

# Remote Control of an Inverted Pendulum System for Intelligent Control Education

Seul Jung  
Intelligent Systems and Emotional Engineering Laboratory  
Department of Mechatronics Engineering  
Chungnam National University,  
Daejeon, 305-764, Korea

and

Jae Kook Ahn  
Intelligent Systems and Emotional Engineering Laboratory  
Department of Mechatronics Engineering  
Chungnam National University,  
Daejeon, 305-764, Korea

## ABSTRACT

This paper presents a remote control task of an inverted pendulum system for intelligent control education. The inverted pendulum moving on the guided rail is required to maintain balancing while it follows the desired trajectory commanded remotely by a joystick operated by a user. Position commands for the inverted pendulum system are given by a joystick through the network. The inverted pendulum system is controlled by a neural network control method. The corresponding control results are confirmed through experimental studies.

**Keywords:** Inverted pendulum system, remote control, intelligent control education.

## 1. INTRODUCTION

Recently, control education has become more important in a control engineering field since the state-of-the art technologies demand more sophisticated control algorithms. It is true that proportional-integral-derivative (PID) controllers have been dominantly used in manufacturing industries for their simplicity and stability. However, as technologies develop rapidly, advanced control methods are gradually demanded for regulating sophisticated and complicated control systems to satisfy highly specified performances.

For effective education of advanced control methods to students, control theories should be explained along with experimental demonstration. Experimental studies will help students to understand control theories with ease and can compensate for the lack of simulation studies for advanced control applications since real world has to deal with many uncertain problems.

The inverted pendulum systems have been dominantly used as a control system test-bed since they have several particular characteristics. Firstly, a single input controls two outputs, an angle and a position, which is considered as a single-input multiple output system. One single input  $u$  has to control two variables, angle  $\theta$  and position  $x$  to satisfy desired position

tracking while balancing. Thus, suitable combinations between angle control and position control provide successful performances. Secondly, the inverted pendulum system is also a nonlinear system, but can be linearized. Lastly, the system is costly effective that can be easily implemented. A single motor and necessary hardware along with a inverted pendulum body are required. Cost of experimental apparatus for education is mostly concerned since many copies of the same system are required.

These characteristics lead to enormous interest of using it as a prototype control system in the control education class. Particular reasons are to illustrate the concept of nonlinearity and challenge of control. In addition, the control concept of balancing mechanism can be applied to many other control applications such as control of balancing humanoid robots, two wheeled mobile robots, single wheeled mobile robots, and other balancing systems.

Many control algorithms have been proposed and applied to various modified types of the inverted pendulum system since its challenging characteristics are evolving. Recently, endowing intelligence to the control systems has been vastly increased along with growing of human desires. One control subject called an intelligent control area has been enormously expanded and has drawn attention of many researchers as technologies develops fast and systems become more complicated.

Two feasible candidates for intelligent control applications are fuzzy logic and neural network. Fuzzy controllers are popularly used in commercial products such as consumer appliances and mechatronics systems. Recently, neural network is gaining many interests since its structure is similar to human brain whose functions are learning and adaption capabilities that system learns and adapts environment gradually by reinforcement.

Many researches on control of inverted pendulum system have been presented in the literature [1-8]. Fuzzy controllers are used to control the nonlinear systems [1, 2]. Rotational inverted pendulum systems have been presented [4]. Inverted pendulum systems with two or three links have been presented [7, 8].

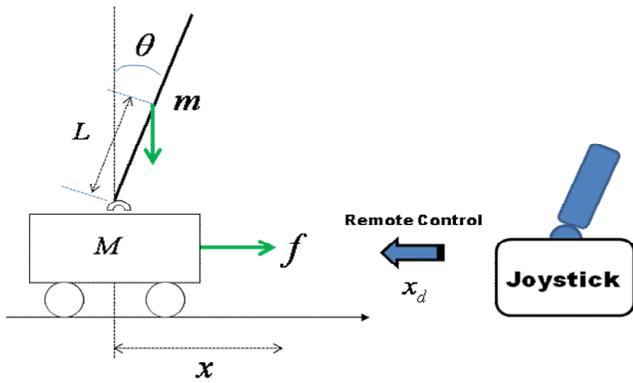


Fig. 1. Teleoperation of an inverted pendulum system

The aim of this paper is to apply neural network to intelligent control education. In the framework of neural network control education, two educational goals, intelligent control and remote control are presented although time delay issue is not considered in this paper [9].

