

# System Response Analysis and Model Order Reduction, Using Conventional Method, Bond Graph Technique and Genetic Programming.

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## ABSTRACT

This research paper basically explores and compares the different modeling and analysis techniques and than it also explores the model order reduction approach and significance. The traditional modeling and simulation techniques for dynamic systems are generally adequate for single-domain systems only, but the Bond Graph technique provides new strategies for reliable solutions of multi-domain system. They are also used for analyzing linear and non linear dynamic production system, artificial intelligence, image processing, robotics and industrial automation.

This paper also describes a unique technique of generating the Genetic design from the tree structured transfer function obtained from Bond Graph. This research work combines bond graphs for model representation with Genetic programming for exploring different ideas on design space tree structured transfer function result from replacing typical bond graph element with their impedance equivalent specifying impedance lows for Bond Graph multiport. This tree structured form thus obtained from Bond Graph is applied for generating the Genetic Tree. Application studies will identify key issues and importance for advancing this approach towards becoming on effective and efficient design tool for synthesizing design for Electrical system.

In the first phase, the system is modeled using Bond Graph technique. Its system response and transfer function with conventional and Bond Graph method is analyzed and then a approach towards model order reduction is observed. The system response and the stability analysis of the system transfer function taken by conventional and by Bond Graph method is compared and analyzed. The approach towards the Genetic Tree formation from the Bond Graph is also developed. The model order reduction using Genetic Tree is in progress.

## Key Words

Model order reduction, Genetic tree, Tree structured transfer function, Bond graph, multi domain system, system response for certain FROI, Genetic programming and Bond graphs equivalent impedances.

## 1. INTRODUCTION:

### System Modeling

A central idea involved in the study of the of dynamics of the real systems is the idea of the model of the system. Models of the systems are simplified, abstracted constructs used to predict their behavior. Bond graph are succinct, pictorial statements of mathematical models. It is a graphical tool which provides enough information to drive other system representation such as transfer function for linear models, linear or non linear state equation [1], etc. The Bond graph and the physical system model preserve the structured information. It preserves model

conciseness, unlike Block diagram representation, only one bond is necessary to represent two types of information: effort/flow.

### Model Order Reduction

Once the model is developed and the input output relationships are studied we move towards the model order reduction of the system. The approximation of high-order plant and controller models by models of lower order is an integral part of control system design. Until relatively recently model reduction was often based on physical intuition. For example, chemical engineers often assume that mixing is instantaneous and that packed distillation columns may be modeled using discrete trays. Electrical engineers represent transmission lines and the eddy currents in the rotor cage of induction motors by lumped currents. Mechanical engineers remove high-frequency vibration modes form models of aircraft wings, turbine shafts and flexible structures. It may also be possible to replace high-order controllers by low-order approximations with little sacrifice in performance [10].

There are several procedures which seek to automate the model reduction process. Suppose a high-order, linear, time-invariant model  $G$  is given, then the Prototype  $H_\infty$  model reduction problem is to find a low-order approximation  $\hat{G}$  of  $G$  such that  $\|G - \hat{G}\|_\infty$  is small. Consider the more difficult problem of selecting  $\hat{G}$  such that the problem of selecting  $W_1$  and  $W_2$  such that  $\|W_2(G - \hat{G})W_1\|_\infty$  is small; the weighting function  $W_1$  and  $W_2$  are used to frequency shape the model reduction error[16]. For example, one might select the weights so that the modeling error is small in the unity gain range of frequency. Model order reduction techniques used here are Balanced Realization and then a final approach to model order reduction by Bond Graph method.

### Genetic Programming

Here we have also tried to model the Genetic tree through tree structured transfer function; the method of model order reduction using this technique is in progress.

Genetic programming is one of a number of evolutionary algorithm follows Darwin theory of evolution and is paraphrased as "survival of the fittest". It basically consist of three operation [3]

1. Reproduction
2. Cross Over
3. Selection technique

It is an effective way to generate design candidates in an open ended but statistically structured, manner. [4]

There are several factors involved in conducting a GP based search they include:

- Embryo

The embryo is an invariant part of a model which contains interfacing or boundary information associated with the problem to be solved.

- Seed Population of candidates

The seed population is an initial set of tree created at rand an using embryo as the starting point.

- Fitness function

The fitness function is a measure of the relative worth of a candidate model based on evolution of its dynamic performance and other characterisation.

- Statistics of evolution

It guides the generation of new candidates, they apply to the basic operations [5].

