (*connotation* and *denotation*), *symbol*, *image*, *contradiction*, *ambivalence*, etc. For purposes of pedagogical exposition, the important point is that all the approaches define literary analysis in terms of open-ended skills, something unknown that has to be analyzed but cannot be mechanically analyzed. It is this open-ended nature of these skills which is why many instructors feel that mastery of literary analysis is not universally attainable by all students.

However, the approach presented in this paper, combining an incremental approach with executive function, would equally apply to any open-ended skill and enable a student at any skill level to achieved mastery. Therefore, in this section, we suffice with analyzing literature based on paradox, irony, ambiguity and tension.

As just remarked, this section continues the theme of this paper that an executive-function syllabus when combined with a step-by-step incremental approach can successfully teach any subject to any student. In previous sections, because of the mathematical content of the subject matter, a strict incremental approach sufficed for supplementing the executive-function syllabus. However, literary analysis is not a formula; you can't simply teach one function argument at a time because there is no literary function that solves the literary analysis of a text. We therefore must expand our concept of *incremental approach.*

The key features of an incremental approach that facilitate mastery of material is the fact that the incremental approach breaks down a complex process into a sequence of steps each of which is specific, clear, almost mechanical, and realistically masterable in a short time. In the educational and business literature, such a breakdown of complex processes is termed goal-setting [18]. Proper goal-setting is instrumental in achieving educational and business objectives.

To appreciate the significance of goal setting, consider the trigonometry example discussed in Section 2. The ultimate goal is to have the student master a function of four parameters – displacement, amplitude, frequency and phase shift. These four parameters *naturally* break down the complex process into four steps, each of which focuses on learning one of the parameters. Each of the steps, is specific, clear, almost mechanical (learning how to vary one parameter), and realistically masterable in a short time by adequate exercises.

We similarly analyzed, in Section 4, a complex statistical process, into a set of specific steps, each of which was masterable even by 4th graders.

In Summary
To teach an executive function syllabus one needs proper goal setting which explores the various component parts of the executive function, the various brain areas, by breaking the task into a sequence of steps each of which is specific, clear, almost mechanical and realistically masterable in a short time.

We plan to more fully explore the relevance of goal-setting to education in a future paper.

Applying these principles to literary analysis we see that to successfully teach any student literary analysis one needs:

- Broad executive-function goals such as the open-ended exploration of tension, ambiguity, paradox and irony.
- Specific, clear, almost mechanical skills, masterable by anyone in a short time, which naturally lead to the open-ended explorations.

To present our ideas we have chosen the Bible which nicely lends itself to literary analysis. We have chosen the literary comments of a biblical commentator, Rashi [8]. Rashi was the first biblical commentator to provide a biblical commentary on both biblical themes as well as individual words and phrases. We selected Rashi because his commentary illustrates both open-ended literary comments as well as specific, clear, almost mechanical comments such as comments on grammar and meaning.

The rest of this section presents a series of examples. Each example is centered on a biblical verse analyzed in two stages. In the first stage, we present component goals that meet proper goal-setting. These component goals will naturally lead in the second stage to explorations of tension, ambiguity, paradox and irony. In this manner, a very complex open-ended, literary analysis, can be made accessible to any student.

To make the exposition smoother we will not differentiate between what Rashi explicitly said and what scholars assumed he had as a reason. Rashi comments are typically very terse and there arose an entire school of Rashi commentators who strove to explain the principles by which he derived his comments. Therefore, to avoid burdening the examples with cumbersome details, we simply present stage I and stage II analysis, it being understood that many of the literary comments in each example correspond to the comments made in the Rashi commentary.