Radial basis function (RBF) neural network is implemented as a neural controller for controlling an angle and a position of the inverted pendulum system at the same time. Back-propagation learning algorithm for the neural network is presented and embedded on the hardware to achieve real-time control. Position of the inverted pendulum system is commanded by a joystick as the desired position. A user controls the joystick to command the movements of the inverted pendulum system remotely. Then the inverted pendulum follows the position command while it maintains balance of the angle.

Experimental environment of remote control of the inverted pendulum system by the joystick is built as shown in Fig. 1. Experimental works of using a neural network controller along with a PID controller are demonstrated and their results are compared.

## 2. PID CONTROL SCHEME

The PID control method is used in controlling an angle and a position. The balancing angle error of the inverted pendulum system is defined as

$$e_{\theta} = \theta_d - \theta, \quad (1)$$

where  $\theta_d$  is the desired angle value which is 0 and  $\theta$  is the actual angle. The angle error passes through the PID controller.

$$u_{\theta} = k_{p\theta}e_{\theta} + k_{d\theta}\dot{e}_{\theta} + k_{i\theta}\int e_{\theta}dt, \quad (2)$$

where  $k_{p\theta}, k_{d\theta}, k_{i\theta}$  are controller gains.

The position error of the inverted pendulum system is defined by

$$e_x = x_d - x, \quad (3)$$

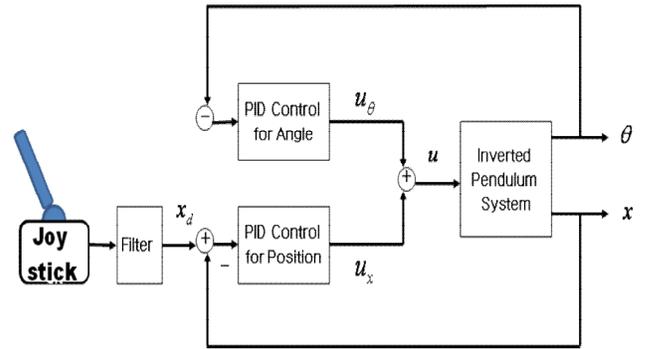


Fig. 2 PID control structure

where  $x_d$  is the desired position value received from the joystick and  $x$  is the actual position. The detailed PID controller output becomes

$$u_x = k_{px}e_x + k_{dx}\dot{e}_x + k_{ix}\int e_x dt, \quad (4)$$

where  $k_{px}, k_{dx}, k_{ix}$  are controller gains.

Each controller output is added together to generate a force input to the system. The total control input is

$$u = u_{\theta} + u_x. \quad (5)$$

The PID control block diagram for the inverted pendulum system is shown in Fig. 2. The joystick command is considered as the desired trajectory  $x_d$  after an appropriate filtering process.

## 3. RBF NEURAL NETWORK STRUCTURE

Neural network has a massive parallel processing structure which requires a fast processor to perform many calculations in on-line. The radial basis function (RBF) network among many neural networks is used since it has a linear output such that nonlinear function is not used in the output layer. There are no weights between the input layer and the hidden layer which is a different structure from the multi-layered perceptron (MLP) network. The RBF network also uses the Gaussian function as a nonlinear function. However, the number of weights to be updated is 4 which is same as the one layered MLP network. The RBF structure is shown in Fig.3.

The Gaussian function used in the hidden layer is

$$\psi_j(e) = \exp\left(-\frac{\|e - \mu_j\|^2}{2\sigma_j^2}\right) \quad (6)$$

where  $e$  is the input vector  $e = [e_1 e_2 \dots e_n]^T$  which is the error vector,  $\mu_j$  is the center value vector of the  $j$ th hidden unit, and  $\sigma_j$  is the width of the  $j$ th hidden unit. The forward  $k$ th output in the linear output layer can be calculated as a sum of outputs from the hidden layer.

