Conflict Resolution and Consensus Development Among Inherently Contradictory Agents Using Fuzzy Linguistic Variables

Terrence P. Fries
Department of Computer Science
Indiana University of Pennsylvania
Indiana, PA 15705 USA

ABSTRACT

Decision making based upon the recommendations of multiple intelligent agents has become common in various applications. However, difficulty arises when the agents have quite different recommendations. Many methods have been proposed that attempt to resolve conflicting opinions multiple, heterogeneous agents in decision making. However, all of these methods require that the agents negotiate until they arrive at a consensus opinion. These do not provide for the cases in which the agents have contradictory opinions that cannot be compromised. In certain cases, agent opinions will conflict due to the nature of the agents' viewpoints. By forcing compromise or neglecting selected conflicting opinions, valuable information may be lost that adversely affect the decision. This paper proposes a method by which a consensus decision can be developed while not requiring that the individual agents abandon their opinions.

Keywords: Multi-Agent Systems, Consensus Development, Fuzzy Linguistic Variables, Fuzzy Aggregation, Fuzzy Ranking

1. INTRODUCTION

The use of multiple agents systems has become widespread for a variety of applications, including electronic business, communications, robot navigation, factory control, scheduling, human-computer interaction, and active networks providing customized packet processing. However, a problem arises when autonomous heterogeneous agents provide contradictory opinions. To address this, researchers have developed a number of methods for conflict resolution among agents [1-5]. The primary approach to conflict resolution is negotiation. These negotiation methods assume that the agents have some common ground on which to adjust their opinions to produce a compromise. This becomes problematic when the agents inherently have conflicting opinions due to their very nature. As an example, consider agents used to route packets in a wide area network. One agent may be concerned with producing the fastest transmission, while another agent has the goal of minimizing the cost of transmission. Inherently, these agents will likely produce contradictory recommendations due the difference in their goals. The opinions provided by both agents are valuable and must be considered in decision making regarding routing. Contradictory opinions cannot be discarded or compromised without losing essential information in the decision making process. However, negotiation methods would compromise the opinions of both conflicting agents. Therefore, alternative methods must be investigated to resolve conflicts without the loss of information.

The conflict resolution approach presented in this research involves the use of fuzzy sets to represent agent opinions. The fuzzy set representation allows the combination of disparate opinions without the loss of information. It also accounts for imprecision of data and incomplete data used by the each of the agents in forming an opinion. The conflict resolution and decision making approach assumes that agents with varying viewpoints will analyze the available data and provide a recommendation for each alternative. Decision making is a two-step process in which the opinions of all agents on a particular alternative are aggregated, or combined, and the resulting recommendations for each alternative are ranked for use in decision making.

Section II presents an overview of fuzzy set theory for those unfamiliar with it. In Section III, a method of developing a consensus among agents with disparate views is presented. Section IV provides a discussion of the effectiveness of the proposed approach and examples of successful implementations of the method presented here.

2. FUZZY INTELLIGENT AGENTS

Fuzzy Set Theory

In many real world situations, concepts cannot be clearly classified into one class, at the total exclusion of all others. For example, the concept of "hot" cannot be defined using mathematical terms due to its varying interpretations by different people. Traditionally, membership of object *x* in a class, *A*, is expressed as a binary value using the membership function in Eq. (1).

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \text{ is an element of set } A \\ 0 & \text{if } x \text{ is not an element of set } A \end{cases} \quad (1)$$

The membership function can also be expressed as a functional mapping which maps a set *A* to 0 or 1 as shown in Eq. (2).

$$\mu_A(x) \quad : \quad A \rightarrow \{0,1\} \quad (2)$$

For example, if *x* is a temperature, then the membership function maps it to be either a member or not a member of the *hot* class. A problem arises in defining the membership function because people have differing opinions as to what temperature is hot. A person from the arctic may consider 30ºC to be a hot day, while someone from an equatorial region may not consider a day to be hot until it reaches 35ºC or higher. Fuzzy logic was introduced by Lotfi Zadeh [6, 7] to address such disparate opinions. A fuzzy set, or fuzzy class, is described by a membership function that defines a degree of membership of an object, *x*, in a set, *A*. The membership function provides a mapping to any real number in the range 0 to 1, inclusive, as shown in Eq. (3).

$$\mu_A(x) \quad : \quad A \rightarrow [0,1] \quad (3)$$

The membership function may be shaped as a ramp, Gaussian curve, sigmoidal shape, or any other continuous function on the interval [0, 1]. For example, the membership function for the concept of *hot* is shown in Figure 1. This membership is be defined as in Eq. (4).

