Analytic Hierarchy Process (AHP) in Ranking Non-Parametric Stochastic Rainfall and Streamflow Models

Masengo Ilunga
Department of Civil & Chemical Engineering,
University of South Africa, Pretoria, South Africa
e-mail: ilungm@unisa.ac.za

ABSTRACT

Analytic Hierarchy Process (AHP) is used in the selection of categories of non-parametric stochastic models for hydrological data generation and its formulation is based on pairwise comparisons of models. These models or techniques are obtained from a recent study initiated by the Water Research Commission of South Africa (WRC) and were compared predominantly based on their capability to extrapolate data beyond the range of historic hydrological data. The different categories of models involved in the selection process were: wavelet (A), reordering (B), K-nearest neighbor (C), kernel density (D) and bootstrap (E). In the AHP formulation, criteria for the selection of techniques are: "ability for data to preserve historic characteristics", "ability to generate new hydrological data", "scope of applicability", "presence of negative data generated" and "user friendliness". The pairwise comparisons performed through AHP showed that the overall order of selection (ranking) of models was D, C, A, B and C. The weights of these techniques were found to be 27.21%, 24.3 %, 22.15 %, 13.89 % and 11.80 % respectively. Hence, bootstrap category received the highest preference while nearest neighbor received the lowest preference when all selection criteria are taken into consideration.

Keywords: Analytic hierarchical process, non-parametric stochastic, hydrological data.

1. INTRODUCTION

Hydrological data such as rainfall, streamflow are main drivers for planning, management and development of water systems. It is often a concern that these data have gaps or are not of sufficient length or are inexistent for hydrological studies. This situation prevails more in developing countries than in developed countries. The uncertain nature/behavior of rainfall and streamflow [1] has been handled from stochastic perspective, e.g. [2], [3], [4], [5]. The use of stochastic models or techniques to generate artificially data records is common in hydrological studies for water resources development, management and operation. For instance stochastic stationary models such as periodic autoregressive and moving average can be used for modeling weekly, monthly and seasonal streamflows [2]. One of the characteristics of stochastic techniques is the preservation of statistical characteristics for both historic and generated data series. A modified K-nearest neighbor (K-NN) was developed for stochastic streamflow simulation [4]. A recent study initiated by the Water Research Commission of South Africa (WRC), [5] summarizes five main categories of non-parametric stochastic hydrological data generation: wavelet, reordering, nearest neighbor, kernel density and bootstrap. For that, five essential criteria: "ability for data to preserve historic characteristics", "ability to generate new hydrological data", "scope of applicability", "presence of negative data generated" and "user friendliness" were used in comparing these categories and in deciding on the most suitable technique for the purpose of rainfall data generation. The selection in the WRC study was focused predominantly on the second criterion as opposed to other criteria. In addition, the selection lacked sufficient transparency when several criteria should be taken into consideration in deciding which category should be prioritized. The order of prioritizing the different techniques, based on a scientific methodology is of great appeal. Hence, this study focuses on ranking/prioritizing the categories of non-parametric stochastic hydrological data models using AHP methodology, through weight allocation. This study does not deal with hydrological data generation, which was already carried out [5]. However, the current study targets on a transparent multi-criteria decision making (MCDM) methodology for ranking non-parametric techniques used for hydrological data generation. For more detail on advantages of non-parametric models over the parametric models, the reader should be referred to [5]. Applications of AHP to hydrological data infilling technique has been reported recently, e.g. [7], [8]. However the literature on ranking non-parametric stochastic models for hydrological data generation does not exist. For multi-criteria decision, AHP has been proven to be a powerful tool with an acceptable level of inconsistency/subjectivity in the judgment process. Subjectivity can be detected by carrying out consistency check during pairwise comparisons. In this study, AHP is formulated and implemented based on a study mentioned earlier [5], where non-parametric stochastic models were used for hydrological data generation. In what follows "model", "generator", and "technique" can be used interchangeably; likewise for "hydrological data generation" and "data generation".

