A Measure for Complex Dynamics in Power Systems

Ralph WILSON, Michael SATTLER, Touria El-MEZYANI,
Sanjeev SRIVASTAVA, Chris EDRINGTON, and Dave CARTES
Center for Advanced Power Systems, Florida State Univsersity
Tallahasse, Florida 32304 U.S.A.

ABSTRACT

In an attempt to quantify the dynamical complexity of power systems, we introduce the use of a non-linear time series technique to detect complex dynamics in a signal. The technique is a significant reinterpretation of the Approximate Entropy (ApEn) introduced by Pincus, as an approximation to the Eckmann-Ruelle entropy. It is examined in the context of power systems, and several examples are explored.

Index Terms: Complexity, Chaos, Power system, Switching converters

1.INTRODUCTION

Complex systems are characterized by highly interdependent components, often in network configurations, exhibiting non-linear dynamics, and other such properties. Non-linear dynamics, such as bifurcations and chaos, have been seen to cause voltage collapse, angle divergence, and other faults in power systems [1-3]. Simply by its nature, power generation and distribution is an inherently a non-equilibrium process (i.e. load changes inducing driven generator responses, swing dynamics, etc) happening in a network structure. Furthermore, the power systems (especially shipboard power systems) of the near future will contain much higher degrees of reconfigurability (structural adaptability), which will allow them to adapt in the case of a fault, or drastic change in load demand [4]. Thus, they will have to be adaptive complex networks.

Since many of the complex characteristics arising in power systems create design and control difficulties, we wish to be able to quantify the complexity of a system. In a previous publication [5], we discuss some distinct potential realms within which to quantify complexity. Namely, we distinguish between component level, configurational, and a global "complex" properties. The component level is based on component dynamics and interactions. The configurational concerns the static, physical causal connections within the system, and its propensity for complex behavior. This would be based on the presence of feedback loops, particular network topologies, and other configuration properties which engender complex behaviors. The global level would concern the emergent properties of the system as a whole, as well as the interaction of the system with its environment. For the work presented here, we focus on the component level, with an eye toward the dynamics and interactions of components.

2. APPROXIMATE ENTROPY AND ITS INTERPRETATION

Measuring Entropy from Data

If a system changes so that a controller would have to incorporate this change into a new control decision, we can say that the amount of information required to determine the state of the system has increased. This is the same as saying that the controller's uncertainty of the system has increased. Entropy (in the sense of Shannon [6]) has long been proposed as a measure of this type of uncertainty in systems. For a system X with n states, the i^{th} state having probability p_i , the Shannon entropy is given as

$$H(X) = -\sum_{i=1}^{n} p_i \log p_i \quad (1)$$

The Renyi entropy is a generalization of the Shannon entropy. The Renyi entropy is given as

$$H_\mu(X) = \frac{1}{1-\mu} \log \sum_{i=1}^{n} p_i^{\mu} \quad (2)$$

It can be shown that as $\mu \to 1$, $H_\mu(X) \to H(X)$, the Shannon entropy. For increasing μ, H_μ is increasingly determined by states with higher probabilities. Lower values of μ weigh the states more equally. We see that from this, we can define infinitely many "entropies" for all positive μ.

Time Delay Embedding: Though in its present formulation, this seems quite abstract, researchers since Shannon have developed ways to apply this notion to experimental data. If an experimenter wants to apply these measures to time series data, they may "reconstruct" the attractor of the system. The most widely used way of doing this is called "time delay reconstruction" [7]. If one has a time series $X=\{x_1,x_2,..x_n\}$, then for an integer time delay τ, one can construct m-dimensional vectors as

$$X_n = \left\langle x_n, x_{n+\tau}, x_{n+2\tau} \ldots x_{n+(m-1)\tau} \right\rangle, \quad (3)$$

for $n \in \{1,2,...N-(m\tau)\}$. Having done this, one has embedded the data in an m-dimensional Euclidian space. We can now think of this as a reconstructed state space of our system. A "state" is now a point or, approximately, a region of the space. The probability of being in certain states can be estimated by looking at the proportion of reconstructed vectors that lie in the corresponding regions of the state space. We can think of these regions as neighborhoods of radius ε, or m-dimensional boxes with sides of length ε, and we can refine the entropy by letting ε get very small. This scheme can be approximated by counting populations in ε -neighborhoods around reconstructed vectors.