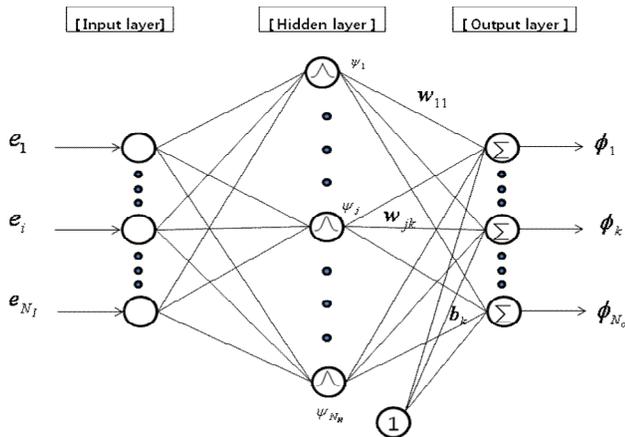


Fig. 3 RBF neural network structure

$$\phi_k = \sum_{j=1}^M \psi_j w_{jk} + b_k \quad (7)$$

where  $\psi_j$  is  $j$ th output of the hidden layer in (6) and  $w_{jk}$  is the weight between the  $j$ th hidden unit and  $k$ th output, and  $b_k$  is the bias weight. In the RBF network, the number of updated weights is 4 such as  $\mu_j$ ,  $\sigma_j$ ,  $w_{jk}$ , and  $b_k$ .

#### 4. NEURAL NETWORK CONTROL SCHEME

##### Reference Compensation Technique(RCT)

PID controllers are dominantly used and work for balancing angle control. But they have the difficulty of control desired position at the same time especially when disturbances are present. Thus, the proposal is to add a neural network along with PID controller to improve tracking performances under outer disturbances. Neural network has the capability of compensating for nonlinear uncertainties in the system.

When neural network is used as a controller, development of an on-line learning algorithm is the most important technique to be solved. One of neural network control algorithms is the reference compensation technique (RCT). Fig. 4 shows the concept of RCT that has neural network at the input trajectory level [3,6,10,11].

The idea of the RCT scheme is for neural network to locate at the input trajectory level to compensate for uncertainties in the system as shown in Fig. 4. The reference input  $r$  is added to the neural network output  $\phi_N$  to generate modified the error  $\varepsilon$ . The modified error  $\varepsilon$  is different from the output error  $e = r - y$ .

The modified error is given by

$$\begin{aligned} \varepsilon &= r - y + \phi_N \\ &= e + \phi_N \end{aligned} \quad (8)$$

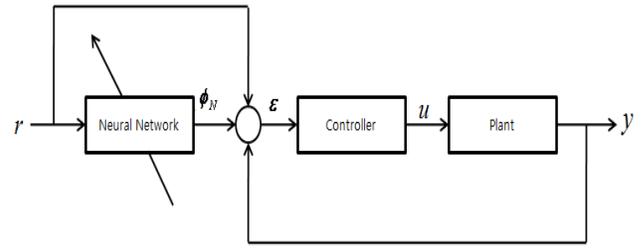


Fig. 4 RCT neural network control block diagram

where  $r$  is the desired input, and  $\phi_N$  is from the neural network output. After passing the controller, we have

$$u = K(e + \phi_N). \quad (9)$$

Define the training signal to be minimized as  $v = Ke$ . Then (9) becomes the closed loop error equation given below.

$$v = f - K\phi_N, \quad (10)$$

where  $f$  is the system dynamic equation and  $K$  is the controller gain.

For deriving learning algorithm, we define the objective function to be minimized as

$$E = \frac{1}{2} v^2. \quad (11)$$

The back-propagation algorithm is required to calculate the gradient function below.

$$\begin{aligned} \Delta w(t) &= -\eta \frac{\partial E}{\partial w} \\ &= \eta K v \frac{\partial \phi_N}{\partial w} \end{aligned} \quad (12)$$

where  $\eta$  is the learning rate. Then weights are updated at each sampling time as

$$w(t+1) = w(t) + \Delta w(t) \quad (13)$$

Therefore, in the learning algorithm point of view, the RCT control scheme becomes eventually same as the feedback error learning control scheme except controller gain [12]. However, compensation at the outer control loop without bothering the internal control structure is one of most important advantages.