2. AHP AND HYDROLOGICAL DATA GENERATION MODELS

Formulated in the 1980's, AHP is a tool for multi-criteria decision making (MCDM) [6]. AHP has been used intensively in several fields e.g. [11], [12]. Generally, AHP methodology can be summarized as follows [6]:

- The problem is modeled as a hierarchy: goal, alternatives and criteria for assessing the alternatives are contained in the hierarchy. Criteria can be split into sub-criteria.
- Priorities are established among the elements of the hierarchy: the pairwise comparisons of the criteria are conducted. During these comparisons, the importance of

criteria is determined using the scale shown in Table 1. Intensities of importance are allocated based on the judgments or experiences of individuals have on a particular topic.

- The overall priorities or preferences are determined for the hierarchy: a comparison matrix summarizes all information obtained from pairwise comparisons. The final decision is based on the normalized principal priority vector (Eigen vector) obtained from the comparison matrix (built matrix). This matrix is also called judgment matrix.
- Consistency check is performed: consistency of decisions made in previous steps is determined by computing a consistency ratio. This ratio should be less than 10%.

In AHP, the goal is reached through a series of pairwise comparisons from a 1 to 9 likert scale. Details of AHP methodology can be traced for instance in [6].

Table 1. Likert scale for pairwise comparisons

Intensity of importance (scale)	**Description of the level of importance**
1	Equal importance/intensity of two elements
3	Moderate importance of one element over the other
5	Strong importance of one element over the other
7	Very strong importance of one element over the other
9	Extreme importance of one element over the other
2,4, 6, 8 are intensities to express intermediates values	

For the judgment to be consistent (valid) during AHP implementation, the computed consistency ratio (CR) should not be more than 10 % (in a percentage form), or less than 1 (in a unit form). Otherwise the judgment is invalid. The consistency ratio CR is the ratio between the constant index CI and the random index RI [6].

The constancy ratio (CR) computed from the judgment matrix by using Eq. (1):

$$CR = \frac{CI}{RI} \qquad (1)$$

Where:

$$CI = \frac{\lambda_{MAX} - n}{n-1} \qquad (2)$$

The values of RI depend on the dimension (n) of the comparison matrix and are displayed in Table 2. λ_{MAX} is the maximum Eigen value of the comparison matrix.

Table 2. Random index (R I) expressed as a function size (n) of the comparison matrix

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The application of AHP has been extended to the field of hydrology and water resources and AHP has recently been used for streamflow/rainfall infilling data problems [7], [8] [9], [10]. However, there is no literature where AHP is applied to the selection of non-parametric stochastic models for hydrological data (e.g. rainfall and streamflow) generation. The current study is an attempt for this specific selection. Since AHP enables to handle complex problems into a hierarchy, AHP can be approached at different levels for the current study as follows:

- Level 1: the main goal in the hierarchy is defined as the selection of a suitable category for non-parametric stochastic technique/model in the process of hydrological data generation for a site in a given catchment.
- Level 2: the criteria set by the modeler (decision-maker or water expert) are defined in the selection of non-parametric stochastic hydrological models. The criteria may be disaggregated into several sub-criteria.
- Level 3: the non-parametric stochastic models are candidates in the selection process for hydrological data generation. Models represent alternatives in the hierarchy structure. The modeler or decision-maker will then make a choice among these alternatives.

3. DATA AVAILABILITY

Data depicted in Table 3 has been used in the current study and was adapted from a previous study [5]. In this previous study, the comparative analysis was just conducted through an observation of the different entries of Table 1, according to the author’s experience [5]. Table 3 displays the different categories of non-parametric stochastic hydrological models for the purpose of rainfall and streamflow data generation. The 5 categories of models considered are: wavelet (A), reordering (B), nearest neighbor (C), kernel density (D) and bootstrap (E). These models will be used as alternatives in the process of data generation (through AHP). The set of criteria for AHP, i.e. “ability for data to preserve historic characteristics”, “ability to generate new hydrological data”, “scope of applicability”, “presence of negative data generated” and “user friendliness” will be used. These criteria will be written as C1, C2, C3, C4 and C5 respectively as shown in Table 3.