Approximate Entropy: A formal definition of entropy for dynamical systems has required some special considerations in the way of ergodic theory, and has produced the notion of Metric entropy (a.k.a Kolmogorov-Sinai entropy) which is "the mean rate of creation of information" [8]. This quantity was shown to be able to be computed via the Eckmann-Ruelle (E-R) entropy, using data in a reconstructed state space. To introduce the E-R entropy, we need some prerequisite definitions. Keeping the vector notation of the time delay embedding in mind, let us introduce

$$C_i^m(r) = \{\text{number of } X_j \mid d(X_i, X_j) \leq r\}, \qquad (4)$$

where $d(X_i, X_j)$ is the maximum of the component-wise differences between X_i and X_j. We can now define

$$\phi^m(r) = \frac{1}{N-m+1} \sum_{i=1}^{N-m+1} \log C_i^m(r) \qquad (5)$$

The E-R entropy is defined to be

$$H_{(ER)} = \lim_{r\to 0} \lim_{m\to\infty} \lim_{N\to\infty} [\phi^{m+1}(r) - \phi^m(r)] \quad (6)$$

An approximation to $H_{(ER)}$, called "ApEn" (first proposed by Pincus in [9]) is defined as

$$ApEn(m,r) = \phi^{m+1}(r) - \phi^m(r) \qquad (7)$$

There are many reasons to approximate the E-R entropy. First, it is infinite for signals with noise of any amplitude [10]. Second, it requires an infinite amount of data. Since we not only have finite data, but also wish to be able to compute with relatively small amounts of data (so that real time analysis would be possible) we would not be able to compute using the E-R definition. When the E-R entropy gives a nonzero finite value, this ensures the existence of deterministic chaos in a signal. In making an approximation of it, we lose this very strong indication. The ApEn statistic does not have this power, as it can not distinguish randomness from deterministic chaos. Though it is finite in the presence of randomness, it is maximal on random data (like all entropy measures). Yet, as we will see in the following discussion, there is reason to believe that, by the novel modification of ApEn proposed here, we may be able to strip away some measure of entropy which is caused by randomness in the system. It is also the case that ApEn has a computational advantage, in that, it has been seen to be robust in use with as little as a few hundred data points.

Interpretation of the ApEn measure

Though the ApEn measure has been widely used in many different physiological settings [10,11], there are some considerations that remain to be settled about its implementations, namely, the choices of parameters used. For a given time series, ApEn is a function of m, r, and N. Though the original authors take care to characterize the appropriate ranges for N, their prescriptions for m and r are given, but not formally justified. The need for further considerations is mentioned by Pincus in [9], in recognition that "guidelines are needed for choices of m and r to ensure reasonable estimates of ApEn(m,r)". It is known that time delay embedding yields false information [12] if the embedding dimension is not high enough to unfold the attractor appropriately. Since the ApEn is dependent upon time delay embedding, it seems reasonable to ensure that m be sufficiently large. We will discuss below the choice of r. An additional consideration that is not mentioned in any of the literature involving ApEn is the question of the time delay (or lag). It is assumed implicitly that τ =1. For some discrete numerical systems, the time delay can be τ =1, but for others, and for discrete samples from experimental signals, the appropriate time delay is dependent upon the sampling rate and can often be quite larger than one [13,14]. Since the ApEn method relies on making a time delay embedding, one must make sure that the time delay is appropriately chosen, just as the embedding dimension should be, in order to reconstruct the attractor faithfully.

Without much consideration of the time delay, and embedding dimension, ApEn has been used successfully in many capacities [9-11],[15] as an indicator of qualitative differences in time series. In this work, we are computing the ApEn measure, with full considerations in choice the appropriate m, r, and τ. There are known solutions to motivating our choices of m and τ. For our choice of τ, we rely on the method developed by Kim et al. which is referred to as the C-C ("C minus C") method [13], and for our choice of m, we rely on the method of false nearest neighbors, or FNN [12].

We have found that, given any fixed m, there is a particular value r_0, such that for $r \geq r_0$, ApEn(m,r) for random signals become zero while, for the same r_0 value, ApEn(m, r_0) on nonrandom, and chaotic signals is nonzero (this will be fully explained in a future work). Thus, for all m, there exists an r, so that we find that ApEn(m,r) is unaffected by randomness, while still achieving some quantification of other dynamical structure present in the data. Much interest lies in this distinction, because one of our main concerns is the detection of chaotic or otherwise complex dynamics.

