NEAR REAL TIME DISCOVERY AND CONVERSION OF OPEN SOURCE INFORMATION TO A REWARD MATRIX

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Abstract: We describe a system model for determining decision making strategies based upon the ability to perform data mining and pattern discovery utilizing open source information to automatically predict the likelihood of reactions to specific events or situational awareness from multiple information sources. Within this paper, we discuss the development of a method for determining actionable information to efficiently propitiate manpower, equipment assets, or propaganda responses to multiple application case scenario experiments. In our experiments, we have integrated open source information linking to human sentiment and manipulated other user selectable interlinking relative probabilities for different reactions to different events based upon current knowledge generated from the situation or event. The goal of our solution, called GlobalSite, is to deliver trustworthy decision making analysis which evaluates situations and potential impacts of such decisions through acquired open source information becoming a vital tool for continuing mission operations, analyst information or other decision making scenarios.

INDEX TERMS—GAME THEORY, RESOURCE MANAGEMENT, OPERATIONS RESEARCH, AUGMENTED DECISION MAKING, RISK MANAGEMENT

I. INTRODUCTION

There is a critical need for augmented decision making methods targeted to improve pattern identification within disparate data sets, are scalable, and reduce human decision making errors in near real time modes. Digital content generation, combined with ubiquitous platforms, has created the “Big Data” challenge in understanding how to make sense of the information generated through multiple sources. Data can be found everywhere and anywhere, be of any type, and be resistant to pattern detection. Human decision making activities performed with data from disparate sources is difficult and a highly time consuming activity in near real time or on demand modes. Human cognition and knowledge base within the decision making process must also be considered as an important factor. There are additional needs for increased information analysis capabilities demonstrating more accurate decisions, planning factors, resource allocation, risk management, and information analysis in a near real time, visually oriented manner with fewer analysts and mission planners.

A major goal within industry and others is to push forward an open architecture framework in order to: inject and fuse data and information from a multitude of sources, contain collaborative environments, provide increased visualization of information (immersion), improve decision making performance in analysis and mission planning, and increase pattern recognition among disparate data sets in order to effectively analyze information. Currently known analytical and planning commercial tools have shortcomings in meeting these goals to include: the inability take in multiple media types; being mostly text based with some multimedia input; issues with scalability, the use of proprietary algorithms and tools which may not incorporate both quantitative and qualitative metrics or predictive measures.

One way to address the decision making process from the human approach for analysts and mission planners is the use of serious games or simulated environments. Serious games can provide simulated virtual learning venues for mitigation of selected biases found within human decision making process [1]. Training and simulations in virtual environments can also allow for immersive simulations and training of real world scenarios thus potentially increasing performance within human decision making process. Although many positive effects have been shown through these methods, these techniques have not yet been extended to data content evaluation within the decision making process.

Further evaluation of industry and academia offerings reveal currently available decision making tools and techniques do not include the ability to manipulate information attributes or weighting factors in near real time for the best or optimal decision derived from open source information, or other data sources. Nor do they include the user knowing adversarial, unknown or known strategies which could potentially impact the outcome of such decision. The GlobalSite solution could allow for the user to interact on demand with the resultant decision made by the user through scalable weighting factors, and incorporates the user’s experience or knowledge base showing the potential effects of their decision through the use of game theoretical concepts and population of a reward matrix in real time.

Automated processing techniques are required to augment tactical intelligence-analysis capabilities by automatically identifying and recognizing patterns. For example, information and patterns of behavior that could provide advance warning of hostile intent are often hidden in a vast background of harmless civilian activity. Additionally, there is a critical need for actionable information for threat prediction [6]. Our paper looks at an example of using linear program game theory to solve for a strategy given a reward matrix for possible actions based upon selected criteria.

In many situations, the opponents know the strategy that they are following. We assume that the players know what actions are available. Maximin equilibrium is often the strategy and is called the Nash theory application of zero or constant sum strategy game. We also consider a constant sum game in which for both player’s strategies, the two players’ reward adds up to a constant value. This means, while both players are in conflict, that there is more to gain than simply having one player’s reward equaling the other player’s loss.