The examples and presentation follow the ideas presented on the Rashi website [23]. Ten specific, clear, almost-mechanical skills are presented on this website which are easily masterable. These ten specific skills are sufficient to enable sophisticated advanced literary analysis of all Rashi comments. The ten skills, which will be explained in the body of the examples are *reference, meaning, grammar, parallelism, contradiction, examples, formatting, database methods, other (non-biblical) disciplines and symbolism.*

Example 1 – Parallelism and Grammar
This example illustrates the grammar and parallelism methods. Grammar, as is well known, deals with all grammatical features of a verse. The nice thing about grammar, is that it is specific, clear, almost mechanical and can be learned with adequate exercises. Parallelism is a feature of biblical poetry. It refers to a phrase that is repeated with slight variations. The recognition of parallelism is an almost mechanical process. Traditionally, the repeated phrase is seen as building on the first phrase and adding to it. There are known collections of parallelism schemas and it is well understood [17].

Figure 7 presents a biblical verse, which will be used for illustrative purposes throughout this example.

The brothers saw that he [Joseph], their father loved more than all brothers;
- *They hated him*
- The couldn't speak with him peacefully

Figure 7: Biblical verse, Gen. 37:3

Stage I: We now proceed to analyze this Rashi comment using two stages. In stage I, as just indicated, we use the grammar and parallelism rule.

- Grammar: The verse in Figure 7 uses the phrase *couldn't speak with him.* When using the verb *speak*, there are several possibilities of connective prepositions. For example, *speak to, speak with, speak of, speak at,* and *speak about.* The grammatical rule of verb connectives is specific, clear, almost mechanical and masterable in a short time provided adequate exercises are presented.

- Parallelism: Figure 7 shows parallel clauses indicated by the two bullets. In Figure 7, hatred (bullet 1) is followed by lack of peaceful conversation (bullet 2). Alternative consequences of bullet 1, hatred, might be a) insult, b) (verbal) attack, or c) ostracization.

Stage 2: Using the grammatical and parallel analysis presented above, we are now ready to derive ambiguity, tension, paradox and irony. Here are some points:

- On the one hand, the brothers hated Joseph; on the other hand, they did not ostracize or insult him, they only abstained from speaking *with* him.

- Furthermore, they did not abstain from speaking *to* him; they only abstained from a conversational relationship, speaking *with* him. One could have expected more.

- The reader feels the tension and ambiguity and even contradiction. Was it a real hatred? If not, why did it express itself in lack of peaceful conversations? We see here the tension that on the one hand he was brother while on the other hand he was hated because father showed favoritism. Rashi's comment captures some of the tensions and ironies: *From the bad traits of the brothers [hatred], you learn their good traits. Although they did hate him, they controlled their hatred and did not fawn him and act hypocritically.*

We have only explored some of the tensions and ambiguities captured in the verse. We could expand more. The focal point of this example is that the tensions and ambiguities naturally arise from the grammar and parallelism which are almost mechanical rules.

Example 2 – Referencing, Parallelism, Database Inquiry

This example presents three methods, referencing, parallelism, and database inquiry. Referencing is simply the technique of citing another verse to illuminate the meaning of a given verse. Database inquiries, refers, as the name indicates, to a traditional database inquiry performed on biblical verses. Reference and database searches are specific, almost mechanical, and easily masterable today since many search engines exist; database and reference methods are considered basic computer skills.

Deut. 25:05 presents the beginning of the poem blessing that a person recites when bringing the first fruit of his field to Jerusalem. The blessing begins *An Aramean* [Laban] *tried to destroy my father* [Jacob];

- *He* [Jacob] *went down to Egypt with a few people*
- *He became there a great nation*

Stage I: Already the parallelism show a tension and contrast between *few people* and *great nation.* Proceeding with our two stage approach to literary analysis we use the database inquiry method and the reference method to answer the following inquiry against the database of biblical verses: *Find verses describing contrasts in the number of Jacob's descendants.*

The results of the database inquiry are presented in Figure 8 along with the references to relevant verses.