## 2.TREE STRUCTURED TRANSFER FUNCTION

### Synthesis of tree structured transfer function

Tree structured transfer function result from replacing typical bond elements with their impedance equivalents, specifying impedance laws for bond graph multiports and recursively substitution element impedance into the impedance law terms. In electrical system impedance is the voltage divided by current. Other physical domains have their own definitions of impedance. Basic bond graph elements, their constitution laws and their generalized effort are provided in following table, where  $f$  represent generalized flow,  $e$  represent generating effort,  $z$  represent impedance and  $s$  represent the voltage [8].

Elements constitutive Impedance

Resistance:  $e = Rf \quad \frac{e}{f} = R$

Capacitance:  $\int f = Ce \quad \frac{e}{f} = \frac{1}{CS}$

Inductor  $\int e = If \quad \frac{e}{f} = IS$

### Synthesis of tree structured TF of high pass filter

The example of high pass filter selected is obtained from the research works of particularly Eri K D, Goodman and Ronald C. Rosenberg. [6], [7].

### Rules for generating tree structured transfer function:-

After obtaining the bond graph, now the tree structured transfer function will be developed that leads to the GP tree generation.

#### Steps:

1. Replace typical bond graph elements with their impedance equations.
2. specify impedance law for 0,1 junctions.

#### One junction impedance:

$$\sum ei = 0$$

Where each of the effort is divided by the flow  $f$  associate with one junction which is same for all attached bond. The following impedance values will result. It should be noted that effort associated with half arrows directed toward a junction are given positive and those associated with arrows directed away from the junction are given negative values.

$$\frac{\sum iei}{f} = \frac{\sum ei}{f_i} = \frac{\sum Z_i Z_o}{\text{one junction impedance}}$$

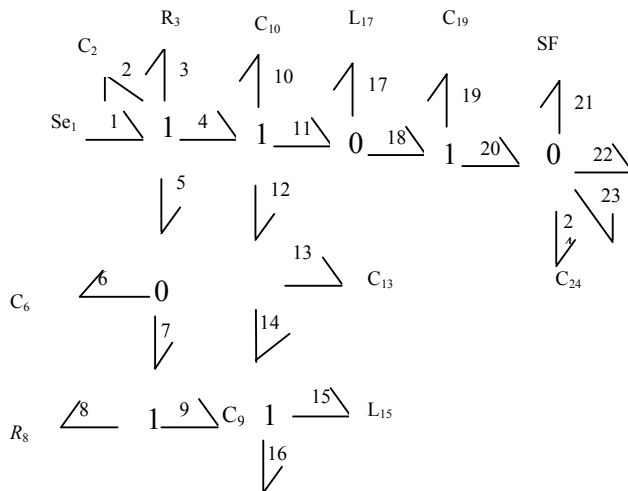
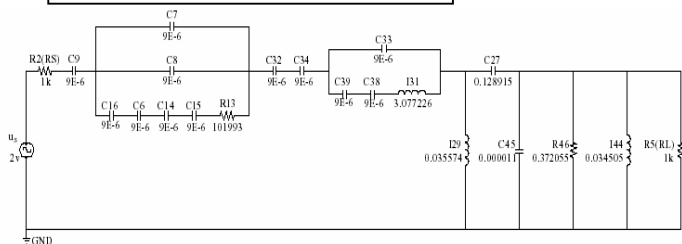


Fig. 1 High pass RC Filter with its Bond Graph

### Zero Junction impedance

In case of a zero junction the attached bond share a common effort, and flow of the bond are sum up to zero. For zero junctions we have

$$\sum fi = 0$$

Where flow associated will half arrows directed towards a junction are given positive values and those associated with arrows direct away from a junction are given negative values. When each of those flows is divided by the effort  $e$  associated with the junction, which is same for all attached bond, the following impedance law is obtained.

$$\frac{\sum i fi}{e} = \sum \frac{fi}{ei} = \sum Z_i^{-1} = 0$$

### Zero junction impedance

### Defining the impedance for every element of bond graph

Now we will define the impedance for each and every element.