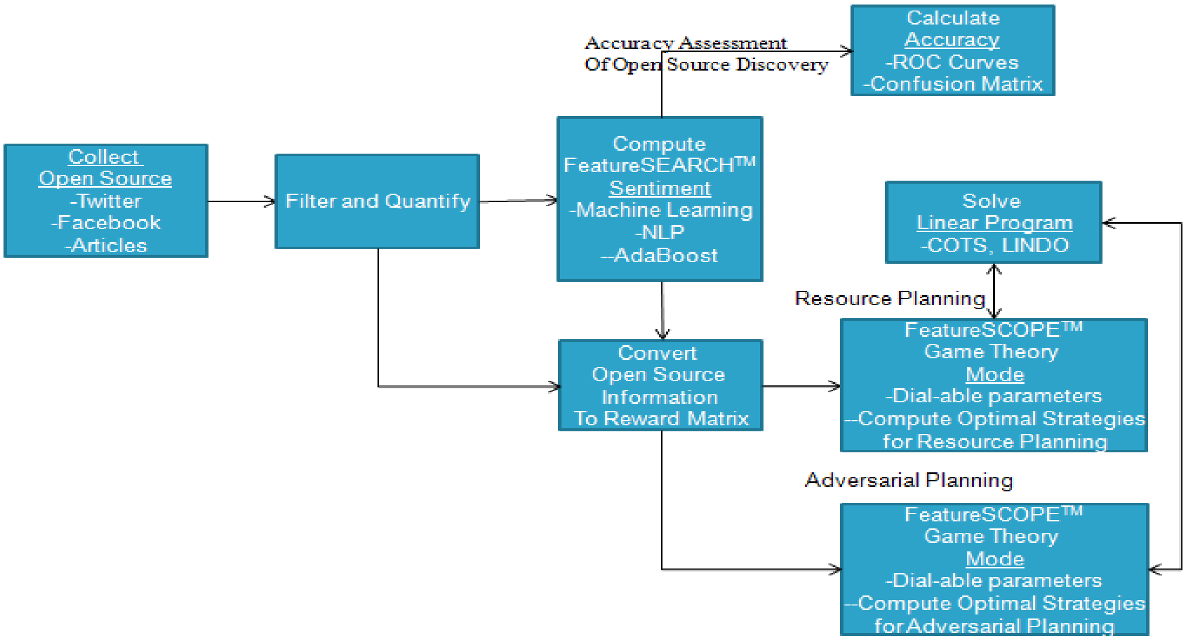

Fig 1: GlobalSite Decision Making Process System Block Diagram Using Open Source Information

II. Decision Making Approach

Current situational awareness efforts seek to incorporate not only geospatial features and structures, but also the human element, especially in urban settings. For example, one group has converted a human geography heat map into a reward matrix for useful game theory analysis [2]. An attempt to predict the likelihood of human reactions to a future event should be based on correct situational analysis. Development of tools for more rapid refinement of flexible plans is required for adapting to a changing operational environment.

Our solution populates a reward matrix in near real time through powerful game theory analysis. Figure 1 shows our techniques which contains a method for processing and assessing the accuracy of discovered open source data which can also be performed through other commercially available algorithms or tools. Once data accuracy is proven through sensitivity analysis, as shown in Figure 3, the information is can either be used as training data or populated into a reward matrix in real time for resource allocation and adversarial planning utilizing game theory analysis.

Our techniques enable a methodical approach to intelligent planning and reaction based upon construction and analysis of a decision model resulting in a structure of the most probable solution. This technique is useful for a number of applications ranging from behavioral economics, war fighter planning, and analysis of information, messaging, and risk management. Our system supports an artificial intelligence (AI) supervised learning approach to quantify information based on user selectable attributes and deriving probabilistic decision outcomes. Our approach also involves training the algorithms and near real time execution.

Our solution integrates multiple data sources into efficient intent analysis processes and uses training data to build the decision trees to predict categories for new events based upon classifiers created for the use case scenario. Given an event, we predict a category and then determine sentiment based on trained data. This information could then be applied during planning in support of course of action (COA) development in the military decision making process (MDMP).

The approach combines the following input: open (unstructured) source, and/or direct user input/modification. In particular, we capture and model “sentiment” and other situational factors through the assignment of positive, neutral and negative values. A reward matrix is then populated using game theoretic concepts such as in a competitive game model. GlobalSite utilize game theory which permits the ability to solve for iterative solutions, instantaneous visual feedback, and interactions by the user on demand. Our output can enable a methodical approach to intelligent planning and reaction including interaction of variables, parameters and attributes by the user resulting in updated probabilities. Game theory is useful for resource management of manpower, equipment, and warnings, etc. [3], since it can show the optimal decision for resource deployment.