Verse Reference	Text of verse or summary	Number contrast
Gen. 32:11	By myself [Jacob] with my staff, I traversed this river Jordan and now I am two camps. Please save me from my brother Esauv [who was trying to kill him]	1 person vs. 2 camps of servants and friends. Jacob's brother Esauv tried to kill him and Jacob fled singly. He is now grown to two camps and is still trying to avoid a confrontation with his brother.
Gen. 31:38-42	Jacob argues with father-in-law Laban for double crossing him and changing salary 10 times and double-crossing promises of daughters for wives	Despite double crosses, Jacob grew from 1 person, to a family (4 wives, 10 children) and much wealth
Gen. 46-08:28, Ex. 12-37	Bible lists the 70 children and grandchildren descending to Egypt. Although enslaved there, they successfully left with 600,000+ people.	70 upon initial descent vs. 600000+ upon exodus. The 600000+ is despite the slavery.

Figure 8: Several verses showing contrasts of numbers in Jacob's household.

Stage II: The almost mechanical methods of stage I each show population growth in the face of adversity. Jacob started as a single fleeing person, grew to a family of 15, went down to Egypt as a family of 70 and despite the slavery grew to a nation of 600,000+.

Although the parallelism method suffices to appreciate the tension and contrast of initial small numbers vs. growth to big numbers, the database inquiry method gives further appreciation of this tension. In each of the examples cited in Figure 8, there is more than a contrast of small vs. big numbers; there is growth in the presence of threats: i) Laban tried to double cross Jacob, ii) Esauv tried to kill Jacob, iii) Egypt tried to enslave Jacob. In each case, despite great threats, Jacob grew in size and wealth. This tension and paradox, how one fleeing person became a nation of 600,000, serves as a basis for the poem blessing

described in Deut. 25:05 recited upon bringing first fruits of one's fields to Jerusalem. The Rashi comment, exploring other verses of numerical contrasts in the face of adversity, facilitates empathy with the poem reciter: The poem reciter recalls the threats and small numbers and how despite this the nation grew. This appreciation of paradoxical contrasts, one to many, is the basis for a poem recitation of thanks. The poem reciter may use Jacob as a personal model for himself; despite difficulties in growing crops, nevertheless, the few [crops] can indeed become a harvest of many.

Example 3 – Meaning and Database Inquiry

Example 3 presents two methods, the meaning and database inquiry method. The meaning method, as the name implies, is simply the collection of methods by which meaning is ascertained. This includes dictionary meaning, but also includes understanding the nuances of synonyms, antonyms and idioms, the creation of metonymies and synecdoches, the skillful use of grammar to creatively change nouns to verbs and many similar associated skills. In this example 3, we use the meaning method to explore the meaning of an idiom. Meaning itself is a clear, specific, almost mechanical skill masterable in a short time provided appropriate exercises are present. Even the more advanced features of meaning – for example, synonyms, idioms, metonymy, etc. – are almost mechanical and quickly masterable.

To present our next example, we first give some background. Ten brothers came down to Egypt to buy food from Joseph, who oversaw all transactions. While there, the youngest of the brothers was caught stealing a royal object. Joseph indicated that the person stealing the object would be punished while the rest of the brothers were free to go. At this point, Judah, one of the remaining nine brothers decides to intercede.

The actual verse, Gen. 44:18, states: *Judah approached Joseph and said: Your servant* [me] *would like to speak into your ear; please do not be angry with your servant since you are like the King.*

The underlined phrase, speak into your ear, sounds strange. It is an idiom. Recall that an idiom is a collection of words whose meaning transcends the sum of the individual words.

Stage I: To ascertain the meaning of this idiom we use the database inquiry method and seek comparable verses where this phrase occurs: We find it used in Ex. 11:2 when Moses asked the Jews who were slaves in Egypt to request gifts from their masters; similarly, it is used in I Sam. 25:24 when Abigail requested from David not to kill her husband for double crossing him; it is also used in Gen. 23:13, when Abraham insists on paying in full for a burial plot which was given to him gratuitously. There are many more such verses.

It emerges from this database inquiry that speak into your ear is an idiomatic phrase used when requesting something you don't deserve and you shouldn't expect to get. Thus i) Joseph had a right to enslave the person stealing royal property and there was no basis for clemency; ii) the Jewish slaves had no rights requesting any property from their masters; iii) David had every right to attack Abigail's husband for double crossing him and iv) Abraham had no right to forcibly pay for a gift.

