Time and Frequency Resource Allocation using Graph Theory in OFDMA Wireless Mesh Networks

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ABSTRACT

A wireless network with a mesh topology works reliably and offers redundancy. In modern broadband wireless mesh networks that use MIMO and OFDMA techniques, the problems of time, frequency, and space resource allocations are different from a cellular system and more complicated due to system architecture and distributed control and management. This paper focuses on the resource allocation problem of the OFDMA system and we define the term of separability order. For simple topologies like the square grid configuration, the allocations are simple and an optimal solution can be shown, but for an arbitrary architecture we need advanced tools and we use Graph Theory tools to present two different algorithmic solutions, to allow frequency reuse.

Keywords: Wireless mesh networks, OFDMA, resource allocation, graph colouring, graph theory, graph algorithms.

1. INTRODUCTION

A wireless mesh network (WMN) [1,2] is a communications network made up of radio nodes organized in a mesh topology usually composed of mesh clients, mesh routers and gateways. The WMN enables rapid deployment with lower-cost backhaul and good coverage. The interest in these networks increases in parallel to the developments of cellular systems and has its own values. A wireless network with a mesh topology is reliable and offers redundancy. Broadband modern networks use multiple antennas at both transmitter and receiver (MIMO - multiple input multiple output) and multicarrier transmission techniques (OFDMA - orthogonal frequency division multiple access) [3].

The allocation of frequency resources in OFDMA WMN has unique characteristics due to WMN spatial architecture which is different from cellular network spatial architecture and due to the fact that the OFDMA channel can be split in sub-bands of subcarriers. Methods of dynamic sub-carrier assignment in OFDMA WMN are shown in [8-10] using cross-layer optimization taking into account interference among links and adaptive power allocation and admission control. In [12] the graph theory is used for resource allocations, in this paper the concepts are reviewed, and new methods and results are added.

2. RESOURCE ALLOCATION

Resources and Separation Order

The resources to be shared considered in this paper are:

- Time: The system may operate with time slots division, requiring network synchronization.
- Frequency: The frequency division can be among different OFDMA bands, *inter-band*, or among different sub-carriers in the same OFDMA band, *intra-band*. A group of subcarriers dedicated to some purpose, such as transmission for a specific user forms a sub-band.
- Space: Frequency reuse can be made using spatial separation:
  - Among geographically separated links.
  - Among different transmission directions using MIMO or directional antennas.
  - Among different reception directions using MIMO or directional antennas.

We define [4] the time frequency separation order (TF-SO) λ_{TF} as the number of combinations of time slots and frequency bands which give practical separation among such combinations (not according to frequency sub bands). If N_T is the number of time slots and N_{FB} is the number of frequency bands, then TF-SO is given by:

$$\lambda_{TF} = N_T \cdot N_{FB} \qquad \text{Eq. (1)}$$

For example, with four time slots and two frequency bands, the TF-SO is $\lambda_{TF} = 8$.

With multiple antennas, the spatial domain can be used to create additional degrees of freedom for communications. Utilizing multiple antennas may result in additional spatial channels. The TF-SO may be modified to include the space separation. Assuming that for each terminal, we have directional antennas that divide the space in N_{sS} sectors, we can extend the TF-SO to time, frequency and space separation order (TFS-SO) λ_{TFS}, according to the number of combinations of time slots, frequency bands, and angular directions that offer practical separation among such combinations. The TFS-SO is given by:

$$\lambda_{TFS} = N_T \cdot N_{FB} \cdot N_S \qquad \text{Eq. (2)}$$

However, since the number of angular directions of each terminal is not fixed, the separation varies with time. Therefore, we must find an effective (average) TFS-SO factor with an effective number of sectors $N_{S.eff}$:

$$\lambda_{TFS.eff} = N_T \cdot N_{FB} \cdot N_{S.eff}. \qquad \text{Eq. (3)}$$

Separation Rules and Separation Order Extension

The OFDMA should be combined with TDMA to allow connectivity among nodes in WMN. There are N_{FB} frequency bands of OFDMA and in each frequency band, N_{FSB} frequency sub-bands and N_T time slots of TDMA. In each slot, all frequencies may be used according to non-collision rules:

- A node may not transmit and receive in the same time slot in the same frequency band.
- All transmissions from a node to other nodes in the same time slot in the same frequency band should have different frequency sub-bands.
- All receptions at a node from other nodes in the same time slot in the same frequency band should have different frequency sub-bands. This includes intentional transmissions to this node as well as interferences, i.e. receptions from transmissions to other nodes received from transmitters in the node's area of reception.

