Resignification of The Concept of Linear Inequality Through Graphic Argumentation. The Case of The Noise Emission Norm

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ABSTRACT [1]

The objective of this research was to analyze the construction of mathematical knowledge associated with the linear inequalities that Chilean students carried out through graphic argumentation, in a modelling process framed in the Socioepistemological Theory of Educational Mathematics. To fulfill this objective, the methodology was qualitative and consisted of a case study in which 28 Chilean 12th grade students participated in a classroom situation where a modelling process was applied through the use of a mobile phone application called Sound Analyzer that designs graphs of a sound in decibels according to time. The students described the number of decibels of a certain song and identified the time intervals in which that song exceeded the number of decibels established by regulation by the Ministry of the Environment of Chile. As a result of the investigation, it was found that the students were able to generate, in a modelling process and through graphic argumentation, a resignification of mathematical knowledge associated with linear inequalities.

Keywords: Graphic argumentation, Inequality, Modelling, Resignification, Socioepistemology.

1. INTRODUCTION

The research carried out in [14] shows that the teaching of the concept of linear inequality in the Chilean curriculum addresses the study of algebraic methods to solve different types of inequalities, the ordering of real numbers and the analysis of the domain and range of different real functions of a one variable. In this way, it is possible to realize that the construction of the concept of linear inequality is dominated by a reasoned justification, which means that this construction is conceived from the same mathematical structure [6]. The foregoing shows a lack of reference frames that allows students to generate a resignification of mathematical knowledge associated with the concept of linear inequality, based on functional justifications, which are opposed to reasoned justifications.

In [7] it is pointed out that reasoned justifications belongs to disciplinary knowledge, whose nature is different from mathematical knowledge that is present in school and, in general terms, in the city, where people make use of their mathematical knowledge, in different specific situations, to build knowledge from functional justifications. In this way, it is important to point out that this type of justification should be rescued by different investigations to provide theoretical elements that allow students to carry out a resignification of mathematical knowledge, understood as the construction of knowledge itself, according to the organization of the human group [5].

From Socioepistemology theory [11], modeling is considered as a practice that is shared and exercised in specific communities and in particular contexts, which when exercised by participants of the didactic system, allows the resignification of mathematical knowledge [10]. In turn, graphic argumentation, understood in the sense of [2], positions graphics as an important element to carry out that resignification. Different investigations have realized that by means of a graphic argumentation, in a specific modeling situation, it is possible to generate a resignification of mathematical knowledge. To mention some of them, in [2] a resignification of the polygon concept was shown in a modeling process, by means of a graphic argument carried out by pedagogy students. In

[1] Acknowledgement is given to Professor Emeritus Thomas Marlowe, Seton Hall University, Department of Mathematics and Computer Science, for his comprehensive and detailed peer-editing of this article.

[1], a resignification of mathematical knowledge by university students through a graphic argumentation was shown in a modeling process within a specific situation in Physics. In [4] classroom situations are proposed that allow generating a resignification of mathematical knowledge through modeling processes in specific situations in Phenology, where graphic argumentation plays a very important role to carry out a construction of the concept of definite integral .

With the purpose of analyzing the construction of mathematical knowledge through a graphic argumentation, the objective of this research will be to analyze the construction of mathematical knowledge associated with the linear inequalities that Chilean students carry out through graphic argumentation, in a modeling process framed in the Socioepistemological Theory of Educational Mathematics [11].

The structure of this article will be as follows: in section 2, the methodology that responds to the proposed research objective will be presented, together with the design of the classroom situation that seeks to promote a modeling process. Later, in section 3, the analysis of the data obtained will be shown, together with the results of this investigation. Finally, in section 4, the conclusions of this research will be presented.

2. METHODOLOGY

To respond to the proposed research objective, the methodology used was qualitative, where the instrumental case study [12] was used to study the particularity and complexity of the case that is part of this research, since it is desired to describe in detail the development of the situation by the students, their ways of interacting, and, mainly, the mathematical arguments that emerge from them. It should be noted that this type of methodology was used in [2], to analyze the construction of mathematical knowledge, in a modeling process, of Chilean students from a graphic argumentation.