Stage II: We now use the idiomatic meaning from Stage I to discover tension, ambiguity, paradox and irony. The tension is already sensed in the idiomatic phrase used to indicate supplication for something undeserved. Building on this tension, we can identify similar tensions in other phrases in the verse:

- *Your servant; my master* - So Judah had no right to ask.
- *Speak into your ear* - There is no basis for clemency for a thief.
- *Please do not get angry* - The mere statement of the request is offensive in and of itself!
- *For you are like the King* - Asking clemency from an ordinary property owner would be unjustified; how much more so asking clemency from a royal person.

Thus the verse describes the emotional tensions and ambiguities Judah must have felt in approaching Joseph. We feel the paradox: on the one hand he had not basis to request anything, and yet on the other hand he tries to obtain some type of clemency.

Example 4 – Contradiction, Other (non-Biblical) Disciplines

Certain almost-mechanical rules naturally expose tension, ambiguity, paradox and irony. Examples 4 and 5 present such rules.

Example 4 presents two methods, the contradiction method and the other discipline method. Quite simply, contradiction occurs when two verses contradict each other. The recognition of contradiction is clear and specific and instantly masterable. The other-discipline method refers to knowledge obtained from other non-biblical disciplines. Although each such discipline is vast, basic principles of other disciplines, as will be shown in this example below, can be quickly obtained by looking them up in an encyclopedia or from a subject matter expert.

The contradictory verses studied in this example are as follows:

- Num. 08:24 This pertains to the Levites: From 25 years of age and above they will come for appointment in Temple service
- Num. 04-03,23,30 Appoint the …Levites …from 30 years of age and above, those who come for appointment to do Temple work.

Stage I: Recognition of the contradiction is clear and immediate: One verse states that Levite Temple service starts at 25 while the other verse states it starts at 30. Already we feel the contradiction.

Sometimes contradictions are resolved through a third verse which synthesizes the other two verses. Sometimes, the contradiction is resolved through our knowledge of outside, non-biblical disciplines. In this case, apprenticeship is a universal phenomenon in many cultures. Apprenticeship typically starts with a training period; upon graduation full vocational service may commence. This pattern is typical for doctors, lawyers, artisans etc.

So one reasonable way to resolve the contradiction is to suggest that:

- The training for Levite work began at 25.
- Actual Levite work began at 30, after completion of a 5 year study program.

Stage II: The literary analysis of this contradiction follows almost immediately from the Stage I analysis.

- We feel the aspirations of a 25 year old Levite who wishes to dive in and fully serve in the Temple. And yet,
- We also experience the tension that he can't yet serve till he trains for five years.

In this case, the contradiction corresponds to the conflict between aspiration and delayed onset of the aspiration. The literary analysis allows us to empathize with the emotional tensions of the aspiring Levite.

Example 5 – Examples
This example presents one method of style, which for lack of a better term, we call examples.

Biblical law by nature is casuistic. This simply means that any simple law is intended to be generalized. For example, Deut. 25:04, prohibits muzzling an ox while threshing. This prohibition is interpreted casuistically and consequently generalized: It is prohibited to prevent any working animal from eating food.

Consequently, if the biblical Narrator wishes that a law be interpreted restrictively, as applying only to that particular case without generalization, then biblical style requires the Narrator to use a General-Specific style.

We have called this method, *examples.* The method deals with whether a specific biblical case law is interpreted as an illustrative example which much be generalized or whether a specific biblical case law is interpreted restrictively as the totality of the law.

With this in mind, we can see that Num. 5:12-13, discussing the case of a jealous husband, uses a general-specific form as follows:

- [General] *When a man's wife behaves foolishly and she desecrates him [by]*
- [Specific] *[his suspecting that] a man was intimate with her*

The biblical text then continues with a description of the jealous-husband ceremony. If the wife is innocent (of intimacy), she passes the test and the couple can be reunited. If the wife is guilty (of intimacy), she will die.