3. ASPECTS OF COMPLEXITY; DYNAMICS, COUPLING, AND EMERGENCE

When system components begin to interact in unanticipated ways, we may consider the potential for emergent behavior. Such unforeseen events potentiate cascades of further effects, possibly on larger scales than the initial interactions. Emergence is a very important characteristic of complex systems, and one which is especially baffling from control and engineering perspectives, as its nature is not generally describable a priori. Though there is not a strict, global definition of emergence, there are distinctions made, such as the distinction between weak and strong emergence [16]. Weak emergence is when higher-level phenomena arise from the lower-level interactions, and while unexpected, truths concerning that [high-level] phenomenon are deducible given the principles governing the low-level domain. As pointed out in [17], even "Dependencies, such as correlations," can be considered weak emergence, as they "arise from the interactions of components and in such a case, can be inferred from the properties of the components and their interactions." This is in contrast to strong emergence, which is the same, except that the truths concerning the high-level phenomena are not necessarily deducible given the low-level principles. Emergence is an instance where the interaction of *n* components can not be described by combining the individual descriptions of the *n* components (whole greater than the sum of its parts).

As of yet, we do not have a direct way to quantify the degree to which our system exhibits general emergent properties, nor do we know a priori what those traits might be. For the moment then, we begin by trying to examine the dynamics on the smallest scale such that system-wide real time data monitoring is feasible, and work our way up. In a power system, this scale is populated by the various power generation, transmission, and distribution components. The relevant dynamical aspects include current, voltage, and frequency. The occurrence of nonlinear (or otherwise unusual) dynamics can be seen as a result of interactions between components, as well as the system responding to global constraints (i.e. hard limits, etc...). Thus, the detection of such dynamics may at least indicate some emergent behavior. It is not clear from small scale considerations what larger scale effects an emergent local behavior will have on the system. So far, it is seen that instability and failure are among effects that may result from such phenomena.

We utilize various dynamical systems to study interesting aspects of non-linear dynamics. We begin in the simplest way by examining the behavior of ApEn in the presence of chaos. We examine the chaotic output of a dynamical system whose behavior is given by an equation, namely the Rössler map. Moving further towards application to power systems, we examine simulated dynamics of a power electronic component when it is coupled with a noise source. Beyond this, we examine local dynamics resulting from a change in the system control in a large scale simulation of a shipboard power system,.

Large power systems exhibit very wide ranges of behavior, including stable dynamics, and less stable nonlinear dynamics, as well as large changes due to control changes, faults, or environmental factors. The latter changes generally cannot be accounted for deterministically. They are examples of non-stationary behavior. One interest is finding out how our ApEn responds to these changes in system behavior. Thus, in proposing any new measurement of system, one must account for how it will react to various states for which it may or may not be as descriptive, or informative. We hope to characterize its responses in a manner that allows us to distinguish stable kinds of dynamics from more non-linear, or otherwise unstable dynamics, whether they are deterministic or non-stationary effects.

Rössler system

Consider the Rössler system,

$$\dot{x} = -y - z$$
$$\dot{y} = x - ay$$
$$\dot{z} = b + z(x - c) \tag{8}$$

with a, b, and c as real-valued parameters. The above equation determines a dynamical system which exhibits nonlinear behavior as dependent upon the parameter values. The system exhibits its nonlinear dynamics, including a period doubling route to chaos. Using the method of Runge-Kutta, we generate a discrete time series behavior for each variable. Let us fix b and c at 2 and 4 (resp.), and vary a, letting a be our “bifurcation parameter”. We see that (figure 1, lower panel), for smaller values of a (near .33) the system is periodic. As a is increased, the dynamics of the sequence bifurcate, becoming multiply periodic. These bifurcations continue in a period doubling cascade until, for a critical value of a (a_c), the sequence becomes chaotic. Similar transitions to chaos have been observed in power systems (and systems from many fields), and so the Rössler map is a natural starting place to examine the behavior in our context.