Description of the context and participants

The study subjects are students of 12th grade, aged from 17 to 18 years, with 13 women and 15 men. They worked in groups of seven to solve the tasks. They who worked cooperatively solving each of the moments set out. After organizing the students into groups, they were given the classroom situation (Figure 1) and the graphs that describe the decibels of the song according to time (Figure 2). Initially, students understood the given task and then began its development. The students through group discussion had total freedom to follow the path they deemed appropriate, making the decisions they consider appropriate to respond to the problem posed. During this process, there was special care not to encourage by any means to give a solution from the teacher, taking a reflective attitude towards the validity of responses that emerged from the students.

Design

The design of the classroom situation (Figure 1) had the intention of generating a modeling process that would allow the students themselves to use graphs that model the number of decibels of a certain song as a function of time, using a mobile phone application called *Sound Analyzer*, which corresponds to a sound level meter which measures the level of noise that exists in a certain place in terms of the number of decibels. The students, when using the application, obtained a graph of 60 seconds, which are subdivided into six sheets distributed in 10 seconds by sheet, which are given to each group (Figure 2).

Considering the concept of linear inequality in the sense of [13], people who understand this concept as an inequality with a real variable where two functions that share the same variable are compared, the classroom situation sought that the students, by means of a comparison between the graph that describes the number of decibels of the song and the graph, built by them, that represents the ministerial norm, they can determine in which time intervals the song exceeds that established by the norm. In this way, the classroom situation sought that the students make use of graphic arguments to answer the questions posed.

Noise emission, such as voice birdsong and ambient noise, can be measured in various ways. One way is to use a sound level meter, which captures the intensity of the sound of the sound waves by means of a graph, which allows us to study the sound.

Decree 38 of the Ministry of the Environment (norm for the emission of annoying noise generated by fixed sources) [9] provides the maximum permitted levels of noise emission generated by fixed sources for the community. It provides that the maximum noise for residential areas is 55 decibels (dB) from 07:00 to 21:00.

To carry out our task, let us carefully read the following context:

"Arturo is on his birthday and is having a party at his house, hiring a DJ who wants to start the party at 7:00 p.m. with the song "Despacito" by Luis Fonsi, at a moderate volume. After a few minutes, Arturo approaches the DJ and very annoyed he tells him that the music is low and that he should turn it up"
A neighbor annoyed by the noises of the party contacts the authorities, causing the arrival of an inspector. If you are the inspector and you have the graphics of the song, put by the DJ.

Moment 1: At what moments does the song exceed the norm established by Ministry of the Environment?
- Underline the intervals that the song exceeds the norm.
- Represent, on a real line, each detected interval.
Moment 2: How long did the DJ exceed the norm established by Ministry of the Environment?
Moment 3: If you were the inspector, what would you say to the DJ or the host of the party?

Figure 1: Classroom situation design.

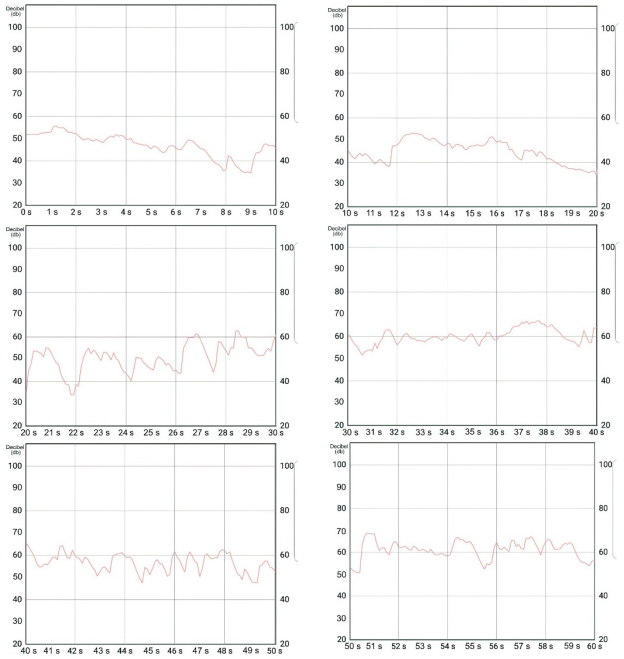

Figure: Graphs with a description of decibels according to the time of the song.