Stage I: The effect of the general-specific clause is to restrict the entire jealous husband ceremony only to the case where the husband suspects his wife of intimacy. That is, the jealous husband ceremony does not apply to *any* desecration of marriage (the general clause); it only applies to desecration by intimacy (the specific clause).

Stage II: Although the general-specific format is purely legal in nature and purpose, it simultaneously serves a literary purpose in empathizing with emotional ambiguities and tensions of the husband.

- The verse explicitly says *if his wife behaves foolishly and she desecrates him.* We empathize with the wishes of the husband to have the jealous husband ceremony apply in *any* case of marriage desecration. After all, intimacy is not the only way to behave foolishly and desecrate a marriage.
- This empathy with the husband's jealously is balanced by the restrictive interpretation that the very serious jealous-husband ceremony only applies if he suspects her of outright intimacy.

We also have irony here: A woman could desecrate her marriage, but without intimacy, and not be subject to the jealous-husband ceremony.

Thus the literary analysis explores the full spectrum of justifiable jealously and the restrictive contrast to those cases where God intervenes with the jealous-husband ceremony.

Example 6 – Formatting
This example studies one rule, formatting. Formatting, refers to use of non-verbal textual features – for example, bold, underline, italics, bullets, paragraphing, consecutiveness, etc. - to communicate unspecified emphasis [11]. The biblical narrator, although not using bold and italics, had equivalent methods, methods using non-verbal textual features, to communicate unspecified emphasis.

The verse we study appears deceptively simple. Deut. 16:19, addressing judges, says,
(1) *Do not pervert justice,*
(2) *Do not recognize presence,*
(3) *Do not take bribes.*

Stage I: There is no formatting in the biblical text but the verse has a three-sentence structure. There are rules for transforming such repeated biblical themes into modern writing. In modern writing we would write as follows:

- *Do not pervert justice*
- *Do not even recognize presence*
- *Do not even take monetary inducements.*

The general rule in such repeated passages is to see each bullet as accepting the previous bullet and adding something additional. Consequently, the verse is read as follows:

- *Do not pervert justice* [e.g. declare the guilty innocent]
- *Do not even recognize presence* [e.g. even if a true verdict is reached, do not differentially treat the litigants]
- *Do not even take monetary inducements.*[e.g. even if a fair trial is given with a correct verdict, do not take monetary gifts for example, to declare someone innocent, when you already have written a decision, but not published it, to declare that person innocent. Although nothing has changed, the acceptance of the gift gives an improper appearance.]

As can be seen, the interpretation of each bullet is accomplished by assuming the previous bullets and adding something. For example, it would be incorrect to interpret *do not take bribes* as meaning *do not take bribes to judge a guilty person innocent* since that is already included in the first clause *do not pervert justice* and would not add anything. Therefore the clause *do not take bribes* is interpreted to mean taking bribes to declare a correct verdict (which the briber was unaware would be done).

Stage II: The climactic evolution of the proper ethical delivery of verdicts allows us to empathize with the judges feelings and experience his tensions.

- Like any other profession, the professional is expected to deliver according to the profession's standards: A guilty person should be found guilty; an innocent person should be found innocent.
- But a judge must be more than a professional. In any other profession you are for example allowed to greet and recognize an important presence; you are allowed for

example to give them a special seat; you are allowed to ask, "How is the family." Not so for a judge. Even though justice is rendered, the judge must act completely impartial to each party.

- Moreover, a judge cannot *appear* to be basing decisions on monetary or other matters. We have irony and paradox here. A judge who has written a decision declaring someone innocent, who is about to deliver that decision, cannot accept a monetary gift to deliver that decision even though it is correct. It is paradoxical that he can't be paid extra to do what he is going to do anyway. Note the contrast with other professions where tipping is not necessarily unethical.

Example 7 – Symbolism

Already, in the introduction to this section, we mentioned alternative approaches to literature such as the symbolic approach. Symbolism is a major literary method. We therefore close this section with an example of a symbolic interpretation.

We again emphasize the theme of this section. Broad open-ended methods can be mastered using proper goal-setting and using a two stage process.