Coupled Buck Converter

The data that we analyzed from a power electronic component is taken from the output of a buck converter, when it is coupled with spurious periodic noise, as reported in [18]. The buck converter is one of the simplest dc-dc converters, and we study it in continuous conduction mode controlled by a pulse-width modulator. A circuit model is used to produce intermittent chaos in a buck converter resulting from the presence of noise within the system. The model uses a negative feedback system with voltage-mode control to automatically correct the duty cycle of the buck converter. In voltage-mode control, the output voltage is compared with a reference signal to generate a control signal which drives the pulse-width modulator by using some typical feedback compensation setup.

The reference voltage v_{ref} is coupled to the spurious signal. This weak periodic signal that is introduced is used to illustrate noise within the feedback control in the converter current output. The difference between the output voltage and the reference voltage is known as the error voltage. The goal of the feedback system is for the error to be small, and it works ideally when the error signal is zero, which occurs when the output voltage equals the reference voltage. This feedback loop introduces the potential for non-linear dynamics in the system. The bifurcation effects of the coupled converter were found to be exactly as in the previous literature [18] (Fig. 2). The bifurcation parameter used is α, which is defined as $\alpha = v_s / V_{ref}$, where v_s is the amplitude of the noise signal, and V_{ref} is the reference voltage.

Shipboard Power system via RTDS

To look at dynamics in a power system, and not just in an isolated component, we consider a set of electrical current measurements obtained from shipboard power system (SPS) simulated in high fidelity on a Real Time Digital Simulator (RTDS). The RTDS is a high speed, real time test system that can be used for control system testing and general power system simulation [19]. The SPS modeled in RTDS consists of a 4.16kV MVAC ring bus, which connects two main generators (MTG, 36 MW each), two auxiliary generators (ATG, 4 MW each), two propulsion motors (PM), a pulsed load and 3MW radar via switchboards. We consider measurements of current in the system operating during a control change, where one control parameter is changed in such a way that the resulting current signal becomes erratic.

4. RESULTS AND DISCUSSION

Systems and Analysis

For all of the data in this paper, the ApEn is computed using a moving window on the time series, as though it were measuring a signal in real time. Each ApEn value is shown plotted directly above the center of its corresponding data window. All data used is discrete, either by generation, or by sampling a continuous wave form.

Rössler system: We compute the ApEn on the dynamics of the variable x generated by the Rössler map, while incrementing the bifurcation parameter a towards, and past, the critical value. The a return map of resulting signal can be seen in the lower panel of Figure 1, showing its bifurcations en route to chaos. The corresponding ApEn values are shown in the upper panels. For fixed point (constant) behavior, or perfectly periodic states, the ApEn gives small values. When the parameter is changed, and the system bifurcates from period one to period two, the ApEn increases. It initially shows a spike response as a result of the use of the moving window. While the window contains data both before and after the bifurcation, the reconstructed state space trajectory contains structure from dynamics before the bifurcation, as well as structure from post bifurcation dynamics. The spikes drop as the window passes the bifurcation, as there is no additional structure being measured. After the spike drops, the values are higher than before the bifurcation. This is because bifurcations add structure to the state space trajectory. This happens in general in the presence of bifurca

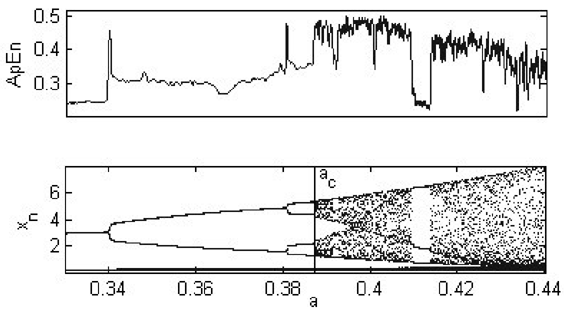

Figure 1. The ApEn is computed for the various behavioral regimes of the Rössler map. The dynamics are dependent upon the parameter *a*. The bifurcation diagram of the Rössler map is in the lower panel, with *a* incremented discretely, while the corresponding ApEn values are plotted in the upper panel. a_C is marked at the point when a reaches a_C.