3. ANALYSIS AND RESULTS

This section will show the productions that the students made during the development of the proposed classroom situation. A description of the different moments that make up this situation will be made, emphasizing how graphic argumentation was a leading element in the construction of mathematical knowledge carried out by the students.

Due to the similarity of the answers given by all the groups, we will present some of the productions (group 5 and 6) that synthesize and better represent what was done by all the groups.

Moment 1
The students organized themselves to develop the classroom situation, and their first procedure was to specify where the 55 decibels were located (which represents the maximum noise established by Ministry of the Environment) in each of the sheets. Then, as a strategy, they drew a horizontal line at 55 decibels (y=55) in all the pictures of the situation (see Figure 3).

Constructing this horizontal line allowed the students to perform a graph comparison to determine the times when the song exceeded the norm and those times when it did not. It can be seen that, in this case, the students put into play the concept of linear inequality through arguments that emerged from the graph (see Figure 3).

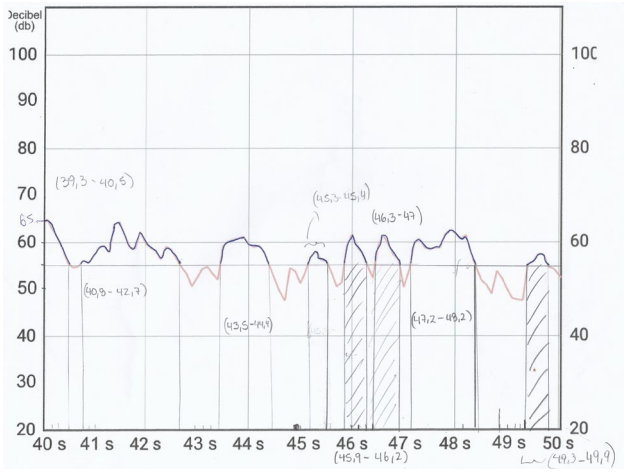

Figure 3: Moment 1 and the norm of the Ministry of the Environment (group 6).

Group 6 carried out a synthesis strategy of the six sheets to be able to construct a real line that represents the moments in which the song exceeded the norm (See Figure 5). In Figure 4 we can find a categorization of five sections, because in plate 2, which described the decibels of the song in seconds 10 to 20, the song did not exceed the norm.

The sections identified by group 6, allowed a glimpse of the elements of an interval, where the lower and upper limits were described, allowing students to identify all the moments in which the song exceeded the norm.

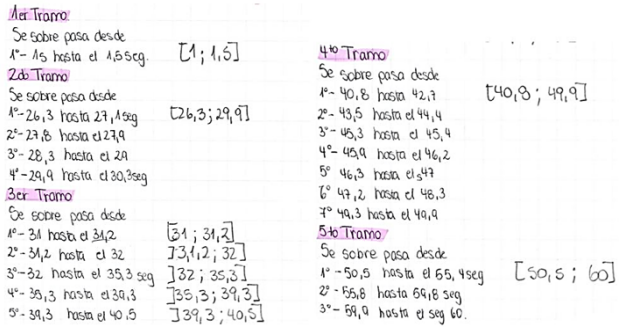

Figure 4: Identification of each moment (group 6).

In the second part of moment 1, the students had to represent each of the moments in which the song exceeded the norm in a real line, an example of this we see in Figure 5, where by means of a real line, it was represented the solution set of all the moments registered above the norm and that belong to the first 60 seconds of the song.

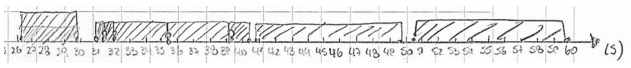

Figure 5: Real line that represents the time intervals in which the song exceeded the norm (group 6)

Moment 2
What was done at moment 1 was helpful for students to be able to answer and address what was required at moment 2.

At this time, it was sought that the students determine an approximation of the time that the DJ exceeded the norm established by the Ministry of the Environment.