Table 3. Comparison of non-parametric stochastic hydrological models as adapted from [5]

Criteria	**Stochastic generation category**				
	Wavelet (A)	Reordering (B)	Nearest neighbor (C)	Kernel density (D)	Bootstrap (E)
Ability for data to preserve historic characteristics (C1)	Non-preservation of skewness coefficient for simple Haar wavelet	Good for within year statistics. However no replication of long-term variability and persistence	Good for within year statistics and inter-annual dependence replication. Current forms of models not designed for replication of inter-decadal variability and persistence.	Good for within year statistics and inter-annual dependence replication. Long term dependence not modeled.	Good for within year statistics for simple methods of fragmentation. Minimum flows may be overestimated. Preservation of inter-annual and longer term statistics by selecting long building blocks.
Ability to generate new hydrological data” (C2)	Full ability to carry out data generation	None	Limited extrapolation ability	Full ability to carry out data generation	Limited for most bootstrap. However, a bootstrap has been developed recently and has shown the ability.
Scope of applicability (C3)	Only Haar wavelet that requires historical data to be a normal distribution. No limitation for generalized wavelet	May not be effective when daily data have many zero for instance.	Wide scope. No limitation.	Wide scope. No limitation.	Wide scope. No limitation.
Presence of negative data generated (C4)	Possible	None	None	Possible. An approach to deal with this possibility has been devised.	None
User friendliness (C5)	Wavelet easy to use while generalized wavelets may not be easy to understand and use.	Easy. Selection of moving window for re-ordering is subjective.	Easy but computations could be intensive.	Complexity and computationally expensive. Selection of kernel is subjective.	Easy to use. Subjective selection of minimum block length.

4. AHP FORMULATION AND IMPLEMENTATION FOR DATA GENERATION

From Table 2, AHP is formulated as follows:

- Level 1. The goal consists in the selection of best non-parametric stochastic hydrological data generator(s). In this case, data considered are streamflow or rainfall. The modeler, the decision maker, or the water expert will make such a selection. Level 1 is the highest level of the AHP structure.
- Level 2. There are 5 criteria as mentioned earlier, i.e. ability for data to preserve historic characteristics (C1), ability to generate new hydrological data (C2), scope of applicability (C3), presence of negative data generated (C4) and user friendliness (C5). This level is the middle level in the decision-making structure based on AHP.
- Level 3: There are 5 categories of non-parametric stochastic hydrological data generators or alternatives among which the decision maker or water expert/model user/developer should select when considering the criteria defined at level 2. As mentioned earlier, the categories of models are wavelet (A), reordering (B), nearest neighbor (C), kernel density (D) and bootstrap (E). Level 3 is the lower level of the hierarchy.

The implementation of AHP for the purpose of hydrological data generation will be done by carrying out pairwise comparisons at level 2 and level 3 in away to achieve the goal defined at level 1.

Pairwise comparisons among criteria

The set of criteria used in the selection of non-parametric stochastic data generators enabled to establish the judgment matrix for AHP. Based on Table 3, and carrying out pairwise comparisons among criteria, a relatively higher intensity (preference) was allocated to the "ability to generate new hydrological data" than the rest of criteria. Such an allocation of intensity was derived from [5], by the author of the current publication. In a similar manner, "ability to generate new hydrological data" was subjectively perceived to be strongly preferred over "presence of negative data generated". For instance, C2 was moderately preferred over C5 and a value of 3 was assigned to this specific pairwise comparison. It means that the decision-maker or water expert prefers the "ability of generating new hydrological data" 3 times more than "user-friendless" of the non-parametric stochastic model. Conversely a value of 1/3 = 0.33 is allocated when C5 is preferred over C2. Likewise, C2 was considered to be strongly important than C3; meaning that decision-maker prefers "the ability of generating new hydrological data" 5 times more than the "scope of applicability" of the non-parametric stochastic model. Conversely a value of 1/5 = 0.25 is allocated when C5 is preferred over C2. In this manner, all pairwise comparisons were carried out and the different intensities (preferences) are summarized finally in Table 4. This table gives the judgment matrix, when the last line is omitted. Each entry in the last row is sum of entries in its column. One of the diagonals of the matrix has values of 1 since each criterion is compared to itself. The pairwise comparisons satisfied the consistency check. The computed value of CR for the judgment matrix was 5.3 %. Hence the judgment was found to be consistent (valid) when carrying out pairwise comparisons among criteria. In this case, subjectivity in making decisions during pairwise comparisons was approximately 5 %. Almost 95 % can be accounted for by objectivity during pairwise comparisons of criteria.