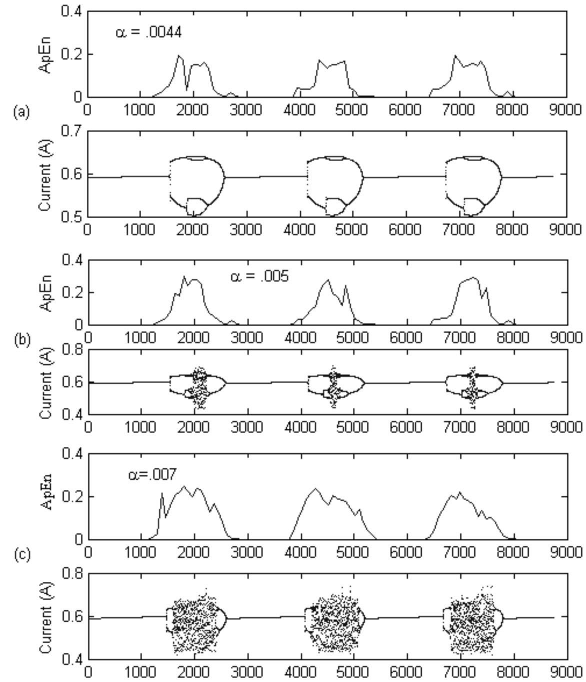

Figure 2. The return maps of the current from the buck converter showing bifurcations (lower panels), with calculation of ApEn (upper panels). Each figure corresponds to a different α value, with α =0.0044, α =0.005, α =0.007 in (a), (b), and (c) respectively.

tions, as we will continue to see. The measure continues to increase as the system bifurcates, and transitions into chaos. Towards the end of the section, we will give more discussion to the general behavior of the ApEn.

Buck Converter: As we can see in Fig. 2, the ApEn responds as we would expect in the presence of bifurcations in the system. While the converter current is perfectly periodic, the ApEn is zero, but once the bifurcations occur, ApEn increases. One can notice the amount of this increase by examining the shape of the ApEn curve in Fig. 2(a). When an initial bifurcation occurs, the increase is in the range of 0.05. When a second bifurcation occurs in cascade, the measure jumps up above 0.1. Instances ofthis can be seen in the rest of Fig. 2. When chaotic dynamics occur, then the ApEn registers a very large increase, spiking to 0.2 and above (Fig. 2(b,c)). Overall, if we restrict our attention to the regions of data which yield ApEn values of 0.1 or higher, we immediately recover the bifurcated regions, as well as the chaotic regions. Beyond these considerations, and more towards understanding how outside (non-deterministic) changes can affect the dynamics of components, we examine the impact of sending a pulse to the buck converter's reference voltage while it is coupled with the noise source (Fig. 3(a)). This has been found to induce isolated bursts of bifurcations, that is to say, not cascades, but one bifurcation, its decay, followed by another, and its decay. It has been found that if the amplitude of the pulse is large enough, the burst exhibits chaos, without a visible cascade of bifurcations as a transition. This isolated bursting occurs for non-zero α, even when α is not large enough to induce bifurcations by itself. We can see from Fig. 3(a) that the initial amplitude increase/phase shift from the pulse, before bifurcation, yields an increase in ApEn value which is below 0.05, indicating sensitivity not only to nonlinear dynamics, but also other changes in signal structure. When bifurcations occur, the ApEn increases more significantly, as before. As we can see, with a 4V pulse amplitude, the ApEn increases above 0.1 in the presence of the multiple large amplitude bifurcations and stays for the duration of the isolated bifurcations. It also picks up on the smaller bifurcations toward the end of the pulse decay. The step like responses to the presence of multiple isolated bifurcations, which can be seen clearly toward the end of Fig. 3(a), seem to indicate some kind of additive response on the part of the ApEn, when confronted with temporally close isolated bifurcations (the response of the ApEn doubles when the window gains an additional bifurcation, and then decreases by half when one passes out of the window).

Shipboard Power system: In looking at the simulation of the shipboard power system, we find that the dynamics are much less simple, in many respects. When the system is periodic, it is not perfectly so, so that the ApEn does not take on zero values during periodic behavior. Since the waveform is more varied, the structure is much more spread out in state space. For our simulation, we look at a stable condition with the introduction of a control change. As can be seen in Figure 3(b), the periodic behavior registers an ApEn relatively small value (<0.2), until the control change occurs about midway through. When the control change occurs, the output of the system becomes erratic, with periodic bursts of aperiodic dynamics. For this condition, the ApEn surges to a sustained value near 1.