The *intra-band* division does not give the same level of separation as different time slots and different frequency bands due to the additional constraints imposed. The extension of the notion of separation order that includes the effect of the sub-bands is denoted as *extended* separation order of time and frequency λ_{xTF}, or as *extended* separation of time, frequency and space λ_{xTFS}. For example for the case when each band has the same number of sub-bands N_{FSB}, the extended separation order is given by $\lambda_{xTF}=N_{FSB}\cdot\lambda_{TF}$ and $\lambda_{xTFS}=N_{FSB}\cdot\lambda_{TFS}$..

The meaning of the extended separation order is the number of combinations of time slots, frequency bands and frequency sub-bands (and of angular directions) and each such combination is denoted as a *resource element* (or briefly - *elements)*.

Perfect Square Grid Example

A perfect square grid serves as an example of resource allocation. We denote the node coordinates as (*i,j*), where *i* and *j* are integers.

Assuming that the communication range R is $1<R<\sqrt{2}$, then only the four closest neighbors can communicate directly, which defines a perfect grid graph.

Lemma The minimum number of resource elements, to be allocated on the edges of the grid, without causing disturbances, is $\lambda_{xTF}=16$.

Proof First regard any edge of the grid, the 6 edges adjacent to its end-vertices, and an additional parallel edge, that is located right beyond that edge. If any pair of these edges is allocated the same element, then the two edges would be in disturbance, therefore, to avoid disturbances, we need to set different elements on those 8 edges. Since every bi-directed transmissions on an edge must use two different elements, then we need to allocate 8×2=16 different elements on those edges. Therefore, we need at least 16 different elements for every disturbance-free allocation of elements on the grid.

To terminate the proof, we present a disturbance-free allocation of one element on every edge of the grid, using 8 different elements (16 elements when doubling the required elements per edge).

***Allocation of elements on horizontal edges**:* For every *i*, *i*=0,1,2,3, we set the element *i* on the edges connecting the two coordinates (*i* mod 4, 0 mod 2) and (*i*+1 mod 4, 0 mod 2), and on the edges connection the two coordinates (*i*+2 mod 4, 1 mod 2) and (*i*+3 mod 4, 1 mod 2) .

Allocation of elements on vertical edges: For every *j*, *j*=0,1,2,3, we set the element *j*+4 on the edges connecting the two coordinates (0 mod 2, *j* mod 4) and (0 mod 2, *j*+1 mod 4), and on the edges connection the two coordinates (1 mod 2, *j*+2 mod 4) and (1 mod 2, *j*+3 mod 4) .

It is easy to verify that no pair of edges is in disturbance on this allocation, and thus, if the number of elements per edge is doubled, we get a non-disturbing allocation of 16 elements on the edges of the grid. □

The resource for a square grid is shown in Figure 1. The numbers 1 to 8 represents each two elements, one for each direction.

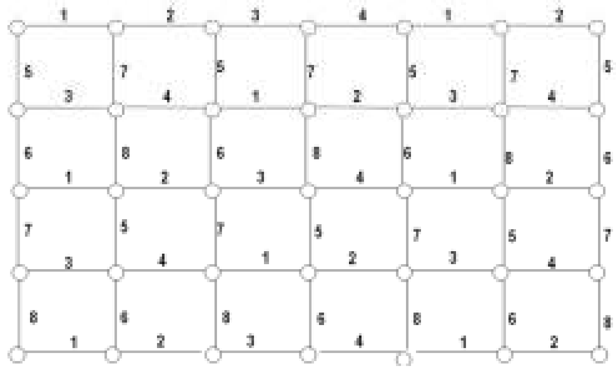

Figure 1. Optimal resource allocation for a square grid. Each of the numbers 1 to 8 represents two elements, one for each direction.

If the resource elements are combinations of time slot and frequency band there are no restrictions to allocations.