Table 4 Pairwise comparisons of criteria C1, C2, C3, C4 and C5

	C1	C2	C3	C4	C5
C1	1	0.25	4	3	2
C2	4	1	5	4	3
C3	0.25	0.2	1	2	0.33
C4	0.33	0.25	0.5	1	0.5
C5	0.5	0.33	3	2	1
	6.08	2.03	13.5	12	6.83

Pairwise comparisons of non-parametric stochastic hydrological data generators

The pairwise comparisons of non-parametric hydrological data models are carried out at level 3 of the AHP hierarchy. The wavelet (A) model is very strongly preferred over the reordering (B) model for the purpose of stochastic data generation, when criterion C1 is considered. It means that a value of 7 is used when A is preferred over to B. Conversely, a value of 0.14 is used when B is compared to A. The wavelet (A) model is strongly preferred over the nearest neighbor (C) model; meaning that a value of 5 is used when A is preferred over C. Conversely, a value of 0.2 is given when C is compared to A. The same procedure is carried out for all pairwise comparisons among non-parametric stochastic models and all results summarized in Table 5a. Table 5a summarizes the pairwise comparisons among non-parametric stochastic hydrological data generators, with respect to the first criterion C1.

When the different pairwise comparisons are carried out among models with respect to criterion C2, the reordering (B) model is more preferred over the wavelet (A) model. In this case value of 2 is used. Conversely, a value of 0.5 is used when A is compared to B. After all pairwise comparsions are carried out with respect to C2, the results can be summarized in Table 5b. Similarly, Tables 5c, 5d and 5e summarize the rest of pairwise comparisons with respect to criteria C3, C4 and C5 respectively.

Table 5a Pairwise comparison of non-parametric stochastic hydrological data generation models with respect to C1

	A	B	C	D	E
A	1	7	5	1	3
B	0.14	1	0.33	0.14	0.2
C	0.2	3	1	0.2	0.33
D	1	7	5	1	3
E	0.33	5	3	0.33	1
	2.67	23	14.33	2.67	7.53

Table 5b Pairwise comparison of non-parametric stochastic hydrologic data generation models with respect to C2

	A	B	C	D	E
A	1	0.5	0.33	0.25	0.14
B	2	1	0.5	0.33	0.2
C	3	2	1	0.5	0.25
D	4	3	2	1	0.33
E	7	5	4	3	1
	17	11.5	7.83	5.08	1.92

Table 5c Pairwise comparison of non-parametric stochastic hydrologic data generation models with respect to C3

	A	B	C	D	E
A	1	3	5	5	0.5
B	0.333	1	3	3	0.142
C	0.2	0.333	1	0.333	0.142
D	0.2	0.333	3	1	0.142
E	3	0.333	7	7	1
	4.733	4.999	19	16.333	1.926

Table 5d Pairwise comparison of non-parametric stochastic hydrologic data generation models with respect to C4

	A	B	C	D	E
A	1	0.2	0.2	0.33	0.2
B	5	1	1	3	1
C	5	1	1	3	1
D	3	0.33	0.33	1	0.33
E	5	1	1	3	1
	19	3.53	3.53	10.33	3.53

Table 5e Pairwise comparison of non-parametric stochastic hydrologic data generation models with respect to C5