- $Z_1 = S_e$
- $Z_2 = \frac{1}{C_1 S}$
- $Z_3 = R_L$
- $Z_4 = \text{Table}$
- $Z_5 = \frac{1}{C_2 S}$
- $Z_6 = \frac{1}{C_3 S}$
- $Z_7 = \text{Table}$
- $Z_8 = R_1$
- $Z_9 = \frac{1}{C_4 S}$
- $Z_{10} = \frac{1}{C_7 S}$
- $Z_{11} = \frac{1}{C_6 S}$
- $Z_{12} = \frac{1}{C_5 S}$

$$Z_{13} = \text{To be Calculated}$$

$$Z_{14} = \text{To be Calculated}$$

$$Z_{15} = \frac{1}{C_8 S}$$

$$Z_{16} = \frac{1}{C_9 S}$$

$$Z_{17} = \text{To be Calculated}$$

$$Z_{18} = \text{To be Calculated}$$

$$Z_{19} = \frac{1}{C_{10} S}$$

$$Z_{20} = \text{To be Calculated}$$

$$Z_{21} = \frac{1}{C_{11} S}$$

$$Z_{22} = I_1 S$$

$$Z_{23} = \frac{1}{C_{12} S}$$

$$Z_{24} = I_2 S$$

$$Z_{25} = \text{To be Calculated}$$

$$Z_{26} = \frac{1}{C_{13} S}$$

$$Z_{27} = \text{To be Calculated}$$

$$Z_{28} = SF_4$$

$$Z_{29} = R_4$$

$$Z_{30} = I_3 S$$

$$Z_{31} = R_3$$

$$Z_{32} = \frac{1}{C_{14} S}$$

Applying one junction impedance equivalent to the bond graph;

$$1. \quad Z_1 = Z_2 + Z_3 + Z_4 + Z_{13}$$

$$2. \quad Z_7 = Z_5 + Z_{10} + Z_{11} + Z_{12} + Z_9$$

$$3. \quad Z_{14} = Z_{15} + Z_{16} + Z_{17} + Z_{18}$$

$$4. \quad Z_{20} = Z_{21} + Z_{22} + Z_{23}$$

$$5. \quad Z_{25} = Z_{26} + Z_{27}$$

Applying 0 junction impedance to the bond graph ;

$$\sum_i \frac{f_i}{e_i} = \sum_i z_i^{-1}$$

Zero junction impedance

$$Z_4 = (z_5^{-1} + z_6^{-1} + z_7^{-1})^{-1}$$

$$Z_{17} = (z_{24}^{-1} + z_{25}^{-1})^{-1}$$

$$Z_{27} = (z_{28}^{-1} + z_{29}^{-1} + z_{30}^{-1} + z_{31}^{-1} + z_{32}^{-1})^{-1}$$

$$Z_{18} = (z_{19}^{-1} + z_{20}^{-1})^{-1}$$

Now using the gyrator impedance low

$$Z_1 = r^2 Z_2^{-1}$$

$$Z_{13} = C_1 Z_{14}^{-1}$$

$$Z_{13} = \frac{C_1}{Z_{14}}$$

Now find out the impedance & represent values of impedance

$$Z_1 = Z_2 + Z_3 + Z_{13}$$

$$Z_4 = (z_5^{-1} + z_6^{-1} + z_7^{-1})^{-1}$$

$$= \left[ \left( \frac{1}{C_2 S} \right)^{-1} + \left( \frac{1}{C_3 S} \right)^{-1} + z_7^{-1} \right]^{-1}$$

$$Z_7 = Z_8 + Z_{10} + Z_{11} + Z_{12} + Z_9$$

$$Z_{14} = Z_{15} + Z_{16} + Z_{17} + Z_{18}$$

$$Z_7 = R_1 + \frac{1}{C_7 S} + \frac{1}{C_8 S} + \frac{1}{C_5 S} + \frac{1}{C_4 S}$$

$$Z_{13} = \frac{C_1}{\frac{1}{C_8 S} + \frac{1}{C_9 S} + Z_{17} + Z_{18}} = \frac{1}{C_{13} S} + Z_{27}$$

$$Z_{27} = (Z_{28}^{-1} + Z_{29}^{-1} + Z_{30}^{-1} + Z_{31}^{-1} + Z_{32}^{-1})^{-1}$$

$$Z_{25} = Z_{26} + Z_{27}$$

$$Z_{25} = \frac{1}{C_{13} S} + \left[ (SF_4)^{-1} + (R_4)^{-1} + (I_3 S)^{-1} + (R_3)^{-1} \right]$$

$$Z_{27} = \left[ (I_4 S)^{-1} + (R_4)^{-1} + (I_3 S)^{-1} + \left( \frac{1}{C_{14} S} \right)^{-1} \right]^{-1}$$

$$Z_{17} = (I_2 S)^{-1} + \frac{1}{C_{13} S} + \left[ (SF_4)^{-1} + (R_4)^{-1} + (I_3 S)^{-1} + (R_3)^{-1} + \left( \frac{1}{C_{14} S} \right)^{-1} \right]^{-1}$$

