A Newsvendor Model
with Initial Inventory and Two Salvage Opportunities

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ABSTRACT

In this paper, we develop an extension of the *newsvendor* model with initial inventory. In addition to the usual quantity ordered at the beginning of the horizon and the usual quantity salvaged at the end of the horizon, we introduce a new decision variable: a salvage opportunity at the beginning of the horizon, which might be used in the case of high initial inventory level. We develop the expression of the optimal policy for this extended model, for a general demand distribution. The structure of this optimal policy is particular and is characterized by two threshold levels. Some managerial insights are given via numerical examples.

Keywords: *Newsvendor* model, initial inventory, lost sales, salvage opportunities, concave optimisation and threshold levels.

1. INTRODUCTION

The single period inventory model known as the *newsvendor* model is an important paradigm in operations research and operations management literature. It has had numerous important applications, as in style-goods products (fashion, apparel, toys, etc.) or in services management (booking on hotels, airlines, etc).

Many extensions have been proposed in order to include specific additional characteristics in the original *newsvendor* model. The literature concerning the *newsvendor* model is thus very large (for extensive literature reviews, see for example [1]). Generally speaking, a *newsvendor* model is characterized by three elements : the objective function, the demand characterization and different financial flow specifications. Most of the studies about the *newsvendor* model focus on the computation of the optimal order quantity that maximizes the expected profit (or minimizes the expected cost) [2]. Nevertheless, some other works consider other criteria, such as maximizing the probability of achieving a target profit [1]. The demand process can be considered exogenous [2] or price-sensitive [3]. Financial flows generally introduced in the *newsvendor* problem are the wholesale price, the selling price, the salvage value and the shortage penalty cost. Many extensions exist, such as a fixed ordering cost [4] or a dynamic selling price [5].

Other authors consider the *newsvendor* problem with a multiplicative neutral independent background risk in an expected utility framework [6]. Some studies treat the *newsvendor* model in a loss aversion or risk-averse framework [7].

In some extensions, other decision variables or parameters have been considered. For example, [8] have analyzed a *newsvendor* model with an initial inventory. In this extension, the decider observes, at the beginning of the selling season, the initial inventory level and fixes his decisions as a function of this initial inventory. These authors have shown that in this case the optimal order quantity can be deduced from the classical model (without initial inventory). [9] has considered a similar model in which the vendor, after observing the demand value, can carry out partial returns or additional orders in the limit of defined levels. [10] has studied the multi-product *newsvendor* problem with value-at-risk considerations.

In the present paper, we develop a new extension of the *initial inventory newsvendor* model in which a part of the initial inventory can be salvaged at the beginning of the selling season. As a matter of fact, when the initial inventory level is sufficiently high, it may be profitable to immediately salvage a part of this initial inventory to a parallel market, before the season. This is an extension of the classical model in which the unique salvage opportunity is placed at the end of the selling season.

In many practical situations, a potential interest exists for such a salvage opportunity before the selling season. For example, if a first quantity is ordered from the supplier a long time before the season, due to very long design/production/delivery lead-times, the demand distribution is not precisely known at the date of the order [11]. In this case, if the demand appears to be particularly low, it could be profitable to return a part of the received quantity to the supplier or sell it to a parallel market, with a return price which is lower than the order price. In this paper, we establish that the optimal policy corresponding to our model is a threshold based policy with two different thresholds: the first corresponds to the order-up-to-level policy of the classical model with initial inventory, and the second threshold corresponds to a salvage-up-to-level policy, and is a result of the salvage opportunity at the beginning of the season. Between the two thresholds, the optimal policy consists of neither ordering, nor salvaging any quantity.

The remainder of this paper is structured as follows. In the following section, we introduce the model, describe the decision process and define the notation used in the paper, the objective function and the model assumptions. In Section 3, we show some of our model properties, we solve the model and exhibit the structure of the optimal policy as a function of the initial inventory level. In section 4 we give some managerial insights via numerical applications. The last section is dedicated to conclusions and presentation of new avenues of research.