	A	B	C	D	E
A	1	0.25	0.33	1	0.33
B	4	1	3	4	3
C	3	0.33	1	3	1
D	1	0.25	0.33	1	0.33
E	3	0.33	1	3	1
	12	2.16	5.66	12	5.66

5. RESULTS AND DISCUSSION

Criteria weights

Table 6 displays, in its last column, the criteria weights calculated from Table 4. Each weight is the average of entries in its respective row. Entries for each column in Table 6 are obtained by dividing respective entries (in Table 4) by the total in the last row (in Table 4). The results shown in Table 6 revealed that criteria C2 and C5 have the highest preference (45.9 %) and the lowest preference (7.4 %) respectively. It can be said that criterion C2 is preferred 6 times more than criterion C5. Hence, the water expert or the modeler will have more preference on the "ability to generate new hydrological data" than on the "scope of applicability". From the decision-maker's point of view, the ability for a non-parametric stochastic model to generate data is preferred twice more than the preservation of statistical characteristics. Similar conclusions can be derived from all pairwise comparisons. Such weights were not derived previously [5]. It should be noted that the difference in weight between criteria C3 and C4 is very small. This means that the decision-maker, the modeler or the water expert will assign more less the same weight to both "scope of applicability" and "presence of negative data generated", in the selection of non-parametric stochastic hydrological data generators. The different preferences are approached from the goal perspective of the hierarchy. As explained earlier, from AHP formulation, there is always a certain level of subjectivity in the results obtained from pairwise comparisons of criteria. Nonetheless the consistency ratio calculated was less than 10 %. This implies an acceptable judgment consistency in AHP formulation (i.e. during pairwise comparisons).

Table 6. Criteria weights

	C1	C2	C3	C4	C5	Criteria weight
C1	0.164	0.123	0.296	0.250	0.293	0.225
C2	0.658	0.493	0.370	0.333	0.439	0.459
C3	0.041	0.099	0.074	0.167	0.048	0.086
C4	0.054	0.123	0.037	0.083	0.073	0.074
C5	0.082	0.163	0.222	0.167	0.146	0.156
	1	1	1	1	1	1

Relative weights of non-parametric stochastic hydrological data generators

The computations of weights of alternatives (or data generation models) with respect to the different criteria were carried out and depicted in Table 7a, 7b, 7c, 7d and 7e. Tables 7b, 7c, 7d, and 7e as presented in Appendix I. The weights of alternatives were computed in a similar way as criteria weights. In any of these tables, each element in the last column corresponds to weights of alternatives respectively. Hence, the weights of non-parametric stochastic models with respect to criterion C1 (in Table 7a) are obtained by dividing the entries (preferences on criteria) in Table 5a, by the sum of entries of each column in Table 5a and finally by averaging values in each row. In a similar way, the weights of non-parametric stochastic models with respect to the rest of criteria (C2, C3, C4, and C5) are summarized in Tables 7b, 7c, 7d, and 7e as shown in Appendix I. The last raw in each of these tables has a unit value, which is the sum of entries for each column.

Table 7a. Weights of non-parametric stochastic hydrological data generators with respect to C1

	A	B	C	C	D	Weight
A	0.059	0.043	0.042	0.049	0.073	0.053
B	0.118	0.087	0.064	0.065	0.104	0.088
C	0.176	0.174	0.128	0.098	0.130	0.141
D	0.235	0.261	0.255	0.197	0.172	0.224
E	0.412	0.435	0.511	0.591	0.521	0.494
	1	1	1	1	1	1

From Table 7a, bootstrap category displays the highest preference (49.4 %) for pairwise comparison among non-parametric stochastic hydrological data generation models, with respect to criterion C1, followed by kernel density, nearest neighbor, reordering and wavelet categories. Hence the modeler or decision maker will have more preference to use bootstrap category of non-stochastic parametric models than other models. Table 7b does not display the same trend. For instance when non-parametric stochastic data generation models are compared with respect to criterion C2, kernel density and wavelet categories scored the highest preference (36. 0%). In Table 7c, when criterion C3 is considered for pairwise comparison among hydrological data generators, the bootstrap and the nearest neighbor categories displayed the highest preference (40.3 %) and the lowest preference (5.1 %) respectively. In Table 7d, the highest preference is noticed for reordering, nearest neighbor and bootstrap categories (28.1 %) when pairwise comparisons between non-parametric stochastic hydrological data generation models are carried out with respect to criterion C4. Finally when criterion C5 is considered, the highest preference (43.8 %) is on reordering category (see Table 7e). The weights of different non-parametric stochastic data generators vary from one criterion to the other. It can be observed that the bootstrap category has the highest preference for C1, C3 and C4. Hence, during the selection process for data generation purpose, the decision maker, modeler or water expert may prefer to use bootstrap category more than other non-parametric stochastic hydrological data generation models.