Often from practical considerations there is restriction to one frequency band. Therefore we have to use different time slots and may use sub division of the band into sub bands and limitations have to be taken into considerations according to the non-collision rules. For this case a solution will be described with four time slots and for sub-bands. We denote the two resource elements related to an horizontal link i as $\overrightarrow{i}$ and $\overleftarrow{i}$ and the two resource elements related to an vertical link j as $j\uparrow$ and $j\downarrow$. The active transmissions in each one of the four time slots are:

1. $\overleftarrow{1}$ and $\overrightarrow{2}$, $\overleftarrow{3}$ and $\overrightarrow{4}$
2. $\overleftarrow{2}$ and $\overrightarrow{3}$, $\overleftarrow{4}$ and $\overrightarrow{1}$
3. $7\uparrow$ and $8\downarrow$, $5\uparrow$ and $6\downarrow$
4. $8\uparrow$ and $5\downarrow$, $6\uparrow$ and $7\downarrow$

In each time slot there are four subband, which are the same subbands, e.g. $\overleftarrow{1}$, $\overleftarrow{2}$, $7\uparrow$, $8\uparrow$ occupy the same frequency subband. Therefore each node is transmitting in two time slots out of the four. The node with the links 1, 2, 8, 5, as an example transmits in slot 1: $\overleftarrow{1}$ and $\overrightarrow{2}$ and in slot 3: $7\uparrow$ and $8\downarrow$; slots 2 and 4 are used for reception. According to the links connected to each node there are 16 types of nodes and each node type have a different template of operation.

3. Formal Resource Allocation Problem

In the rest of the paper we use Graph Theory methods and terminology [5,6,7], where each terminal, having a transmitter and a receiver, becomes a vertex, and the links become edges.

Let V be a set of topologically allocated vertices, and R be an upper bound on the distance between any two communicating vertices, i.e. if the distance between any two vertices does not exceed R, then they can communicate, and thus we connect them by an edge.

To avoid collisions, for every two vertices u, and v, different resources are being used to broadcast from u to v and to broadcast from v to u. Hence, there cannot exist simultaneous broadcasting from u to v and from v to u , and thus, for every slot of time, we may disregard the direction of the broadcasting, and use undirected graphs to describe the connections between the vertices of V. Therefore, the set of vertices, V, and the upper bound on the distance, R, induce an undirected graph $G = (V , E(V,R))$ were E denotes the set of edges connecting the vertices V within the distance R.

A resource allocation on G defines a set of *resource elements*, such that every edge of G uses one resource element of the set in each direction. A resource element stands for any combination of elementary resources: It can be regarded as a combination of time slots, band frequencies, sub-carriers, etc. Hence, it can represent pairs of the type {*time, band*}, {*time, sub-band*} etc.

The resource elements allocated on the edges of G must follow the following rules:

- For every vertex $v\in V$, all the simultaneous broadcasting from v, and to v, are mutually disturbing, and thus should be allocated by different resource elements.
- Moreover, if a vertex x transmits to a vertex y, then y is disturbed by the transmissions of x to other vertices, and it may disturb y to receive transmission from other vertices.

A more restrictive rule may be added to allow autonomous management of links between two nodes by the nodes it links by allowing the resources allocated to the link to be used in both directions according to the traffic between these nodes.

Therefore, the edges adjacent to x and the edges adjacent to y must all be allocated by different resource elements. We thus define two edges (u,v), and (x,y), with resource elements f_1, and f_2 respectively, to be *mutually undisturbed* if and only if they satisfy the following demands:

- Vertices $u, v, x,$ and y, are all distinct, and
- Vertices u and v are not neighbours of vertices x, and y.

Obviously, if edges (u,v), and (x,y), are not mutually undisturbed, then they are *mutually disturbed*, and every pair of transmissions, set on this pair of edges, is also *mutually disturbed*.

Our goal is to allocate as few different resource elements as possible to the set of edges of the graph G, such that no two transmissions are mutually disturbed.

In the following two chapters, Chapter 4 and Chapter 5, we present two algorithms of two distinct approaches for solving this problem (however none of the algorithms is preferable). The first algorithm paints closed vertices (neighbours and neighbours of neighbours) with different colours, and allocates different resource elements to their edges. The second algorithm takes advantage of the radius of the *reach range* to allocate different elements to the edges in that range, while farther edges repeat using the same set of elements (and their vertices are regarded as being differently coloured).