The scope of our paper is as shown in Figure 2, in which a Venn diagram illustrates the currently models developed and tested for applications in mission planning and resource analysis experimental scenarios. There are three possible decompositions of solution types for this particular challenge: dominant, saddle, and mixed strategy which is addressed within one design. Outside this model space are more challenging models including imperfect knowledge, non-rational players, asymmetric, and cooperative models.

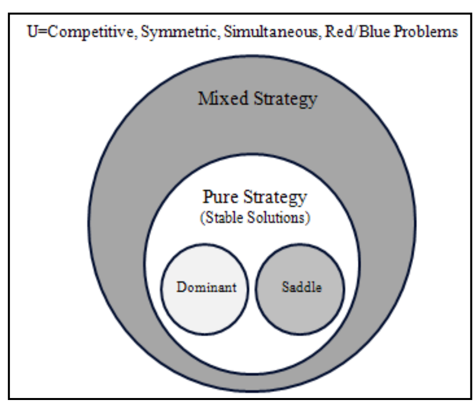

Fig 2: Scope Addressed

III. ACCURACY ASSESSMENT OF OPEN SOURCE DISCOVERY

An actual tweet with Twitter data contains a maximum of 140 characters and is only a fraction of the data returned. Additional information may be included in results such as: User ID; Time of Tweet; Geo Coordinates; URLs; Hashtags; User Mentions; and a Link to Profile Image. The Twitter search Application Programming Interface (API) returns roughly a maximum of 1500 tweets for a search request, are rate limited, and are indexed back roughly one week. Searches can specify subject (can be AND/OR), geographic region, and language. A real time stream of 1% of Twitter traffic provides about 10 GB of information per day. This stream can be filtered by User IDs, Keywords, or Location and cannot be combined with User IDs or Keywords.

Results of our sensitivity analysis experiments are shown in Figure 3. Our test case used a small data set of 152 open source news articles, based on sentiment values of positive, neutral and negative (+1, 0, -1) values, toward U.S. forces over the past decade. The attribute factors chosen for comparison were unemployment rate, country, religion, data year, inflation rate, and sentiment confidence score. These factors were chosen for their high correlation with sentiment values to beliefs, culture, and lifestyle.

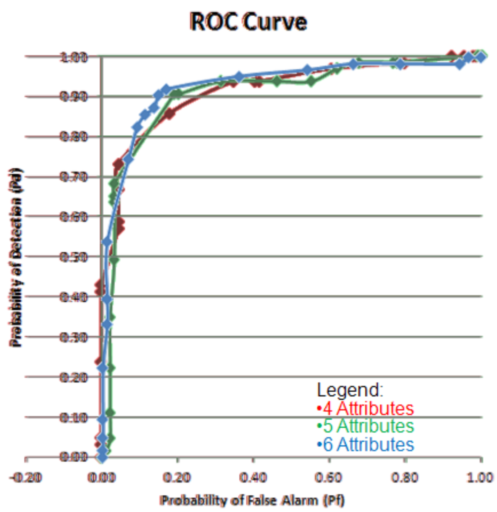

Fig 3: Sensitivity Analysis of Open Source Information Discovery

Receiver operating characteristics (ROC) curves are then used to measure the probability of detection of correct sentiment and false alarms from the Twitter data. ROC curves can be generated when a floating point confidence measure is calculated from our AI algorithm. To compare classifiers, one may want to reduce the ROC performance to a single scalar value representing expected performance. Comparisons between the tests are based on differences between (estimated) areas under curve (AUC). Our results, which extend our previous work [4], displays the accuracy assessment of correctly discovering open source articles on "sentiment," correctly determining the likelihood of automatic determination of positive or negative sentiment and increasing in accuracy as additional attributes were added. Our accuracy assessment has been performed with textual data at rest and was English language based. We investigated available corpuses to see how they have categorized language use (verbs, adjectives, pronouns, nouns, modals) and made adjustments to our setup by adding additional features. Through open source information, the experimental goal was to understand the environment in which the best decision is made for resource allocation based upon sentiment indicators found through Twitter and online news articles. We validated experimental ROC curves and calculated false positives and false negatives. Experimental results revealed: religion and country attributes dominated sentiment measurements and through additional attributes; increased sensitivity was observed within the reward matrix generated with fewer errors detected when additional attributes where incorporated.