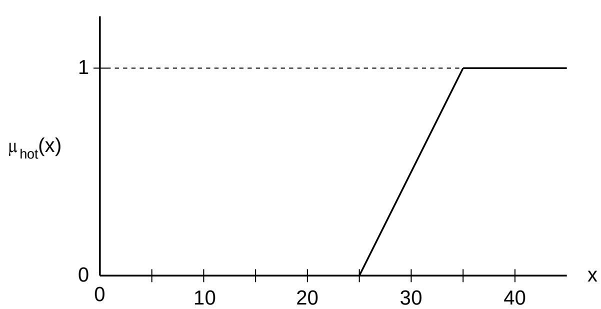

Figure 1. Fuzzy membership function for hot

$$\mu_{hot}(x) = \begin{cases} 0 & \text{if } a \le 25 \\ (x\text{-}25)/(x\text{-}35) & \text{if } 25 < x < 35 \\ 1 & \text{if x} \ge 35 \end{cases} \quad (4)$$

For this membership function, a temperature of 25 has no membership (*x* value of 0.0) in *hot*, while 30 has a 0.5 degree of membership, and 35 has full membership (*x* value of 1.0). In a similar manner, linguistic variables such as hot, far, near, fast, and costly can also be expressed as fuzzy sets. This allows the expression of membership to employ linguistic variable that are much more understandable to humans who are developing and evaluating the applications.

Special Case Fuzzy Sets

Researchers have found it convenient to limit membership functions to a specific shape to simplify their use in a particular application. The most common membership functions are triangular and trapezoidal fuzzy sets.

A triangular fuzzy set has a triangular-shape convex membership function and is denoted by (*a*, α, β) and defined as where a is the center of the triangle and α and β define the left and right vertices, respectively, or the left and right spreads as shown in Figure 2. A triangular fuzzy number membership function is defined as

$$\mu_A(x) = \begin{cases} 1-|a-x|/\alpha & \text{if } a-\alpha \le x \le a \\ 1-|a-x|/\beta & \text{if } a \le x \le a+\beta \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

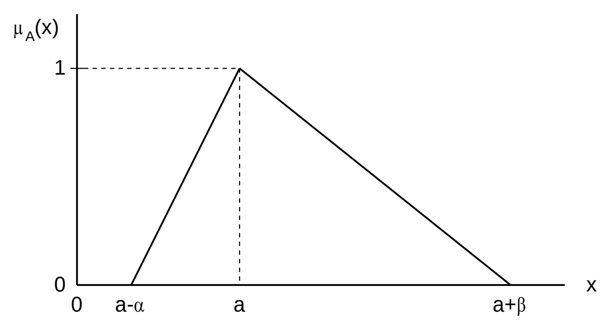

Figure 2. Triangular fuzzy number denoted by the tuple (a, α, β)

A trapezoidal fuzzy number (TFN) is a convex trapezoid denoted by the 4-tuple (*a*, *b*, *c*, *d*) where *a*, *b*, *c*, and *d* denote the critical points of the trapezoid as shown in Figure 3. The membership function of a trapezoidal fuzzy number is defined in Eq. (6).

$$\mu_A(x) = \begin{cases} (x-a)/(b-a) & \text{if } a < x < b \\ 1 & \text{if } b \le x \\ (d-x)/(d-c) & \text{if } c < x < d \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

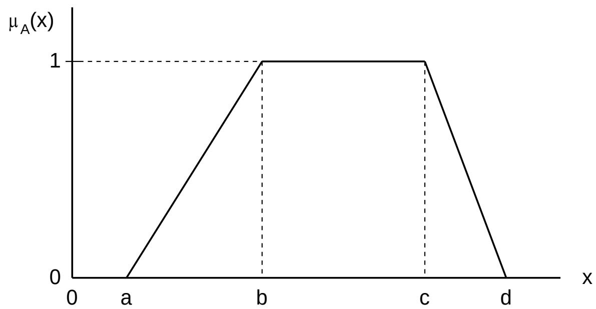

Figure 3. Trapezoidal fuzzy number (TFN) denoted by the 4-tuple (a, b, c, d)