4. First Resources allocation algorithm

The first algorithm, (A1), uses the Graph Theory term "Coloring of Vertices" to set elements only on edges that are "far enough" from each other.

Algorithm 1 (A1)

Input:

- A set V of topologically allocated vertices V,
- An upper bound on the distance R
- A set F of elements (Assume that F is large enough)

1) *Build the graph $G = (V,E)$ from V according to R (as suggested on the problem description)*
2) *Build the graph $G' = (E,E')$ from G such that for every $e_1,e_2\in E$, $(e_1,e_2)\in E'$ if and only if e_1,e_2 are mutually disturbed.*
3) *Colour the edges of E by using as few colours as possible, such that the end vertices of every edge of G' are coloured differently. (Notice that even if the vertices of G' are randomly coloured properly, then every vertex is set by a colour that is different from the colours of its neighbours, and thus no more than $\Delta(G')+1$ colours are needed to colour properly the vertices of G' , where $\Delta(G')$ is the maximum degree of G' [5,6]).*
4) The colours of E in G' are the colours of E in G .

Output: A resource element on every edge of G .

Notice that every pair of edges, that was set by the same element, is mutually undisturbed, since G' was built accordingly

Obviously, if the graph G' is properly coloured by the minimum number of colours, then the algorithm may set as few elements as possible to the edges of G , since that colouring of G' ensures that the same elements are set properly as many times as possible. Unfortunately, the problem of properly colouring the vertices of an undirected graph with the minimum number of colours, is known to be NP-hard [11], that is, mathematicians believe that no polynomial time algorithm exists for solving that problem. We believe that this is also the status of our proposed Resources Allocation Problem. Therefore, we must settle with the random colouring of the vertices of G', that ensures the use of no more than $\Delta(G')+1$ colours. The result $\Delta(G')+1$ is a small enough number of elements to be set on the graph G .

The following Example, (E1), illustrates how Algorithm (A1) runs on a given undirected graph. Notice the two suggested colourings of E of G' in step 3: The first randomly uses the alphabetic names of E, and needs 10 colours to colour E properly , while the second detects the existence of a triangle of G' with a "crowded" neighbourhood, (i.e. many edges are adjacent to the vertices of the triangle), and starts the colouring there, resulting the optimal 9 colours for the proper colouring of E. (Notice that the vertices of that triange - emphasized by yellow - and its neighbours, must all have distinct colours. Therefore, the 9 colours, needed to colour them, are the minimum number of colours in a proper colouring of E) .

Unfortunately, detecting maximum cliques (i.e. subgraphs of maximum number of vertices, where every pair of edges is connected by an edge), is also known to be NP-hard [11], therefore it is pointless to add the detection of crowded maximum cliques to the algorithm.

Example 1 (E1)