$$Z_{18} = [Z_{19}^{-1} + Z_{20}^{-1}]^{-1}$$

$$= \left[ \left( \frac{1}{C_{10} S} \right)^{-1} + Z_{20}^{-1} \right]^{-1}$$

$$Z_{21} = Z_{21} + Z_{22} + Z_{23}$$

$$= \left[ \frac{1}{C_1 S} + I_1 S + \frac{1}{C_{12} S} \right]$$

$$Z_{18} = \left[ \left( \frac{1}{C_{10} S} \right)^{-1} + \left\{ \frac{1}{C_{11} S} + I_1 S + \frac{1}{C_{12} S} \right\}^{-1} \right]^{-1}$$

$$Z_{13} = \frac{C_1}{\frac{1}{C_8 S} + \frac{1}{C_9 S} + \left\{ (I_2 S)^{-1} + \left( \frac{1}{C_{13} S} \right)^{-1} \right\} + \left\{ (SF_4)^{-1} + (R_4)^{-1} + (I_3 S)^{-1} + (R_3)^{-1} + \left( \frac{1}{C_{14} S} \right) \right\} + \left\{ \frac{1}{C_{11} S} + I_1 S + \frac{1}{C_{12} S} \right\}}$$

$$Z_1 = Z_2 + Z_3 + Z_6 + Z_{13}$$

$$Z_7 = [Z_5^{-1} + Z_6^{-1} + Z_7^{-1}]^{-1}$$

Now creating the final Tree Structured transfer function.[11]

$$\frac{R_2 + C_1}{1 + C_1 S + R_2 + \frac{1}{C_2 S} + \frac{1}{C_3 S + R_1} + \frac{1}{\frac{1}{C_7 S} + \frac{1}{C_6 S} + \frac{1}{C_5 S} + \frac{1}{C_4 S} + \frac{1}{\frac{1}{C_8 S} + \frac{1}{C_9 S} + \frac{1}{(I_2 S) + \frac{1}{C_{13} S} + SF_4 + R_4 + \frac{1}{I_3 S + R_3 + \frac{1}{C_{14} S} + \frac{1}{C_{10} S + \frac{1}{C_{11} S + I_1 S + \frac{1}{C_{12} S}}}}}}$$