IV. RESOURCE MANAGEMENT

In regards to related work in this area of data fusion and prediction, Chen et al. proposed a data fusion approach for asymmetric threat detection and prediction based on advanced knowledge infrastructure and stochastic (Markov) game theory. Game theory considers the effect of a player's decision on other decision makers. Two or more decision makers choose an action and that affects the rewards earned by the players. In general, game theory is useful for making decisions in cases where the decision makers have conflicting interests [5].

For example, if a reward matrix exists, then the equilibrium point is the one where the reward is the smallest value in its row and the largest number in its column. This equilibrium point is also called a saddle point since it is like the center point in a horse's saddle and is also known as the Nash Equilibrium [7]. The saddle point is the local minimum in one direction (row) and a local maximum in another direction (column) [3] such as:

$$\max_{\text{all rows}} (\text{row min}) = \min_{\text{all columns}} (\text{column max}). \qquad (1)$$

This left half of (1) presents the basic applied theory to decision making of our model under uncertainty. Our model has manpower and an equipment output. For a possible action, one consideration is to choose the "best" worst outcome [3]. The maximin criterion suggests that the decision maker should choose the alternative which maximizes the minimum payoff for the decision maker. This pessimistic approach implies that the decision maker should expect the worst to happen. The maximin criterion is concerned with making the worst possible outcome as pleasant as possible.

The right half of (1) represents minimax regret criterion which uses the concept of opportunity cost to arrive at a decision. The regret of an outcome is the difference between the value of that outcome and the maximum value of all the possible outcomes. For any action and state, there is opportunity for loss or regret. The decision maker should choose the alternative that minimizes the maximum regret he could suffer. These decision making criteria discussed are reasonable, however many decisions are made without using any analysis [5].

"Objects"	"Attributes"					
	Deaths	Injury	Damage	Structure	Empathy	Money
Guatemala	1400	261	73	117	6	2
Japan	117	58	71	138	22	75
Mexico	236	32	201	28	6	13
Myanmar	1031	250	280	42	39	69

Saddle Point

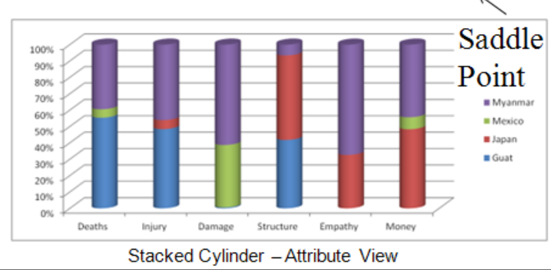

Fig 4: Resource Planning

When the reward matrix contains no saddle point, we can use a linear program solver. Our resource planning experiment was performed with textual data at rest. Open source information that lends itself to matrix creation is foreign aid assistance based upon recent earthquakes. For example, when allocating resources, where is the maximum reward for all parties engaged or adversarial positions? In order to solve this question, we can build a reward matrix based upon probabilities or weightings from data obtained. The purpose of the reward matrix is to calculate optimal strategies to determine what resources to send based upon open source information for war fighter, analyst, or others. Figure 4 shows some experimental results for determining which country to send foreign aid. In this case a saddle point exists and the best choice to help is Myanmar, with the worst aspect of choice being Empathy. Therefore, send Propaganda last or red player would send Propaganda first. Blue player would send manpower first (corpsmen).

V. ADVERSARIAL PLANNING EXPERIMENTAL RESULTS

Game theory, as a model of conflict, suffers from several limitations. Players are assumed to always maximize their outcomes. Not all of the payoffs or situations can be quantified in a reward matrix. Game theory is not applicable to all types of problems. However, game theory offers important insights and demonstrates superiority of cooperation over competition. Game theory models the heuristics people use in managing their conflicts and helps to explain why rational decisions often miss opportunities for mutual gain [8].

Imperfect information may still be useful to help make decisions. Opponent modeling works by observing the opponent's actions and building a model by combining information from a pre-computed equilibrium strategy with the observations [9]. Previous work performed in the community includes computing robust optimization equilibrium by methods analogous to those for identifying Nash Equilibrium of a finite game with complete information [10].

There is much attention given to simultaneous-move, one-shot, normal form games with complete information. Each player or agent has a private payoff known only to that agent. The payoff to an agent x is not only a function of all the agents' actions (as in the usual complete information game) but also of the realized private-type of agent x. The type of an agent may be discrete or continuous. Each agent's realized type is chosen independently from some commonly known distribution over types, and the payoff matrices for the agents are also common knowledge. These games have incomplete information because each agent must choose its strategy, i.e., its probability distribution over its actions, without knowing the realized types of all the other agents [11].