Representing Fuzzy Opinions in Intelligent Agents

It has become common to represent agent opinions in a multi-criteria decision making system using fuzzy sets [8-10]. For this research, agent opinions are assumed to be represented by trapezoidal fuzzy numbers (TFNs). TFNs were chosen because they reduce the computational complexity of numerical calculations required for aggregation and ranking of agent opinions. In addition, they provide membership functions that can express full membership ($\mu_A(x) = 1$) for all or any portion of the universe of discourse, A, as desired. An agent's recommendation for a particular alternative can be expressed using the linguistic variables: *very unlikely*, *unlikely*, *possible*, *likely*, and *very likely*. These linguistic variable can be represented as TFNs as shown in Figure 4. Other sets of linguistic variables may also be used. For example, in a test case using multiple fuzzy agents for robot navigation (see Section 4), the following set of linguistic variables was used to describe the various potential directions of movement for the robot: *left*, *left-center*, *straight*, *right-center*, *right*, *back*, etc.

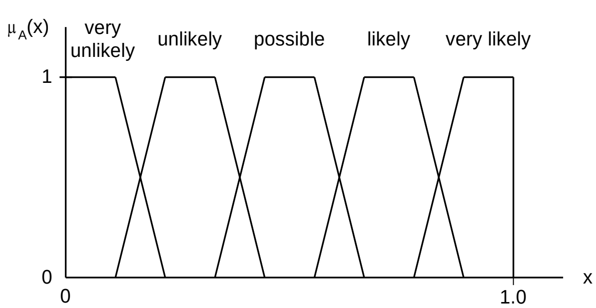

Figure 4. Representing agent opinions as linguistic variables using TFNs

3. CONFLICT RESOLUTION

In developing a consensus among intelligent agents, it is assumed in this research that a real time response is required. In many multiple agent applications, it is infeasible to wait long periods of time for decisions to be made. For example, the routing of data packets or access to critical information cannot wait for extensive computation. Therefore, a restriction is placed on the decision making process that it provides a real time response and, as a result, have limited computational complexity.

The conflict resolution and decision making approach presented in this paper assumes that agents with varying viewpoints will analyze the available data and provide a recommendation for each alternative. Decision making is a two-step process: (1) agent opinions are aggregated for a particular alternative are aggregated, or combined, and (2) the resulting recommendations for each alternative are ranked for use in decision making.

First, the recommendations of each agent for a particular alternative must then be combined into a single opinion on that alternative. Using the fuzzy recommendations, the aggregate opinion will be expressed in terms of a fuzzy number. Second, the aggregated recommendations for each alternative must be ordered so that the best one is selected based on the agent opinions. The best alternative is the consensus of the agents. The ordering of fuzzy numbers is not straight forward as with crisp numbers, such as 2, 5, and 8, due to the varying degrees of membership. Aggregation and ranking approaches are presented below for determining the best alternative based on the consensus of agent opinions.

Aggregation of Fuzzy Opinions

A number of aggregation approaches have been proposed [8-13]. Some aggregation methods require that the fuzzy opinions have some intersection so that they are not totally out of agreement. If the opinions do not have some agreement, the agents negotiate until they can arrive at a consensus. However, the agents assumed in this research may purposely have disparate recommendations due their divergent viewpoints. Other methods that have been proposed are computationally complex which violates the real-time response requirement. Therefore, none of these methods are appropriate.

Hsu and Chen [14] present an aggregation method using a similarity matrix that exhibits the similarities between the opinions of experts. The matrix operations are significantly faster than other approaches that rely on complex equations. However, Hsu and Chen require that all opinions for a particular option have a common intersection at some α-level cut. This research modifies Hsu and Chen's aggregation method to allow opinions that do not intersect.

When at agent opinions for an alternative intersect at some α-level cut, the similarity matrix approach is used to aggregate the intersecting opinions. When there is no common intersection among agent opinions, weighted linear interpolation is used to aggregate the opinions for each alternative. Each agent, i, is assigned a rating, r_i. The most important agent is given a rating of 1 and the others are given ratings less than one in relation to their importance.