Table 8 of Appendix II displays the overall preferences (weights) in the selection of non-parametric stochastic models when all the criteria are taken into consideration. The selection process in AHP is achieved with respect to the goal of the problem at hand. The results showed that bootstrap category has the high level of preference (i.e. 27.2 %), followed by kernel density (24.3 %), wavelet (22.1 %), reordering (13.9 %), and nearest neighbor (10.6 %) categories. This order of preference is the ranking of the non-parametric stochastic hydrological data generators when all criteria are considered. The validity of this ranking is restricted to the 5 criteria defined at level 2 of the hierarchy. The increase in the number of criteria may influence the results. Similarly to [5], the overall preference was shown to be for bootstrap category; however [5] did not show the ranking in the selection process nor a sound methodology was used. Unlike the previous study [5], the current study demonstrates that a transparent multi-criteria decision making (MCDM) technique can be used in the selection of non- parametric stochastic hydrological data generators.

6. CONCLUSION

AHP has been proven to be a versatile tool since it can be used to a variety of problems. In particular, it has been used for the first time in the selection of non-parametric stochastic hydrological data generation models. The overall model selection process was made consistently and transparently using pairwise comparisons. The overall highest preference was shown to be on bootstrap category. However, the preferences for kernel density and wavelet categories did not substantially differ from the bootstrap category. The decision-maker, model user or water expert could make sound choices among several techniques for the purpose of hydrological data generation. Further work could include the application of AHP to more categories of stochastic models and other criteria. Specific non-parametric stochastic models of these categories should be tested for AHP process.

7. REFERENCES

[1] N.K. Ajami, G.M. Hornberger & DL. Sunding, "Sustainable Water Resource Management under Hydrological Uncertainty". **Water Resources Research**, Vol. 44, W 11406, DOI: 10.1029/2007WR006736, 2008, pp.1-10

[2] J.D. Salas, J.A. Ramirez, P. Burlando, R.A. Pielke, **Stochastic Simulation of Rainfall and Streamflow Processes**. In Handbook of weather, Climate and water: Atmospheric Chemistry, Hydrology and Societal Impacts, Eds. T.D. Potter & B.R. Colman. ISBN 471214892, John Wiley & Sons, Inc, pp. 607-640, 2003.

[3] J.R. Prairie, B. Rajagopalan, U. Lall & T. Fulp, "Stochastic Non Parametric Technique for Space-time Disaggregation of Streamflow", **Water Resources Research**, Vol. 43, W03432, DOI: 10.1029/2007WR004721, 2007, pp.1-10

[4] J.R. Prairie, B. Rajagopalan, Fulp, T.J., & E. A. Zagona, "Modified K-NN Model for Stochastic Streamflow Simulation", **Journal of Hydrologic Engineering**, Vol. 11, No. 4, July 1, 2006, pp. 371-378.

[5] J. Ndiritu & J. Nyaga, **A Non Parametric Multi-Site Stochastic Rainfall Generation with Applications to Climate Change**. Water Research Commission (WRC), Report No. 2148/1/13. ISBN 978-1-4312-0515-8, 2014.

[6] T.L. Saaty, **The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation**, ISBN 0-07-054371-2. McGraw-Hill, 1980

[7] M. Ilunga, **Analytic Hierarchy Process (AHP) in Selecting Artificial Neural Networks (ANN) for Ungaged Streamflow Prediction**, World Multi-Conference on Systemic, Cybernetics and Informatics WMSCI 2014 ISBN-13: 978-1-936338-97-9, 2014.

[8] M. Ilunga, **Analytic Hierarchy Process (AHP) in Ranking Rainfall Stations In-Filled with Radio Basis Function Artificial Neural Networks (RBFN)**. World Multi-Conference

on Systemic, Cybernetics and Informatics: WMSCI 2014, ISBN-13: 978-1-936338-97-9, 2014

[9] M. Ilunga and E.K. Onyari, **Application of Hierarchical Process (AHP) for ANN Model Selection in Streamflow Prediction**. World Multiconference on Systemics, Cybernetics and Informatics (WMSCI 2011), Orlando, USA, July 19 th-22 nd, 2011

[10] M. Ilunga, **Analytic Hierarchy Process (AHP) in Ranking Watersheds for Streamflow Data Infilling Models**, The 17 th World Multi-Conference on Systemic, Cybernetics and Informatics (WMSCI 2013), USA, Orlando, July 2013

[11] Ashari, M. A., Zaiton, S., Hashim, M. & Samsudin, S., "Using Fuzzy Analytic Hierarchy Process for Southern Johor River Ranking", **International Journal of Advances in Soft Computing Applications**, Vol. 1, No. 1, 2009, pp. 63-77.