2. THE MODEL PARAMETERS

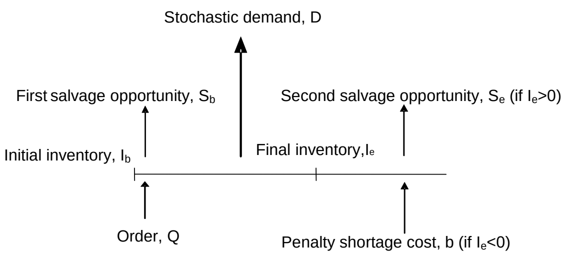

Figure 1: Ordering and selling process

A manager has to fill an inventory in order to face a stochastic demand. The ordering and selling processes are as depicted in Figure 1.

Before occurrence of the demand, an initial inventory is available. Without loss of generality and because it is more coherent with the main idea of the present paper, this inventory is assumed to be positive. Note however that a model with a negative initial inventory can be also developed, which would correspond to situations with some *firm orders* received before the beginning of the selling season. At the beginning of the season, the manager can make two decisions: first, he can sell a part of this initial inventory to a parallel market and/or second, he can order a new quantity to complete the initial inventory in order to better satisfy the future demand. After demand has occurred, the remaining inventory, if any, is salvaged or the unsatisfied orders, if any, are lost and, in this case, a shortage penalty cost is paid.

The decision and state variables corresponding to this problem (according to Figure 1) are denoted as follows: I_b: the initial inventory level, available at the beginning of the selling season; Q: the ordered quantity, which is to be received before the demand occurs; S_b: the quantity salvaged at the beginning of the season, before the demand occurs; I_e: the inventory level at the end of the selling season; S_e: the quantity that is salvaged at the end of the selling season.

We also define the following parameters:

D: the random demand, which is characterized by a continuous probability density function $f(\cdot) : [0, \infty[\to \mathbb{R}^+$ and by the cumulative distribution function $F(\cdot) : [0, \infty[\to [0, 1]$; p: the unit selling price during the season; s_b: the unit salvage value for the quantity S_b; c: the unit order cost for the quantity Q; s_e: the unit salvage value of the quantity S_e; b: the unit shortage penalty cost.

As mentioned above, the objective function of the model consists of maximizing the total expected profit, denoted as $\Pi(I_b, Q, S_b, S_e)$. This expected profit, with respect to the random variable D, is explicitly given by

$$\begin{aligned} &\Pi(I_b, Q, S_b, S_e) = \\ &s_b S_b - cQ + s_e S_e + p\int_0^{I_b+Q-S_b} Df(D)\,dD \\ &+p(I_b + Q - S_b)\int_{I_b+Q-S_b}^{\infty} f(D)\,dD \\ &-b\int_{I_b+Q-S_b}^{\infty} (D - I_b - Q + S_b)f(D)\,dD. \qquad (1) \end{aligned}$$

The different terms can be interpreted as follows: $s_b S_b$, is the profit generated by salvage at the beginning of the season; cQ, is the order purchase cost; $s_e S_e$, is the profit generated by salvage at the end of the season; the fourth and fifth terms are the expected sales; the last term is the expected shortage penalty cost.

It is worth noting that equivalent models can be built with a cost minimization criterion ([1] and [12]). The decision variables have to satisfy the following constraints,

$$0 \leq Q, \qquad (2)$$

$$0 \leq S_b \leq I_b, \qquad (3)$$

$$0 \leq S_e \leq I_e. \qquad (4)$$

Some assumptions are necessary to guarantee the interest and the coherency of the model, as in the classical *newsvendor* model. These assumptions can be summarized in the following inequations:

$$0 < s_e < s_b < c < p \qquad (5)$$

Note that s_b and s_e can be negative, which corresponds to a situation where a cost is charged in order to dispose of the material.