##### Reference Compensation Technique for Inverted Pendulum

According to the previous RCT control scheme [6,13], the detailed PID controller outputs for the inverted pendulum system are described as

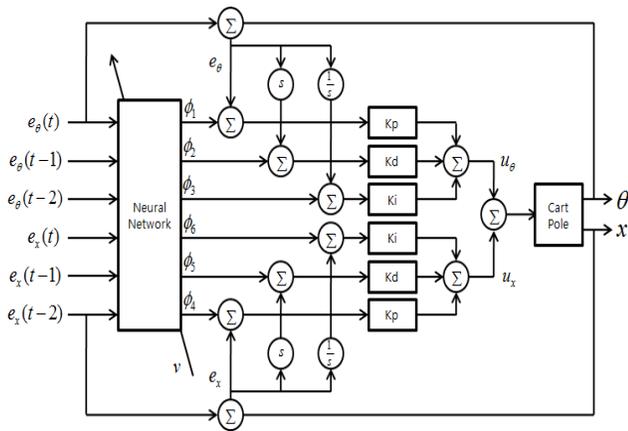


Fig. 5. Neural network control structure

$$u_{\theta n} = k_{p\theta}(e_\theta + \phi_1) + k_{d\theta}(\dot{e}_\theta + \phi_2) + k_{i\theta}(\int e_\theta dt + \phi_3) \quad (14)$$

$$u_{xn} = k_{px}(e_x + \phi_4) + k_{dx}(\dot{e}_x + \phi_5) + k_{ix}(\int e_x dt + \phi_6) \quad (15)$$

where  $\phi_i$  is the  $i$ th neural network output.

The control input to the system is

$$\begin{aligned} u &= u_{\theta n} + u_{xn} \\ &= v + \Phi_\theta + \Phi_x \end{aligned} \quad (16)$$

where,

$$v = u_{c\theta} + u_{cx}$$

$$\begin{aligned} u_{cx} &= k_{px}e_x + k_{dx}\dot{e}_x + k_{ix}\int e_x dt, & u_{c\theta} &= k_{p\theta}e_\theta + k_{d\theta}\dot{e}_\theta + k_{i\theta}\int e_\theta dt \\ \Phi_\theta &= k_{p\theta}\phi_1 + k_{d\theta}\phi_2 + k_{i\theta}\phi_3, & \Phi_x &= k_{px}\phi_4 + k_{dx}\phi_5 + k_{ix}\phi_6. \end{aligned}$$

The training signal  $v$  is defined as

$$v = u - (\Phi_\theta + \Phi_x) \quad (17)$$

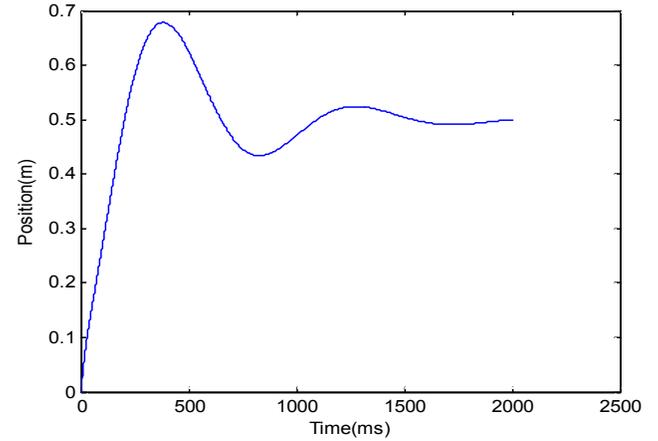
To minimize the error  $v$ , the objective function defined in (11) is minimized. The detailed back-propagation algorithm requires differentiating (11) with respect to the weight vector  $w$  as

$$\frac{\partial E}{\partial w} = \frac{\partial E}{\partial v} \frac{\partial v}{\partial w} = v \frac{\partial v}{\partial w} = -v \left( \frac{\partial \Phi_\theta}{\partial w} + \frac{\partial \Phi_x}{\partial w} \right) \quad (18)$$

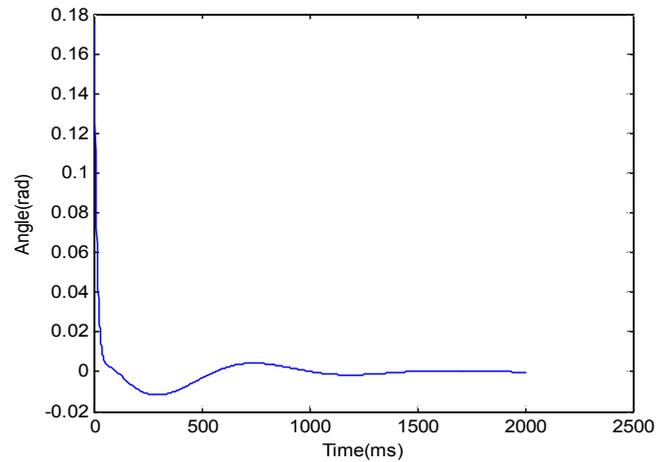
The detailed neural network control structure for the inverted pendulum system is shown in Fig. 5. Delayed errors are inputs of neural network and neural network outputs are added to each component of PID controllers. The internal PID controlled block has the outer loop of compensating for uncertainties to form the closed loop controlled system.