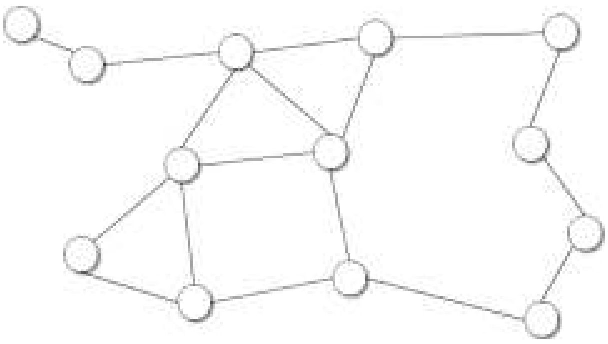

Figure 2. *Algorithm 1, step 1: The given graph G=(V,E)*

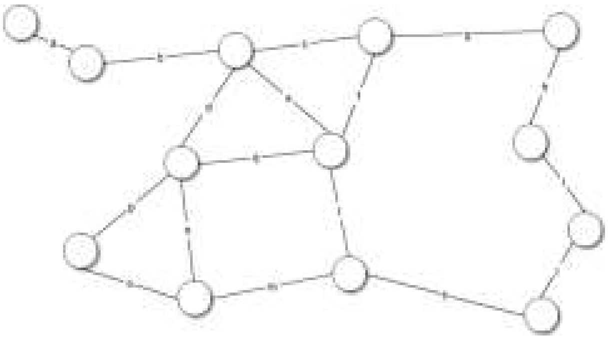

Figure 3. *Algorithm 1, step 2: Numbering the edges of G*

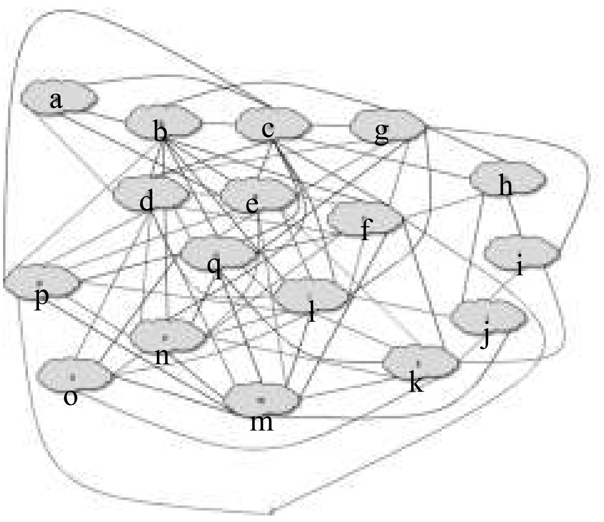

Figure 4. *Algorithm 1, step 2: The graph G'*

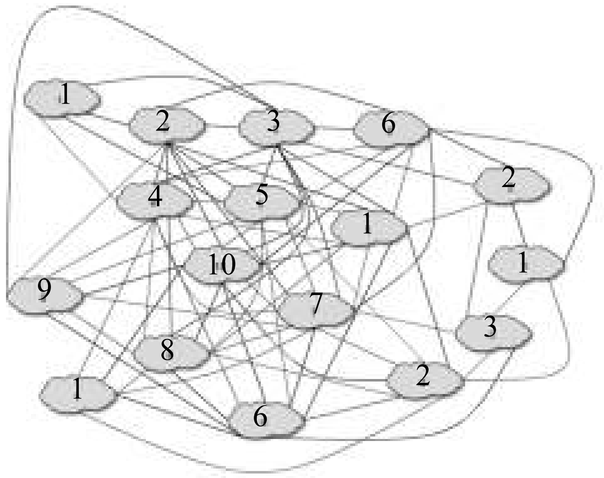

Figure 5. *Algorithm 1, step 3: Colouring E in G' in alphabetic order – Colouring with 10 colours*

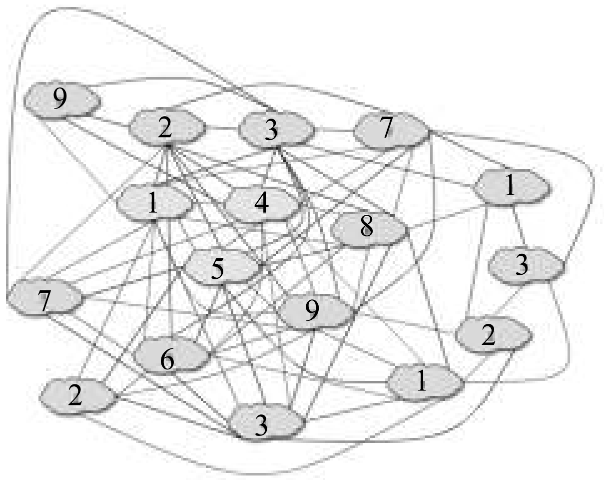

Figure 6. *Algorithm 1, step 3: Colouring E in G', starting with the emphasized triange and its neighbours – Optimal colouring with 9 colours*

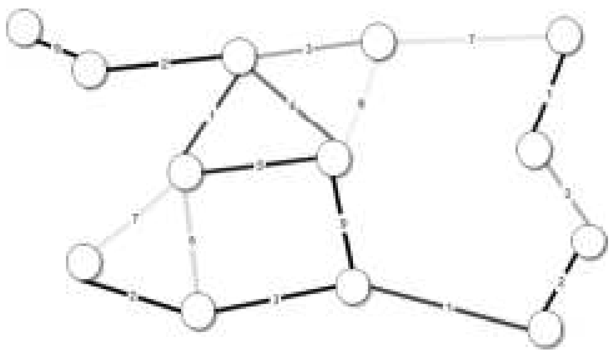

Figure 7. *Algorithm 1, step 4: The resulting allocation - Optimal colouring of E in G*

5. SECOND RESOURCES ALLOCATION ALGORITHM

Our second algorithm, (A2) , is a topological algorithm, that uses an algorithm we denote as *Common Elements Algorithm* (CEA), to draw as many non tangent circles of diameter R as possible, on the area of the graph G. Every two edges of different circles, cannot be mutually disturbed, and thus can be set the same element. Therefore, the same scope of elements can be set to the edges of each such circle.