The degree of importance normalizes the ratings and is defined in Eq. (7).

$$w_i = \frac{r_i}{\sum_{i=1}^{n} r_i}, \quad i = 1,2,\ldots,n \tag{7}$$

The aggregated fuzzy opinion for alternative k is formed as a TFN tuple (a^*, b^*, c^*, d^*) using the formulas in Eq. (8).

$$a^* = \sum_{i=1}^{n} w_i a_i$$

$$b^* = \sum_{i=1}^{n} w_i b_i \tag{8}$$

$$c^* = \sum_{i=1}^{n} w_i c_i$$

$$d^* = \sum_{i=1}^{n} w_i d_i$$

where

n	is the number of agents with opinions on alternative k
w_i	if the degree of importance of agent i
(a_i, b_i, c_i, d_i)	is the TFN opinion of agent i for alternative k

The resulting aggregated opinion, (a^*, b^*, c^*, d^*), can be defined as in Eq. (9).

$$\tilde{R} = \sum_{i=1}^{n} w_i (\cdot) \tilde{R}_i \tag{9}$$

where $(\cdot)$ is the fuzzy multiplication operator and $\tilde{R}_i = (a_i, b_i, c_i, d_i)$.

Ranking of Fuzzy Opinions for Decision Making

Once the opinions of the agents have been aggregated to produce a consensus opinion for each alternative, the best alternative must be selected. However, the opinions are expressed as fuzzy numbers and cannot be immediately compared. Researchers have proposed a number of methods for ranking fuzzy numbers [15-19]. Many fail to distinguish between fuzzy numbers with identical modes and symmetric spreads. While others cannot distinguish between fuzzy numbers with identical modes and symmetric spreads, thus, they favor numbers with larger spreads over smaller spreads. This is counterintuitive, since larger spreads indicate more uncertainty in the opinion. All but Nakamura's [19] method lack a mechanism to adjust favoritism toward larger or smaller spreads.

This research modifies Nakamura's [19] fuzzy preference function so that it can differentiate between fuzzy numbers with identical modes and symmetric spreads, and uses it to rank fuzzy opinions. Nakamura's approach compares each pair of fuzzy opinions using a fuzzy preference function which takes into account the Hamming distance of each fuzzy number to the fuzzy minimum and to the fuzzified best and worst states. The pairwise comparisons are then used to rank the fuzzy opinions. The new fuzzy preference function compares each fuzzy opinion to an "ideal" fuzzy number which represents the case where the opinion is "very likely." This eliminates the problem Nakamura's method suffers when comparing fuzzy numbers with identical modes and symmetric spreads. Elimination of the pairwise comparisons significantly reduces the number of calculations required for Nakamura's method to n calculations for n nodes using the new method presented in this research. This research has shown that the order of computational complexity is reduced from $O(n^2)$ for Nakamura's method to $O(n)$ using the new method. The new method also simplifies the ranking of the opinions. Nakamura's method only provides preferences for pairs of fuzzy numbers, therefore, the preference function for each pair of fuzzy numbers must be evaluated. It is then necessary to evaluate all of the pairwise comparisons to provide a ranking. Since, in the new method, the fuzzy opinions are compared with a "very likely" fuzzy number, they all already ranked in comparison to this value and the process of determining the ranking based on pairwise comparisons is eliminated. The result of each fuzzy preference calculation for each node provides its ranking. The new fuzzy preference function comparing opinion A_i and the very likely mode, V, replaces the second fuzzy opinion with V and is defined in Eq. (10).

$$\mu_p(A_i, V) = \begin{cases} \frac{1}{\Delta_\alpha}[\alpha\, D(\underline{A}_i, \underline{A}_i \wedge \underline{V}) \\ \quad + (1-\alpha)\, D(\overline{A}_i, \overline{A}_i \wedge \overline{V})] & if\ \Delta_\alpha \neq 0 \\ 1/2 & if\ \Delta_\alpha = 0 \end{cases} \tag{10}$$

where

$$\Delta_\alpha = \alpha[D(\underline{A}_i, \underline{A}_i \wedge \underline{V}) + D(\underline{V}, \underline{A}_i \wedge \underline{V})] + (1-\alpha)[D(\overline{A}_i, \overline{A}_i \wedge \overline{V}) + D(\overline{V}, \overline{A}_i \wedge \overline{V})]$$

The notation $\bar{A}$ is the greatest upper set of A defined in Eq. (11).

$$\mu_{\overline{A}}(y) = \sup_{\{x|x \geq y\}} \mu_A(x), \qquad \forall y \in U \tag{11}$$

$\underline{A}$ is the greatest lower set of A defined in Eq. (12).

$$\mu_{\underline{A}}(y) = \sup_{\{x|x \leq y\}} \mu_A(x), \qquad \forall y \in U \tag{12}$$

$A_i \wedge V$ is the extended minimum defined in Eq. (13).

$$\mu_{A_i \wedge V}(z) = \sup_{\{x,y|x \wedge y = z\}} [\mu_{Ai}(x) \wedge \mu_V(y)], \forall z \in U \tag{13}$$

and $D(A_i, V)$ is the Hamming distance between A_i and V, defined by Eq. (14).