## 5. SIMULATION

Position and angle control of the inverted pendulum system are simulated by using a neural network control method. Initial angle is 0.18radian and position is 0 m. Then the inverted



(a) position



(b) Angle

Fig. 6 Simulation results

pendulum system is required to move to 0.5 m while balancing. Fig. 6 shows the simulation results. We see that the balancing angle and the cart position are well regulated.

## 6. EXPERIMENTS

### Experimental setup

Control environment of the inverted pendulum system has been setup as shown in Fig. 7. The system is composed of an inverted pendulum actuated by a dc motor, a computer, a joystick, and control hardware. The pendulum moves on the guided rail. At the end of guided rail, limit switches are attached to protect movement beyond the allowable distance.

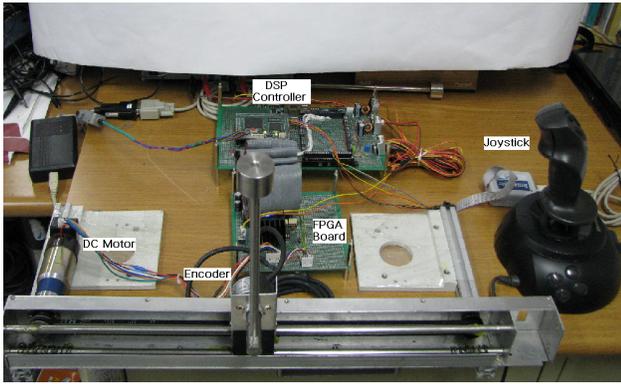


Fig. 7 Experimental setup

The positional command from the joystick can be obtained from a PC and the PC sends it to a DSP through serial communication. Then, the DSP performs control algorithms, PID and neural network to generate a control input torque signal. A FPGA board generates PWM signal and send it to the motor driver to drive a dc motor. Control hardware block diagram is shown in Fig. 8. Main processors are a DSP and an FPGA. There are two encoders, one for the position and another for the angle. The FPGA processor receives two encoder data, counts them, and communicates with the DSP to generate a pulse-width-modulation (PWM) signal to the motor driver.

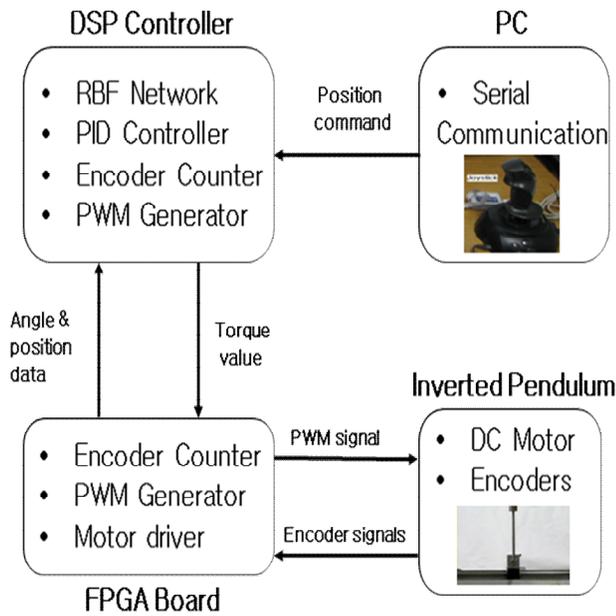


Fig. 8 Overall structure

### Experimental results

Position tracking control experiments of the inverted pendulum system are conducted. Initially, the pendulum balances itself as shown in Fig. 9. Then a user moves the joystick to generate desired position commands. Figure 10 shows the control

performance of the inverted pendulum system by the joystick movements such that the pendulum follows the desired position while balancing.