Algorithm 2 (A2)

Input:

- A set V of topologically allocated vertices V ,
- An upper bound on the distance R
- A set F of elements (Assume that F is large enough)

1) Build the graph $G = (V,E)$ from V according to R (as suggested on the problem description)
2) As long as $E \neq \varnothing$,
a) Run the following algorithm, (CEA) , on G
b) Subtract the elements, being used by (CEA) of previous step, from F
c) Delete the edges, that received elements by (CEA) of previous step, from E
d) Delete vertices, that became isolated after the previous step

Output: The elements on all edges of G .

Following is the sub-algorithm that is used by (A2):

Common Elements Algorithm (CEA)

Input:

- An undirected graph $G=(V,E)$
- A set $F = \{f_1,f_2,\ldots,f_r\}$ of elements (Assume that F is large enough)

1) As long as $E \neq \varnothing$,
a) Choose a random vertex $x \in V$,
b) Denote by B the area of the circle centered in x with the radius R
c) Denote by A the set of edges of E with at least one vertex in B
d) Set on each edge of A a different element from $\{f_1,f_2,\ldots,f_{|A|}\}$ (If sub-bands are used, then regard their vertices as being differently coloured) .
e) Denote by U the set of all end vertices of the edges of A
f) Denote by H the set of edges of E with at least one vertex in U
g) $V \leftarrow V\text{-}U$, $E \leftarrow E\text{-}H$

Output: Subgraphs $G_1=(V,H)$, and $G_2=(V,E\text{-}H)$, such that elements are allocated to the edges of H and no elements are allocated to the edges of $E\text{-}H$.

Notice that each iteration of (CEA) determines a new circle of diameter R, and allocates the edges of that circle, the same scope of elements as were set to the circles of the previous steps of that algorithm. Since two such circles are "far enough" from each other, no two edges, with a common element, are mutually disturbed.

Moreover, every call of (CEA) uses a different scope of elements, and thus two adjacent edges are allocated with different elements, and thus are not mutually disturbed.

The following Example, (E2), illustrates how Algorithm (CEA) runs on a given undirected graph.

Example 2 (E2)