Decision theory studies decision making in situations where the consequences of one's action are uncertain. The classical decision theoretic scenario is that of a single agent having to choose among a set of actions, the consequences of which depend either on certain states of affairs about which the agent is not completely informed, i.e., subjective uncertainty, or on the result of some random processes that are independent of her, i.e., objective uncertainty [12].

A general classification that categorizes algorithms by the cross-product of possible strategies and possible beliefs about the opponent's strategy could be performed. A possible strategy can be classified based upon the amount of history it has in memory. Given more memory, more complex policies can be formulated, since policies are maps from histories to action distributions [13].

Harsanyi proposed a method for transforming uncertainty over the strategy sets of players into uncertainty over their payoffs. The transformation appears to rely on an assumption that the players are rational. Without a common belief of rationality, such implications are not necessarily maintained under a Harsanyi transformation. Under the belief system model, such implications can be maintained in the absence of common belief of rationality [14].

A large class of sequential decision making problems under uncertainty with multiple competing decision makers/agents can be modeled as stochastic games. Non-cooperative games can be solved in which each decision maker makes his own decision independently and each has an individual payoff function. In stochastic games, the environment is non-stationary and each agent's payoff is affected by joint decisions of all agents, which results in the conflict of interest among decision makers [15].

Generally, players may not possess full information about their opponents. In particular, players may possess private information that others should take into account when forming expectations about how a player would behave. To analyze these interesting situations, a class of games with incomplete information was created as use case scenarios (i.e., games where at least one player is uncertain about another player's payoff function) which are the analogue of the normal form games with complete information similar to Bayesian games (static games of incomplete information). Although most interesting incomplete information games are dynamic (because these allow players to lie, signal, and learn about each other), the static formulation allows focusing on several modeling issues [16].

	Manpower	Equipment	Propaganda	Funding
Manpower	$1281w_1$	$[1/4(1281)+3/4(322)]w_1$	$[1/4(1281)+3/4(39)]w_1$	$[1/4(1281)+3/4(69)]w_1$
Equipment	$[1/4(322)+3/4(1281)]w_2$	$322w_2$	$[1/4(322)+3/4(39)]w_2$	$[1/4(322)+3/4(69)]w_2$
Propaganda	$[1/4(39)+3/4(1281)]w_3$	$[1/4(39)+3/4(322)]w_3$	$39w_3$	$[1/4(39)+3/4(69)]w_3$
Funding	$[1/4(69)+3/4(1281)]w_4$	$[1/4(69)+3/4(322)]w_4$	$[1/4(69)+3/4(39)]w_4$	$69w_4$

	Manpower	Equipment	Propaganda	Funding
Manpower	1281	562	350	372
Equipment	1041	322	110	132
Propaganda	970	251	39	62
Funding	978	259	46	69

Fig 5: Adversarial Planning Reward Matrix

Our adversarial planning experiment was performed with textual data at rest; it is English language based and serves as a method to complete unknown information in a reward matrix. Our purpose is to demonstrate what resources to send based upon open source information for war fighter, analyst, or others for adversarial planning (known and mixed) using recent earthquake generated Twitter data in order to answer the question of what assistance to send to which country. The diagonal values from Twitter included: Manpower = Dead + Injured; Equipment = Damage + Structure; Propaganda = Empathy; and Funding = Money. The cost function is based on distance in miles to country of interest as shown in Fig 5. The experimental results revealed best choice for blue is manpower using a linear program solver with the best choice for red is propaganda.

A red player may be 3 times closer to Myanmar than the blue player and therefore easier to influence in terms of cost. Therefore, we used ¾ for red and ¼ for blue as the cost function. The distance of the cost function can also be "distance between beliefs or world view" such as religion, culture, and government type or any other distance cost. We initially assume all attributes are equally weighted, but can add an additional layer of complexity by using weights relative to attributes; however we assumed $w_{1,2,3,4} = 1$.

Weights are determined with respect to blue player's evaluation of which attributes are more important, since in this case the blue player is driving the decision and the red player is trying to minimize the blue player's reward. Figure 6 shows the result of weighting the blue player's preference by valuing propaganda over all other attributes by a factor of 9. Weighting changes our results to the best choice for blue is propaganda using a linear program solver. The best choice for red is propaganda.