Group 5 calculated the time in an approximate way, using the positive differences of the upper and lower limits of each of the intervals, a situation that can be observed in Figure 6.

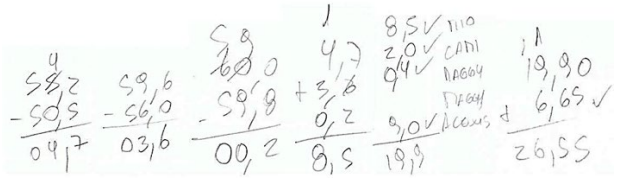

Figure 6: Calculation of the time in which the song exceeded the norm (group 5).

The students made the approximate calculation of what was requested, by adding all the differences of the limits of each interval, reaching, in the case of group 5, the result of 26.55 seconds in which the song exceeded the norm.

Moment 3
The argumentation that was born from the graphic allowed the students to make decisions about what they would say to the DJ or the host of the party, if they were the inspectors.
The responses of the students were varied, many alluded to the DJ receiving a fine, and others determined the importance of respecting the community by not emitting annoying noises and some referred to the consequences of noise pollution.
In accordance with [10], this research adopts the idea that in a specific situation a resignification of mathematical knowledge occurs when the participants develop a mathematics that is functional, giving evidence of this when studying the use of knowledge, which is developed in the situation or scenario in question. It should be noted that the concept of functional mathematics is understood in the sense of [5], where reference is made to that mathematics that serves students in training areas and professional settings to help them build and transform their lives. In this sense, the use of graphs allowed students, in a modeling context promoted by the designed classroom situation (Figure 1), to generate arguments to determine, through the graph comparison process, the time intervals in where the song exceeded the number of decibels established by standard (when $f(x)>g(x)$, where $f(x)$ is the function that models the decibels of the song, as a function of time, and $g(x)=55$), also constructing the solution set of this linear inequality on a real line (Figure 5). The foregoing allowed to show the development of a functional mathematics by the students and, therefore, a resignification of mathematical knowledge associated with linear inequalities.

4. CONCLUSIONS

The development of the classroom situation, proposed in this research, allowed the students themselves to build through the Sound Analyzer application, the graph of a function that describes the number of decibels of a certain song as a function of time. Subsequently, in a modeling process and through graphic argumentation, the students generated a resignification of mathematical knowledge associated with the concept of linear inequality, from functional justifications that emerged from them. The foregoing allows to provide elements for the construction of frames of reference that allow students to generate a resignification of mathematical knowledge associated with the concept of linear inequality, based on functional justifications.

The use of the graph as a source of argumentation allowed the graph to be given a different status from that traditionally established in school mathematics, which considers the graph as a representation of a function. From this, it is concluded that graphic argumentation should have a privileged status in teaching the concept of linear inequality, since this type of argumentation, in a modeling process, is a very useful tool for students to develop a functional mathematics, which has not been developed by the current educational system [10].

The situated nature of the classroom situation allows the student not only to resignify mathematical knowledge, but also to understand other types of knowledge inherent in the situation, such as the use of technology, the understanding of environmental laws and regulations and collaborative work, among others.

One element that led to the success of the designated classroom situation were the meanings and interpretations that the students gave to the resulting

regions after drawing lines on the Cartesian plane, since this allowed them to respond to the situation from the study of variation. sound. In this sense, the classroom situation allowed to promote meanings of the concept of linear inequality that traditionally do not appear in the teaching processes of said concept. In this sense, the classroom situation allowed to promote meanings of the concept of linear inequality that traditionally do not appear in the teaching processes of said concept.

It remains as a task for the educational community, the search for mechanisms that allow the incorporation of graphic argumentation and the theoretical modeling perspective presented in this research, in the teaching processes of the concept of linear inequality, to make this knowledge functional for the students.

Finally, it is expected that the educational community will continue with the development of research that will allow teachers to design and implement, in their mathematics classes, modeling processes that allow students to carry out a resignification of mathematical knowledge. In this direction, what has been done in research such as those carried out in [4] and [8] offer interesting socioepistemological perspectives on mathematical modeling that could serve as a reference for the development of future researches.

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