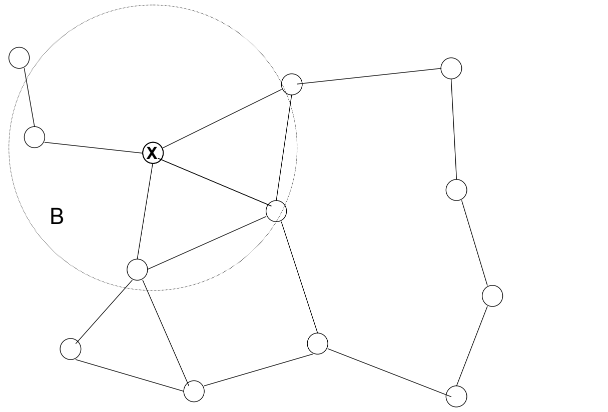

Figure 8. *Algorithm 2, steps a,b: The circle B around the centre x*

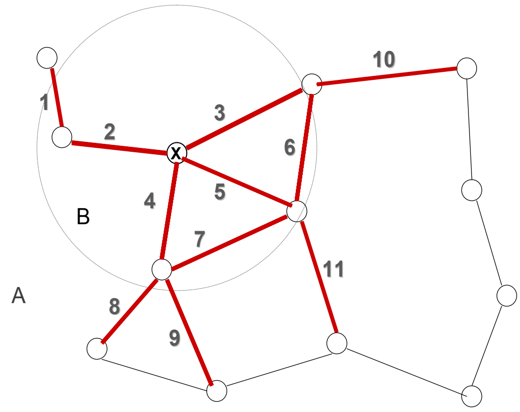

Figure 9. *Algorithm 2, steps c,d: The set A and the different allocations*

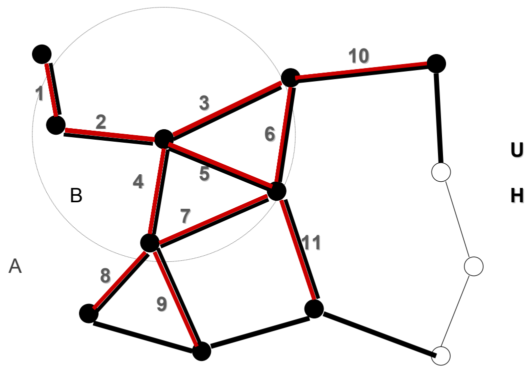

Figure 10. *Algorithm 2, steps e,f: The sets U and H*

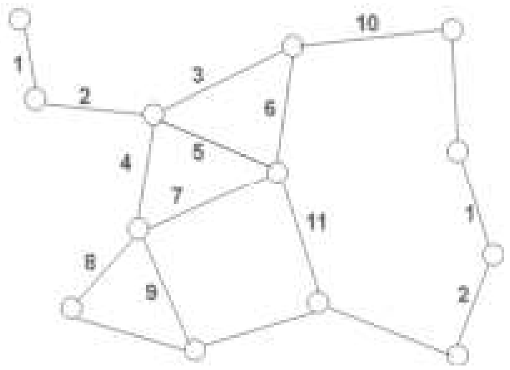

Figure 11. *Algorithm 2, results for first run of Algorithm CEA*

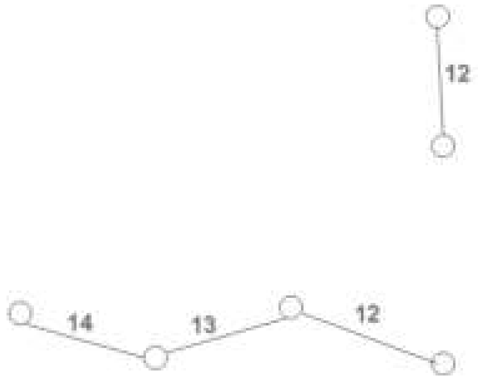

Figure 12. *Algorithm 2, results for second run of CEA*

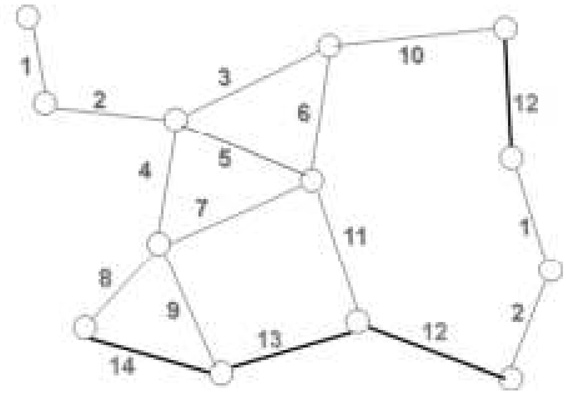

Figure 13. *Result of Algorithm 2 : Union of results of all CEA*

6. SUMMARY

The paper is focused on time, frequency and space resource allocations in WMN and defines the separation order of a system. The main contribution of the paper is in presenting algorithms, based on graph theory, that provide solutions to frequency and time resource allocations that can be used in various ways in OFDMA WMN.

As previously mentioned, the results of the algorithm can be used in several ways: One option allocates the same elements set of the form *{time, band}* to the edges of all the vertices with the same colour. Another option allocates the same combination of time slot and band, *{time, band}*, to all vertices coloured with the same colour, while their edges share the same set of sub-bands, for transmissions directed from the vertices to their neighbours. Therefore, each colour presents a different combination of a time slot and a band. Therefore, the first options sets pairs *{time, band}* to the edges, while the second option sets pairs *{time, sub-band}* to the edges of the graph. In both cases the pairs stand for the *elements.*

Note that each edge can be used to transfer to both directions, but not in the same time slot or not using the same band. Thus it should be allocated by two different elements. Therefore, regard every element *f* of the resulting algorithms, as a pair of elements *(f, f ')*, one for each direction of the edge.

Although Example (E1), on the first algorithm, results in the optimal solution for the graph of the example, these two algorithms are not necessarily optimal, but they result in a small number λ_{TF} of combinations of time slots and frequencies, i.e. a small number of pairs of the form *(time, frequency)*.

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