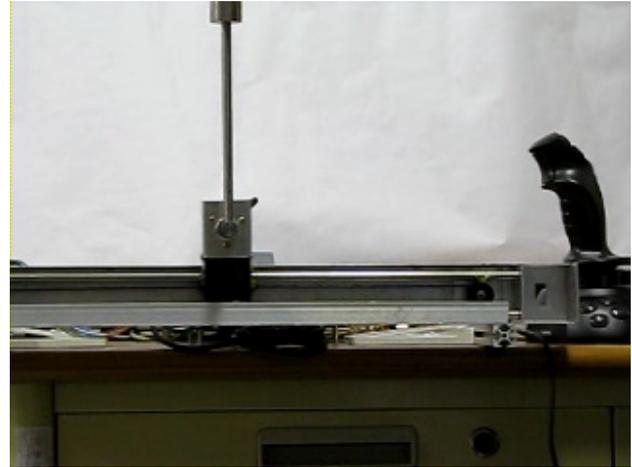


Fig. 9 Balancing control experiment at the beginning

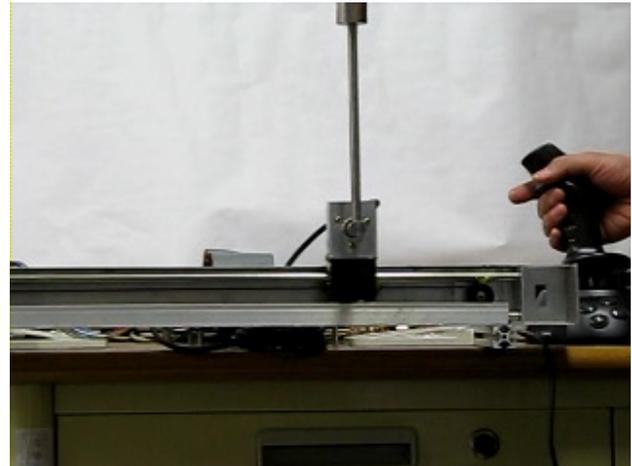
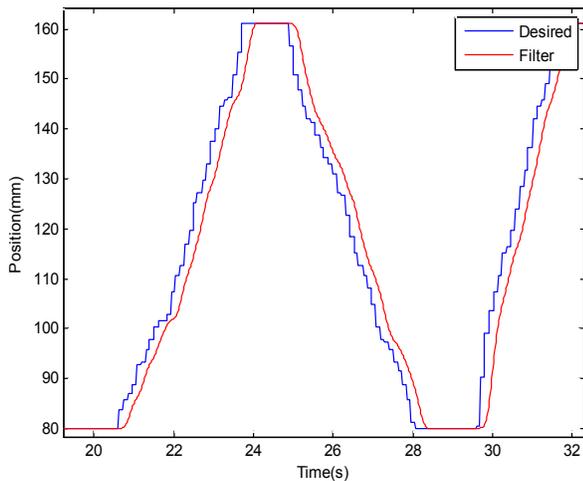


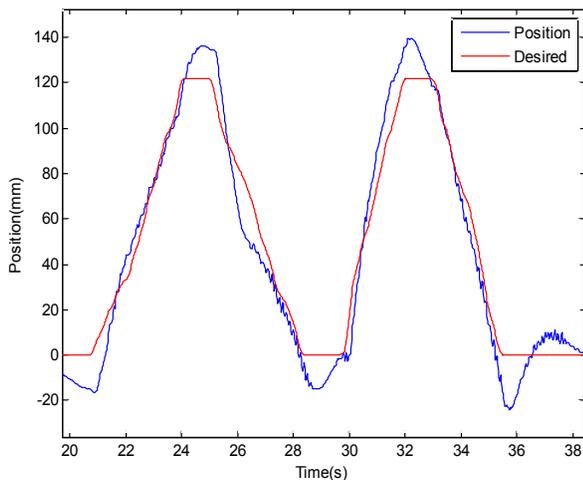
Fig. 10 Joystick command control for inverted pendulum

Figure 11 shows the corresponding plots of experimental results. Figure 11 (a) shows the desired trajectory commanded by the joystick and filtered commands. Since the command from the joystick is not smooth and rough, it is required to be filtered out to generate smooth trajectory command for the pendulum to follow. Figure 11 (b) shows the actual tracking results with desired commanded trajectory. In Figure 11 (b), the inverted pendulum maintains balancing without the joystick commands at the beginning, and then it follows the joystick commands of twice joystick operations of push and pull. Each push and pull joystick operation takes about 7 seconds. Although there are small tracking errors, the pendulum successfully follows the desired trajectory while balancing.