For this example we use Pharoh's dream mentioned in Gen. 42:2-5. The dream is presented in bulleted fashion in Figure 9.

A.	*I [Pharoh] am standing by the Nile*
B.	*From the Nile, 7 cows arise*
C.	*[cows that are] good looking and of healthy flesh*
D.	*They [The cows] grazed by the meadow*
E.	*And behold, seven other cows*
F.	*Arose after them from the Nile*
G.	*Bad looking and thin-fleshed*
H.	*They [the bad cows]\ stood next to the [other] cows on the border of the Nile*
I.	*The bad looking cows consumed the good looking cows*

Figure 9. Pharoh's dream in bulleted presentation (Gen. 42-2:5).

Stage I: Symbolism is the most complicated of the literary methods. It has many subrules. In this case we use an *equation method* to understand the symbolism. The key symbolic equations for understanding the dream are presented in Figure 10.

Cows	=	Years
Nile	=	Economy

Figure 10. Symbolic equations governing the interpretation of the dream in Figure 9. The results of the interpretation are presented in Figure 11.

A.	*I [Pharoh] am standing by the economy*
B.	*From the economy, 7 years arise*
C.	*[years that are] good looking and of healthy flesh*
D.	*They [The years] [allowed us to] grazed by the meadow*
E.	*And behold, seven other years*
F.	*Arose after them from the economy*
G.	*Bad looking and thin-fleshed*
H.	*They [the bad years]\ stood next to the [other] years on the border of the economy*
I.	*The [pain of the] bad looking years consumed the [memory of the] good looking years*

Figure 11. The interpretation of the dream in Figure 9, based on the symbolic equations in Figure 10.

The equation method is implemented by *substituting* the equational values into the dream. For the convenience of the reader we have underlined, in Figure 9, words that must be translated using the symbolic equations in Figure 10. The results of this substitution is presented in Figure 11.

As seen in Figure 11, it was necessary to insert some parenthetical explanations to supplement the equational substitution.

The symbolic equation method is *one* method of interpreting symbolism. There are many others. The method can be mastered quickly and is almost mechanical.

Stage II: Although symbolism is an independent literary method, it is fruitful to explore for ambiguity, tension, paradox and irony in the symbolic interpretation. The dream interpretation has clear tensions, ambiguities, and even ironies.

- The Egyptians could enjoy themselves well with healthy flesh and meadow grazing for seven years. One can easily empathize with the desire to enjoy one's economy.
- But the joy would not last forever. There would follow seven painful years with lack of food.
- Moreover, the experience of the painful years would ironically erase the fond memories of the good years. So in the end it would not be worth it to rejoice for seven years.
- We also experience the tension: Should we enjoy while we can even if it is erased afterwards; or, should we plan to avoid the starvation years even though we might have to forfeit the joy of the good years.

Summary

In this section, we have presented seven examples that illustrate how use of ten almost-mechanical rules can lead to advanced literary analysis exploring tension, ambiguity, irony and paradox. Although advanced literary analysis is not immediately accessible to everyone, the ten almost-mechanical rules are clear, specific and masterable in a short time. Furthermore, they enable sophisticated open-ended analysis of literature.

6. CONCLUSION

This paper has examined use of an incremental approach based on proper goal-setting in implementing an executive-function based syllabus. Section 2 explored textbook problems and showed that although a syllabus topic such as periodic functions may lend themselves to rich verbal problems with multiple (in this case four) parameters, nevertheless, textbooks may water down the syllabus and devote an excessive proportion of problems to two-parameter non-verbal problems. Then, Section 3, showed how executive function with its emphasis on multi-component problems can facilitate tutorial sessions focusing, not on content, but on organizational issues such as doing each component separately or sharing information between components. Section 4 showed that an incremental approach could succeed in teaching difficult college level topics even at the elementary school level. Finally, Section 5 showed how an incremental approach could facilitate teaching subjects such as literary analysis, which are normally perceived as based on ability rather than exclusively on skills. These ideas should prove useful in a variety of instructional settings for improving student performance.

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