Interpretation of Results

This signal measure is zero or small on perfectly periodic and constant signals, which have minimal Shannon entropy. It is also zero on random signals, which have maximal Shannon entropy. This allows it to be seen as a measure of complexity that is a convex function of disorder. It attains its maximum value on signals whose entropies are between the minimum and maximum. This has been seen [20] as an important characteristic of measures of complexity, since complex systems necessarily exist somewhere between complete order, and complete disorder.

Our preliminary uses of the ApEn methodology as a dynamic, "real-time" observation tool suggests that we may be able to distinguish "non-trivial" changes in system behavior. This is done by detecting geometrically "non-trivial" changes in the reconstructed state space trajectory of the system. What is meant by a non-trivial change, the results suggest, is one which disturbs the stable nature of the system, either by transitions into multiperiodicity (bifurcations), complete aperiodicity (chaos),

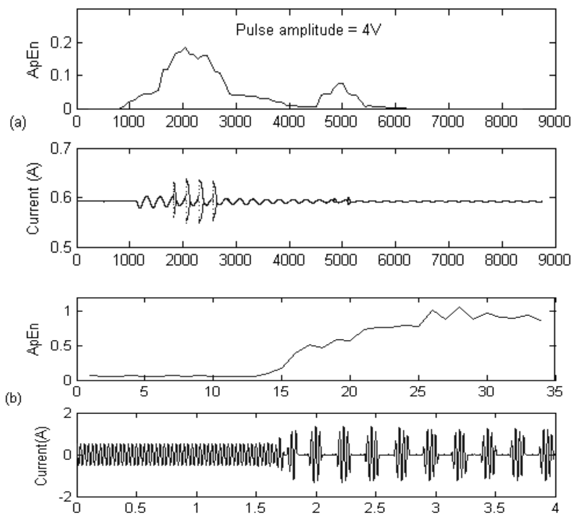

Figure 3. (a) Bifurcation diagram of buck converter response to a pulse with an amplitude of 4V (lower panel). The ApEn is calculated as the buck converter evolves (upper panel). (b) The shipboard power system undergoes a control change during a stable running condition (lower panel). The corresponding ApEn, computed with a moving window, evolves with the signal (upper panel).

or general perturbation. What it seems that we are observing is the ability of a modified ApEn to detect the increase in structure of a system's state space trajectory (not necessarily an attractor). When a deterministic system transitions from periodic to chaotic, the corresponding attractor begins (topologically) as a discrete collection of points (discrete systems) or a circle (continuous systems). The system attractor then transitions (most often) into a fractal manifold or some other relatively space filling structure. Many aspects can be measured of these structures (fractal dimension, Lyapunov exponents, etc.), but these kinds of chaotic quantifications are more meaningful on measurements on stationary systems, or during stationary periods of a system, as they give specific geometric information about attractor structure and dynamics. The systems of interest will not always be stationary, and thus, there will not always exist an inherent attractor to be studied. Luckily, non-deterministic changes still correspond to shifts in the reconstructed data's structure in state space, and seem to be able to be detected by the modified ApEn.

5. CONCLUSION

The nature of power systems is such that outside conditions create changes which are not given by any deterministic aspect of the system components, or their interactions. Thus, power systems investigators, and controllers, may easily find themselves in a situation where they are less interested in the specific aspects of chaotic or otherwise unstable behavior, and more interested simply in the presence of it. This is especially likely when one takes into account the fact that the more specific information is computationally more expensive, thus delaying a control response time. One important example is that of switching between centralized and decentralized control. There is growing evidence [21,22] suggesting that distributed control may be more appropriate for use in complex systems. Yet, when systems are stable, centralized control is more efficient. Thus, the ApEn methodology presented here might yield information as to when to switch from centralized to decentralized control.

We seem to be able to use the modification of the ApEn, as was done here, to make dynamically meaningful distinctions in the output of a system. It will continue to be seen for which state changes the ApEn takes on small values. By doing this, we can begin to fathom precisely what can be determined through the measure, and what eludes it. It seems thus far that, if we consider the complexity of a system as inherently linked to the manner in which it traces out a history in its state space, then the ApEn measure may be of value in determining when a system is deviating from past behavior, as well as when it does not seem to be settling into some new stable behavior. We suggest that this may be an appropriate point to begin suspecting that the system behavior, or state, may be deserving of being called "complex".