$$D(A_i, V) = \int_S |\mu_{Ai}(x) - \mu_V(x)|\, dx \tag{14}$$

The new fuzzy preference function can be simplified by showing that $D(A_i, V)$ when V is a TFN defined as (a, b, 1, 1). Therefore, if V is represented by (a, b, 1, 1), the revised new fuzzy preference function used to compare opinion A_i with the very likely mode, V, is defined as

$$\mu_p(A_i, V) = \begin{cases} \frac{1}{\Delta_\alpha} \alpha\, D(\underline{A}_i, \underline{A}_i \wedge \underline{V}) & if\ \Delta_\alpha \neq 0 \\ 1/2 & if\ \Delta_\alpha = 0 \end{cases} \tag{15}$$

where

$$\Delta_\alpha = \alpha[D(\underline{A}_i, \underline{A}_i \wedge \underline{V}) + D(\underline{V}, \underline{A}_i \wedge \underline{V})] + (1-\alpha) D(\overline{V}, \overline{A}_i \wedge \overline{V})$$

4. IMPLEMENTATION AND TESTING

Comparison of Ranking with Current Methods

In section 3, the basic limitations of many current ranking methods were discussed. The proposed ranking method was compared with some of the more commonly used methods, each of which suffers from significant shortcomings. Yager's F1 [20] and F3 [21] indices, Kerre's method [22], and Nakamura's fuzzy preference function [19] are unable to distinguish between fuzzy numbers with identical modes and symmetric spreads as shown in Figure 5. Though Yager's F4 index [21] is able to distinguish between fuzzy numbers with identical modes and symmetric spreads, it provides a more favorable rank to those with larger spreads. This is counter-intuitive because larger spreads indicate a greater uncertainty resulting in high ranking for opinions which exhibit a great deal of skepticism. In this case, higher ranking of the narrower spread is more commonly preferred because that opinion has greater certainty. A more preferable option would be to allow the ability to modify favoritism for wider or narrower spreads based upon the particular application and characteristics of the agents expressing those opinions. Only Nakamura's method provides the ability to adjust favoritism toward smaller or larger spreads; however, it suffers from an increased computational complexity..

Yager's F1, F3, and F4 indices, Kerre's approach, Nakamura's method, and the new fuzzy preference function were used to rank five test cases shown in Figures 5-9 and results are shown in Table 1. In Figure 5 (Example 1), only Yager's F4 and the new preference function are able to correctly distinguish between fuzzy numbers with identical modes and symmetric spreads. However, the F4 index favored the larger spread which represents greater uncertainty in the fuzzy number. The other methods rank the 2 fuzzy numbers as equal. The new method correctly ranked the fuzzy numbers in Figure 6-8 (Examples 2-4). The case in Fugure 9 (Example 5) compared a fuzzy number, A, with a large spread and higher mode against one with a smaller spread and lower mode, B. The F3 and F4 indices and the new approach all favored the smaller spread which represented greater certainty in the opinion expressed by the expert. These tests demonstrate that the method correctly ranked the fuzzy more numbers in a manner superior to the others.

Table 1. Comparison of Ranking Methods

Method	Examples				
	1	**2**	**3**	**4**	**5**
Yager F1	A=B	A<B	A<B	A>B	A>B
Yager F3	A=B	A<B	A<B	A>B	A<B
Yager F4	A>B	A<B	A>B	A>B	A<B
Kerre	A=B	A<B	A<B	A>B	A>B
Nakamura with α=0.5	A=B	A<B	A<B	A>B	A>B
New FPR	A<B	A<B	A<B	A>B	A<B

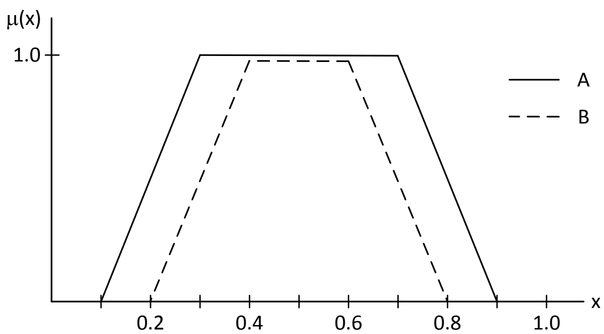

Figure 5. Example 1 - TFNs with identical modes and symmetric spread.