### 3.GENERATING GENETIC PROGRAMMING TREE FROM BOND GRAPH TREE STRUCTURED TRANSFER FUNCTION

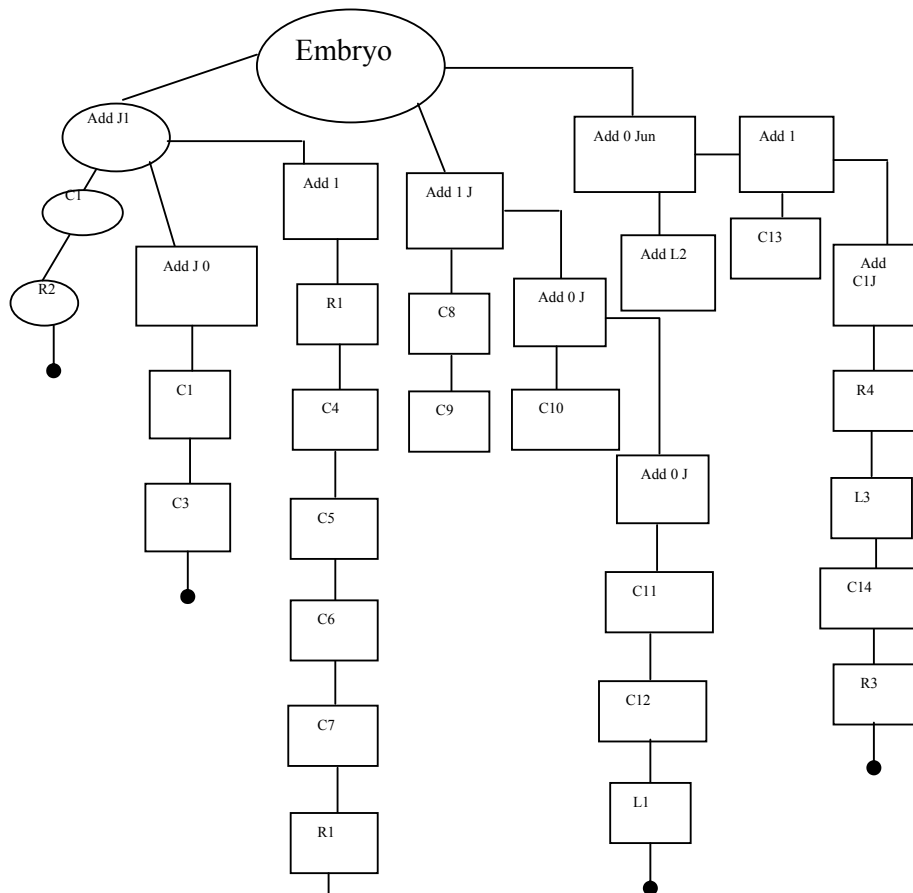


Fig. 2

**4.DEVELOPMENT OF STATE SPACE FROM BOND GRAPH**

**a =**

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11
x1	-111.1	-55.56	0	-222.2	-111.1	0	-0.007757	-90.91	0	0	0
x2	-111.1	-56.1	-4.358	-222.2	-111.1	0	-0.007757	-90.91	0	0	0
x3	0	-0.5447	-4.358	0	0	0	0	0	0	0	0
x4	-111.1	-55.56	0	-222.2	-111.1	0	-0.007757	-90.91	0	0	0
x5	-111.1	-55.56	0	-222.2	-111.1	0	-0.007757	-90.91	-0.325	0	0
x6	0	0	0	0	0	0	0	0	-0.325	-28.11	0
x7	-111.1	-55.56	0	-222.2	-111.1	0	-0.007757	-90.91	0	-28.11	0
x8	-111.1	-55.56	0	-222.2	-111.1	0	-0.007757	-2.445e+005	0	0	-28.99
x9	0	0	0	0	111.1e+005	2.222e+005	0	0	0	0	0
x10	0	0	0	0	0	0	.444e+005	-90.91e+004	0	0	0
x11	0	0	0	0	0	0	0	-90.91e+004	0	0	0

**b =**

	u1
x1	0.001
x2	0.001
x3	0
x4	0.001
x5	0.001
x6	0
x7	0.001
x8	0.001
x9	0
x10	0
x11	0

**c =**

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11
y1	0	0	0	0	0	0	0	0	-1	0	0

**d =**

	u1
y1	0

**5.TRANSFER FUNCTION DEVELOPED FROM CONVENTIONAL METHOD**

Transfer function:

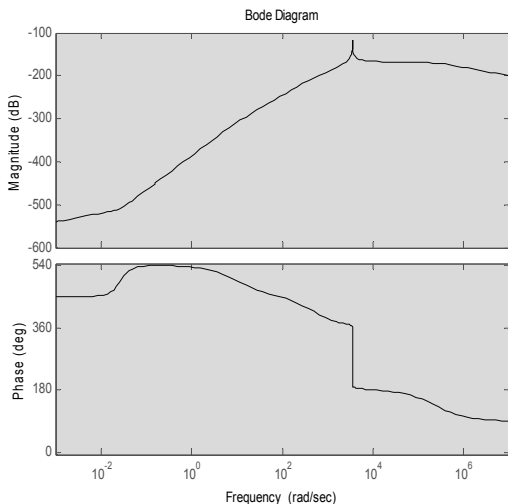
$$\frac{-0.001 s^{10} - 0.004902 s^9 + 36.11 s^8 + 177 s^7 + 9.913e-008 s^6 - 0.0007268 s^5 - 0.003563 s^4 - 8.696e-010 s^3 + 2.675e-021 s^2 - 1.876e-035 s}{s^{11} + 2.45e005 s^{10} + 1.411e008 s^9 + 3.055e012 s^8 + 1.569e015 s^7 - 8.668e016 s^6 - 6.904e019 s^5 - 1.059e021 s^4 - 3.525e021 s^3 - 8.712e008 s^2 + 0.01201 s + 1.129e-015}$$

**6.SYSTEM RESPONSE USING BOND GRAPH TREE STRUCTURED TRANSFER FUNCTION AND CONVENTIONAL METHOD.**

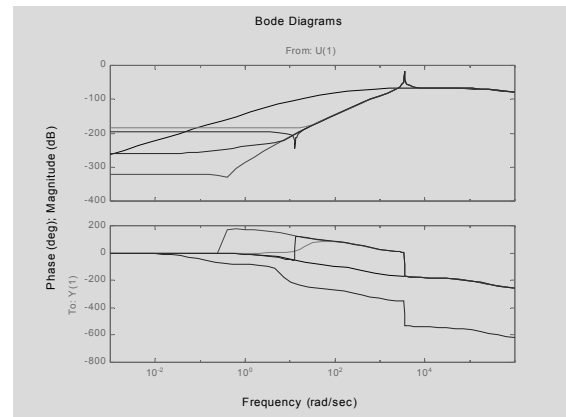
System Frequency response using Bode Plot for a selected FROI.