It is true that faster movements of the joystick cannot satisfy position tracking performances since the bandwidth of the inverted pendulum is low compared with that of human movements.



(a) Joystick command trajectory



(b) Enlarged position tracking result for the joystick command

Fig. 9 Position tracking performance

## 7. CONCLUSIONS

The remote position control of the inverted pendulum system is presented. Remote control test-bed is developed and integrated for intelligent control education based on neural network control application. Neural network along with PID controllers is used and implemented on the hardware to control the angle and the position simultaneously in on line fashion. The reference compensation technique control algorithm is used for the RBF network is to improve performances of the cart position of following the trajectory commanded by the joystick remotely. The test-bed can be used for experimental studies of students who take an intelligent control subject.

Although remote control of the inverted pendulum system is successful, time delay issue which is the most important problem in the remote control applications is not considered in this research. Further improvement on intelligent control

performance and student evaluations on this test-bed will be investigated.

## ACKNOWLEDGEMENT

This paper has been presented in World Multi-conference on Systemics, Cybernetics and Informatics (WMSCI 2010) and recommended for the submission to Journal of Systemics, Cybernetics and Informatics. This research was financially supported by Korea Research Foundation (KRF) through the basic research program (R01-2008-000-10992-0).

## REFERENCES

- [1] T. H. Hung, M. F. Yeh and H. C. Lu, "A PI-like fuzzy controller implementation for the inverted pendulum system," **Proc. of IEEE Conference on Intelligent Processing Systems**, 1997, pp. 218-222.
- [2] M. E. Magana and F. Holzapfel, "Fuzzy-logic control of an inverted pendulum with vision feedback," **IEEE Trans. on Education**, vol. 41, no. 2, 1998, pp. 165-170.
- [3] S. S. Kim and S. Jung, "Hardware implementation of a real time neural network controller with a DSP and an FPGA board", **IEEE ICRA**, 2004, pp. 4639-4644.
- [4] M. W Spong. "The swing up control problem for the acrobat", **IEEE Control Systems Magazine**, 15, 1995, pp. 72-79.
- [5] W. White, and R. Fales, "Control of double inverted pendulum with hydraulic actuation : a case study", **Proc. of the American Control Conference**, 1999, pp.495-499.
- [6] S. Jung and H. T. Cho, "Decentralized neural network reference compensation technique for PD controlled two degrees of freedom inverted pendulum," **International Journal of Control, Automations, and Systems**, vol. 2, no. 1, 2004, pp. 92-99.
- [7] H. Fer and D. Enns, "An application of dynamic inversion to stabilization of a triple inverted pendulum on a cart", **IEEE Conf. on Control Applications**, 1996, pp. 708-714.
- [8] J. Shen, A. K. Samy, N. Chaturvedi, D. Bernstein, and H. McClamroch, "Dynamics and control of a 3 D pendulum", **IEEE Conf. on Decision and Control**, 2004, pp. 323-328.
- [9] H. J. Choi and S. Jung, "Neural network-based Smith predictor design for the time-delay in a tele-operated control system", **Artificial Life and Robotics**, vol. 14, no. 4, 2009, pp. 578-583
- [10] G. H. Lee, J. S. Noh, and S. Jung, "Decentralized Implementation and experiment of neural network controllers for intelligent control system education" **International Journal of Fuzzy Logic and Intelligent Systems**, vol. 7, no. 4, 2007, pp. 267-273.
- [11] D. H. Song, G. H. Lee, S. Jung, "Neural network compensation technique for standard PD-like fuzzy controlled nonlinear systems, **International Journal of Fuzzy logic and Intelligent Systems**, vol. 8, no. 1, 2008, pp.68-74.
- [12] M. Miyamoto, M. Kawato, T. Setoyama, and R. Suzuki, "Feedback error learning," **Neural Networks**, vol. 1, 1988, pp. 251-265.
- [13] S. Jung, H. T. Cho, and T. C. Hsia, " Neural network control for position tracking of a two-axis inverted pendulum system: Experimental studies," **IEEE Trans. on Neural Networks**, pp. 1042-1048.