	Manpower	Equipment	Propaganda	Funding
Manpower	1281	562	350	372
Equipment	1041	322	110	132
Propaganda	6030	2259	351	558
Funding	978	259	46	69

Fig 6: Weighted Reward Matrix

Some tools use "strategies" measured in different units in the same reward matrix and can be problematic. Examples include use of manpower (count of people) mixed with propaganda (not necessarily units of people). If all strategies in a given decision model reward matrix are not in the same (equalized) units, then use of game theory and mini-max or maxi-min functions can provide misleading results. We can create purely dominant and incorrect solutions just due to relative size of unit measures. Our solution addresses this properly and uniformly for any decision model. We equalize all strategies (in a given decision model) to the same unit. This is a key point to the application of game strategies to a general class of decision problems. An adjustable "equalization" factor has the purpose to convert all strategy measures to the same unit (e.g., cost, time) and must be done for any decision model. The equalization factor for our solution is independent of additional (importance) weights that may be applied.

	Manpower	Equipment	Propaganda	Funding
Manpower	0.059976	0.1515	0.996721	0.619632
Equipment	0.01247	0.0315	0.209836	0.128834
Propaganda	1	1	1	1
Funding	0	0	0	0

Fig 7: Normalized Reward Matrix

Figure 7 shows the result of normalizing column values using the same cost function and attributes. Normalization changes our results to the best choice for blue player is propaganda with the best choice for red player being manpower. It is interesting to note that strategies have flipped from Figure 5 results based on equalizing the units.

	Manpower	Equipment	Propaganda	Funding
Manpower	1281w_1	[½(1281) + ½(322)]w_1	[½(1281)+ ½(39)]w_1	[½(1281)+ ½(69)]w_1
Equipment	[½(322) + ½(1281)]w_2	322w_2	[½(322) + ½(39)]w_2	[½(322) + ½(69)]w_2
Propaganda	[½(39) + ½(1281)]w_3	[½(39) + ½(322)]w_3	39w_3	[½(39) + ½(69)]w_3
Funding	[½(69) + ½(1281)]w_4	[½(69) + ½(322)]w_4	[½(69) + ½(39)]w_4	69w_4

Change cost function to [½ and ½]

	Manpower	Equipment	Propaganda	Funding
Manpower	0.1151	0.425263	1	1
Equipment	0.023932	0.089123	0.218241	0.207921
Propaganda	1	1	0.49674	0.806931
Funding	0	0	0	0

Fig 8: Reward Matrix with Modified Cost Function

Figure 8 shows the result of modifying the cost function from [3/4, 1/4] to [1/2, 1/2] revealing the best strategy for blue is to choose propaganda twice as often as manpower, with the best choice for red selecting propaganda twice as often as manpower. The purpose of showing these results with different choices is to highlight the ability and need for a tool which can be used to allow the user to dial and modify modeled parameters of the reward matrix to model "what if" scenarios.

VI. TOOL IMPLEMENTATION

Our tool implementation has the ability to manipulate factors "on the fly" with near real time results. GlobalSite decision model file stores parameters of the decision model and problem space. It does not store a solution, but is only a model of the problem space and includes everything needed to calculate a solution.

Figure 9 shows an example prototype graphical user interface (GUI) of our GlobalSite decision making tool. Features of our solutions contain: Normalize Displayed Values; GUI support for sorting options; Add Alternate Red/Blue View; addition of probabilities; Dials for changing values; Add display for relative "distance"; Add clarity for specifying attribute vs. (1-Attribute); Probability for Red, Unknown strategy; and real time visual feedback.