[12] C.W. Chang, C.R. Wu, C.T. Lin, C.H. Chen, "An Application of AHP and Sensitivity Analysis for Selecting the Best Slicing Machine", **Computers and Industrial Engineering**, Vol. 52, 2007, pp. 296-307.

[13] Young, K. D, Yamin, Y. & Dymond, R.L. "Applications of the Analytic Hierarchy Process for Selecting and Modelling Stormwater Best management Practices", **Journal of Contemporary Water Research & Education,** Issue 146, 2010, pp. 50-63.

ACKNOWLEDGMENTS

The use of data extracted from WRC Report No. 2148/1/13 for the formulation of AHP is acknowledged.

8. APPENDICES

Appendix I. Determination of w*eights of non-parametric stochastic hydrological data generators*

Table 7b. Weights of non-parametric stochastic hydrological data generators with respect to C2

	A	B	C	C	D	Average
A	0.375	0.304	0.349	0.375	0.398	0.360
B	0.052	0.043	0.023	0.052	0.027	0.040
C	0.075	0.130	0.070	0.075	0.044	0.079
D	0.375	0.304	0.349	0.375	0.398	0.360
E	0.124	0.217	0.209	0.124	0.133	0.161
	1	1	1	1	1	1

Table 7c. Weights of non-parametric stochastic hydrological data generators with respect to C3

	A	B	C	C	D	Average
A	0.211	0.600	0.263	0.306	0.260	0.328
B	0.070	0.200	0.158	0.184	0.074	0.137
C	0.042	0.067	0.053	0.020	0.074	0.051
D	0.042	0.067	0.158	0.061	0.074	0.080
E	0.634	0.067	0.368	0.429	0.519	0.403
	1	1	1	1	1	1

Table 7d. Weights of non-parametric stochastic hydrological data generators with respect to C4

	A	B	C	C	D	Average
A	0.053	0.057	0.057	0.032	0.057	0.051
B	0.263	0.283	0.283	0.290	0.283	0.281
C	0.263	0.283	0.283	0.290	0.283	0.281
D	0.158	0.093	0.093	0.097	0.093	0.107
E	0.263	0.283	0.283	0.290	0.283	0.281
	1	1	1	1	1	1

Table 7e. Weights of non-parametric stochastic hydrological data generators with respect to C5

	A	B	C	C	D	Average
A	0.083	0.116	0.058	0.083	0.058	0.080
B	0.333	0.463	0.530	0.333	0.530	0.438
C	0.250	0.153	0.177	0.250	0.177	0.201
D	0.083	0.116	0.058	0.083	0.058	0.080
E	0.250	0.153	0.177	0.250	0.177	0.201
	1	1	1	1	1	1

Appendix II. Determination of overall weights non-parametric stochastic hydrological data generators

Table 8. Computation of overall weights of models

X	Ci	A	B	C	D	E	A	B	C	D	E
0.225	C1	0.053	0.088	0.141	0.224	0.494	0.012	0.020	0.032	0.050	0.111
0.459	C2	0.360	0.040	0.079	0.360	0.161	0.165	0.018	0.036	0.165	0.074
0.086	C3	0.328	0.137	0.051	0.080	0.403	0.028	0.012	0.004	0.007	0.035
0.074	C4	0.051	0.281	0.028	0.107	0.281	0.004	0.021	0.002	0.008	0.021
0.156	C5	0.080	0.438	0.201	0.080	0.201	0.012	0.068	0.031	0.012	0.031
Overall weights ==➔							**0.221**	**0.139**	**0.106**	**0.243**	**0.272**

X: column representing weight criteria. The overall weight for A is obtained by multiplying each entry in X by its corresponding entry in column A and by summing up the new entries obtained from multiplication. Ci’s are the different criteria.