Acknowledgments
This work was supported by the Office of Naval Research, under Grant Contract # N00014-09-C-0144

6. References

[1] Y. Yu, H. Jia, P. Li, J. Su, "Power system instability and chaos", **Electric Power Systems Research**, 65, 2003, pp. 187-195

[2] I. Dobson, "Computing a Closest Bifurcation Instability in Multidimensional Parameter Space", **J. Nonlinear Sci**., 3, 1993, pp. 307-327

[3] H. Wang, E. Abed, A. Hamdan, "Bifurcations, Chaos, and Crises in Voltage Collapse of a Model Power System", **IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications**, 41(3), 1994, pp. 294-302

[4] R. Sarfi, M. Salama, A Chikhani, "A survey of the state of the art in distribution system reconfiguration for system loss reduction", **Electr Power Syst Res**, 31, 1994, pp. 61-70

[5] T. El-Mezyani, R. Wilson, S. Srivastava, D. Cartes, "Complexity Assessment in All Electric Ship", **Proceedings of ASNE Day 2010**, Arlington VA, April 2010 (to be published)

[6] C. Shannon, "A Mathematical Theory of Communication", **The Bell System Technical Journal**, Vol 27, 1948, pp. 397-423

[7] F. Takens, "Dynamical systems and Turbulence, Warwick, 1980" **Lecture Notes in Mathematics** 898 (Springer, Berlin 1981) , 1980, p.366

[8] J. Eckmann, D. Ruelle, "Ergodic theory of chaos and strange attractors", **Rev. Mod. Phys**. 57, pp. 617-656

[9] S. Pincus, "Approximate entropy as a measure of system complexity", **Proc. Natl. Acad. Sci**., 88, 2001, pp. 2297-2301

[10] S. M. Pincus, I. M. Gladstone, R. A. Ehrenkranz,, "A Regularity Statistic for Medical Data Analysis", **Journal of Clinical Monitoring**, 7(4), 1991, pp335-345

[11] J. D. Veldhuis, M. L. Johnson, "Impact of pulsatility on the ensemble orderliness (approximate entropy) of neurohormone secretion", **Am J Physiol Regulatory Integrative Comp Physiol**, 281, 2001, pp 1975-1985.

[12] H. D. Abarbanel, M. B. Kennel, "Local false nearest neighbors and dynamical dimensions from observed chaotic data", **Physical Review E**, 47(5), 1993, pp. 3057-3068

[13] H. S. Kim, R. Eykholt, J. D. Salas, “Nonlinear dynamics, delay times, and embedding windows”, **Physica D** 127, 1999, pp. 48-80

[14] R. Wilson, N. Podder, "Observation of period multiplication and instability in a dc glow discharge", **Physical Review E**, 76(4), 2007, 046405-(1-8)

[15] S. Pincus, B. H. Singer, “Randomness and degrees of irregularity”, **Proc. Natl. Acad. Sci** 93, 1996, pp. 2083-2088

[16] D. J. Chalmers, “Strong and weak emergence”, **The re-emergence of emergence: the emergentist hypothesis from science to religion**, Oxford University Press, New York, 2006, pp. 224-256

[17] Y. Bar-Yam, “A mathematical theory of strong emergence using multiscale variety”. **Complexity**, *9*, 2004, pp. 15–24

[18] C. K. Tse, Y. Zhou, “Intermittent chaos in switching power supplies due to unintended coupling of spurious signals”, **IEEE Power Electronics Specialists Conf. Record**, Vol 2, 2003, pp. 642-647

[19] R. Kuffel, J. Geisbrecht, T. Maguire, R. P. Wierckx, and P. G. McLaren, “RTDS-a fully digital power system simulator operating in real time,” **Proc. 1995 IEEE Conf. on Communications, Power, and Computing**, WESCANEX, vol. 2, 1995, pp.300-305

[20] D. Feldman, J. Crutchfield, “**Measures of statistical complexity: Why?**”, Phy. Lett. A 238 (1998), pp. 244-252

[21] N. Baker, W. Harris, J. Le Goff, R. McClatchey, “CICERO: developing complex distributed control systems using object technology”, **International Seminar on Client/Server Computing Proceedings**, Vol 1, October 1995, pp. 11/1-7

[22] R. Jennings, S. Bussmann, “Agent-based control systems: Why are they suited to engineering complex systems?”, **IEEE control systems**, Vol 23, no 3, 2003, pp. 61-73