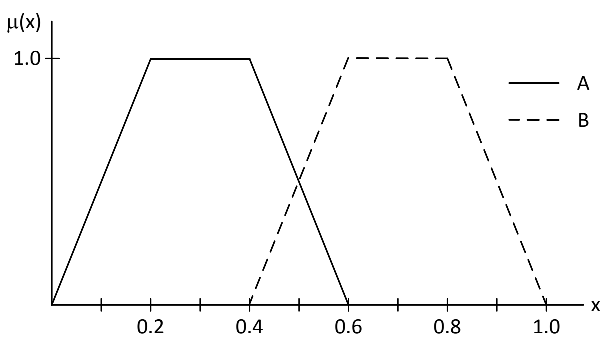

Figure 6. Example 2 - TFNs with different modes

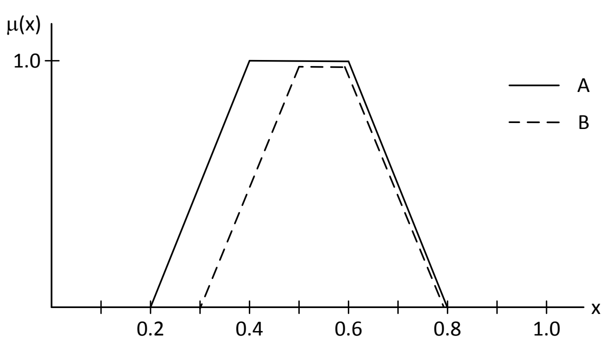

Figure 7. Example 3 - TFNs with same right-hand-side

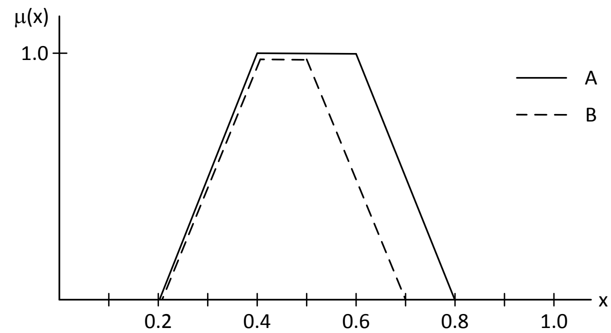

Figure 8. Example 3 - TFNs with same left-hand-side

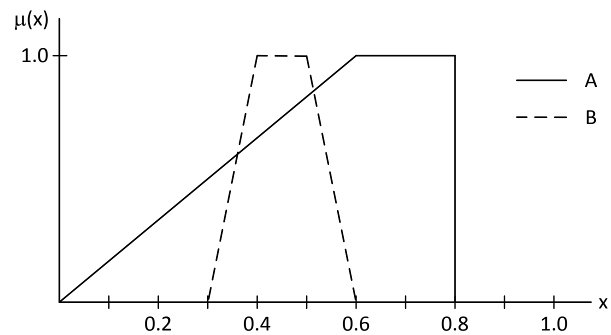

Figure 5. Example 5 - TFNs different modes and spreads

Case Studies

In order to test the validity of the proposed conflict resolution and ranking approach, the method was introduced into several multiple criteria decision making case studies. Each case uses multiple fuzzy agents which may produce disparate opinions.

An agent-based diagnostic system utilizing fuzzy linguistic variables and the conflict resolution method described in the previous section has been implemented and tested. The diagnostic system was implemented for a manufacturing testbed consisting of two conveyors on which pallets are transported. The workcell had two robots that interact with the system and seven stations to simulate manufacturing processes including assembly, material handling, and inspection. The agents used in this diagnostic system were employed with expertise that considers recency of faults, frequently occurring faults, minimization of the resource cost to examine possible fault sources, mean-time-to-failure of components, and cyclic failures. The agent-based diagnostic system increased diagnostic accuracy, while reducing by an average of 91% the time to make a successful diagnostic determination [23].

A navigation system for a Pioneer 2-DX mobile robot has been implemented using the conflict resolution method presented in this paper. It uses autonomous agents that express their opinions using linguistic fuzzy numbers. This system reduced the total distance traveled by 18% over a similar system not using the conflict resolution method [24].

5. CONCLUSIONS

The conflict resolution method for multiple agent systems presented in this paper has been shown to perform well in several test applications. The approach allows the inclusion of disparate agent opinions in the consensus. Additional work is in progress on developing a method that allows implementation of various negotiation approaches without sacrificing the ability to retain and utilize disparate information.

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