Model Order Reduction[9]

Comparison of Frequency Response of Original System And 9<sup>th</sup> Order Reduced Model by Balanced Realization, Hankle Norm, Schmidt Technique and Bond Graph Method.



**Fig: 3**



**Fig. 4**

Comparison of Frequency Response of Original System And 8 Order Reduced Model by Balanced Realization, Hankle Norm, Schmidt Technique and Bond Graph Method.

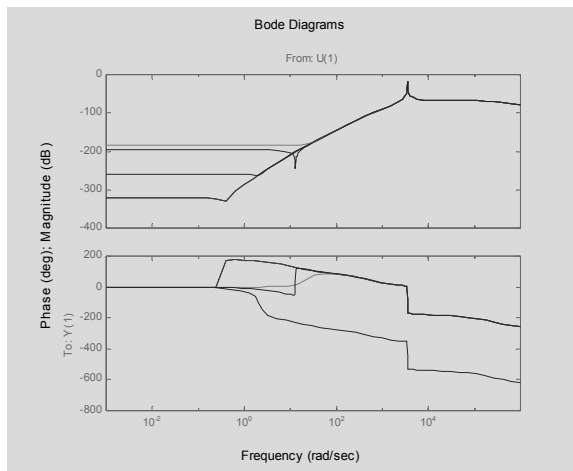


Fig. 5

Model Order Reduction with respect to errors

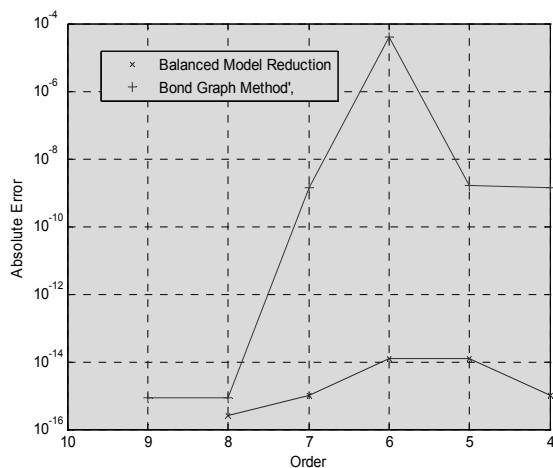


Fig. 6

System Stability using Root Locus

Location of closed loop poles are,  $-56.3 + 3.91e+003i$ , Damping = 0.0144 and Natural Frequency is  $3.91e + 003$  rad/sec.

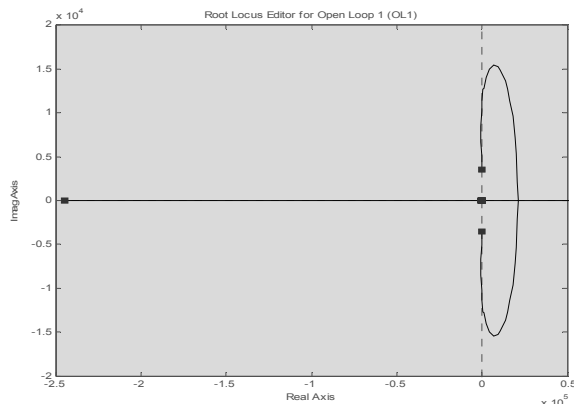


Fig. 7

RESULTS AND CONCLUSION

The system responses obtained through Tree Structured Transfer Function derived from Bond Graph and the one obtained from conventional method is exactly the same, in addition to it there is no change in the stability performance.

The BG/GP approach, which combines the capability of genetic programming to search in an open ended design space and the merits of bond graph, proves to be the promising method to do system level synthesis of multi domain system.

This research paper has explored and explained the bond graph tree structured transfer function as a design methodology to genetic the genetic tree for electrical system, by taking advantage of genetic programming as a search for competent design and the bond graph as a representation for dynamic system.

A new Modeling Approach using Bond Graph Method is applied. The second conclusion leads toward model order reduction. The error plot indicates the least errors due to Bond Graph approach. The final model retains structural information.

The BGM combining with genetic programming offers a tremendous opportunity in artificial intelligence. This technique of BG/GP can be very useful for model order reduction.

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