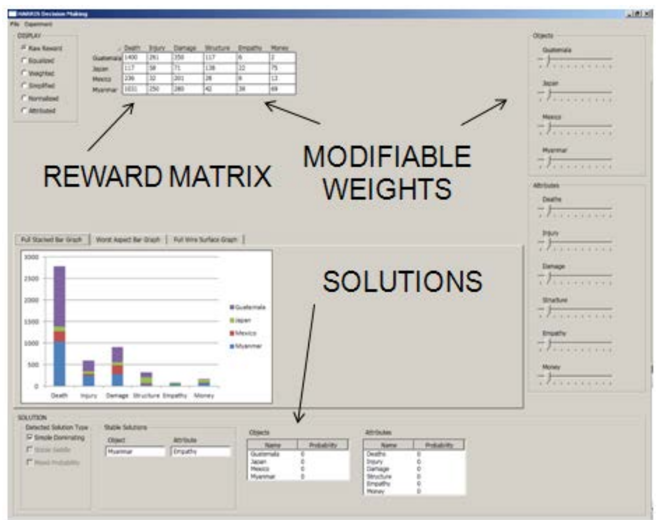

Fig 9: Prototype GlobalSite Decision Making GUI

We have a method to simplify the reward matrix, before applying a linear program solver through removal of "non-dominant" rows/columns which is performed as the first action. We use the standard convention of considering blue player strategies as rows, and red player choices as columns. The removal of dominated strategies is accomplished as follows: If all elements of a row "i" are less than or equal to corresponding elements of another row "j," then "i" is a completely dominated strategy since it will not contain the solution. All cells of dominated rows are marked as redundant by graying out these values within the user modifiable GUI. If all elements of a column "i" are greater than or equal to corresponding elements of another column "j," then "i" is a completely dominated strategy since it will not contain the solution, is redundant, and cells will be grayed out. The tool continues validating (non-dominated) cells of remaining columns and rows, since elimination of redundant cells may create other dominated rows/columns – until there is no change. The resulting simplified matrix (non-redundant cells) will either contain a single solution, multiple rows, or columns and may be saddle or mixed solution.

Several studies provide discussion and attempts to integrate and validate usefulness of the application of game theory models. The strategy action game is not only applicable in the field of commercial negotiation; subsequent research can extend further into the fields of education, marketing, finance, risk management, and society. The competition and cooperation relationship between manufacturer and distributor in other applications are delicate, allowing room for other methods besides strategy action game, such as series bargaining game and mean difference. Studies have been performed on the analysis aiming at the strategy application, and intervention into the negotiation harmonization with the manufacturer or distributor. On one hand, it insists on an objective observation attitude; on the other, it may also produce the deviation of unscrambling the behavior of game participants subjectively [17].

VII. Conclusion and Future Work

No decision is ever 100% correct; however, understanding the effects of algorithmic decisions based upon multiple variables, attributes, or factors and strategies with probability assignments can increase the probability for the best decision for a particular situation or event. GlobalSite can perform open source discovery and data mining activities to parse information found from disparate, non-obvious, and previously unknown data sources and allows for the user to dial the weighting factors based upon their knowledge or domain expertise.

Direct user input or modifications, additions, or deletions of attributes of interest and their associated probabilities can be modeled for a multitude of scenarios from the reward matrix. The initial step in the process utilizes a decision tree learning method that is used for classification, weighted factors and probabilistic prediction based upon the information obtained from multiple and disparate data sources. Optimal strategies are then calculated to increase the likelihood of making the best decision available using game theory in a constant sum game for a resource allocation scenario. Data fusion and visualization techniques provide the user with a useful tool to interact with the results generating near real time decisions. Our system can be extended to other applications such as course of action planning, strategy, resource management, risk assessment, and behavioral economics, and is not tied to proprietary feeds, inputs, or outputs. Our solution can have multiple algorithms/inputs/outputs based upon user needs and requirements and allow for human interaction with the resulting decision made by the system to show changes based upon different decisions made in near real time.

We have defined a solution to determine strategies based upon game theory using our model and have proven out these models using open source information. We have shown experimentally using open source information that we can calculate optimal strategies and resource allocation to provide the best decision using either opportunity costs or reward matrix. By utilizing the reward matrix, we can make "sense" of data by assisting users in making the best decision possible.

Automated processing techniques are needed to augment tactical intelligence-analysis capabilities by identifying and recognizing patterns, weighting them appropriately, providing near real time objective decisions where the user can interact with the information based upon their experiences and knowledge base. GlobalSite is a probabilistic decision solution which allows for users to interact with information in near real time using game theory to provide a reward matrix of the best possible outcomes. GlobalSite can host additional algorithms for filtering techniques, has an open architecture framework, and is not data, attribute, or factor limited.

As stated within this paper, game theory does have its limitations. Not all payoffs or situations can be quantified in a reward matrix (e.g., flash mob). However, this may be a separate pattern recognition algorithm which could be combined within our solution. Future work considered will include addition of alternate red/blue view models, investigating how to accommodate for unknown strategies, incorporation of data reduction algorithms, working with larger data sets for scalability including live and static data sources, multiple media file types, and investigation of importing heat maps to align information to provide a more visual context to the user.

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