3. THE MODEL

The Model Properties

In this section we consider the model described in Section 2 and we show the concavity of its expected objective function with respect to the decision variables, which permits to explore the structure of the optimal policy.

Property 1 *The objective function $\Pi(I_b, Q, S_b, S_e)$, defined in Eq. (1) is a concave function with respect to Q, S_b and S_e.*

Proof 1 *The hessian of $\Pi(I_b, Q, S_b, S_e)$ with respect to Q, S_b and $S_e$is given by*

$$\nabla^2\Pi(I_b, Q, S_b, S_e) = -(b+p)f(I_b + Q - S_b)\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad (6)$$

From the model assumptions (Eq. (5)), for each vector $V = (V_1, V_2, V_3) \in I\!R^3$ we find

$$V^T \nabla^2 \Pi(I_b, Q, S_b, S_e) V = -(b+p) f(I_b + Q - S_b)(V_1 - V_2)^2 \leq 0, \qquad (7)$$

which proves that the matrix $\nabla^2 \Pi(I_b, Q, S_b, S_e)$ is semi-definite negative. Consequently, the objective function $\Pi(I_b, Q, S_b, S_e)$ is jointly concave with respect to Q, S_b and S_e.□

Lemma 1 *The optimal value of the decision variable S_e is given by*

$$S_e^* = \max(0; I_e) \qquad (8)$$

Proof 2 *It could be easily shown that the first partial derivative of the expected objective function $\Pi(I_b, Q, S_b, S_e)$ with respect to S_e is given by*

$$\frac{\partial \Pi(I_b, Q, S_b, S_e)}{\partial S_e} = s_e. \qquad (9)$$

From assumption (5), one concludes that $\Pi(I_b, Q, S_b, S_e)$ is an increasing function in S_e. Thus the optimal value of S_e, considering the constraint (4), is $\max(0; I_e)$.□

From Lemma 1, one concludes that the optimal value of S_e depends on I_e. However, at the beginning of the selling season, I_e is a random variable. Hence, if S_e is substituted by its expected optimal value in Eq. (1), then the expected profit function for the model described in Figure 1 becomes

$$\begin{aligned} \Pi(I_b, Q, S_b) &= s_b S_b - cQ + \\ & s_e \int_0^{I_b+Q-S_b} (I_b + Q - S_b - D) f(D)\, dD \\ & + p \int_0^{I_b+Q-S_b} D f(D)\, dD \\ & + p(I_b + Q - S_b) \int_{I_b+Q-S_b}^{\infty} f(D)\, dD \\ & - b \int_{I_b+Q-S_b}^{\infty} (D - I_b - Q + S_b) f(D)\, dD. \qquad (10) \end{aligned}$$

It is worth noting that this expected objective function depends only on I_b, Q and S_b. Therefore $Q^*(I_b)$ and $S_b^*(I_b)$, the optimal values of the decision variables Q and S_b, are the solution of the optimisation problem

$$(Q^*(I_b), S_b^*(I_b)) = \arg\left\{\max_{0 \leq Q, 0 \leq S_b \leq I_b} \{\Pi(I_b, Q, S_b)\}\right\}, \qquad (11)$$

where $\Pi(I_b, Q, S_b)$ is given in Eq. (10).

Since the objective function $\Pi(I_b, Q, S_b)$ is concave with respect to the decision variables Q and S_b, hence one could use the first order optimality criterion in order to characterize the optimal policy.

Optimality Conditions for Q^*

Consider the partial derivative of $\Pi(I_b, Q, S_b)$ with respect to Q

$$\frac{\partial \Pi(I_b, Q, S_b)}{\partial Q} = -c + b + p + (s_e - b - p) F(I_b + Q - S_b) \qquad (12)$$

For any given S_b value satisfying $0 \leq S_b \leq I_b$, the optimal ordering quantity $Q^*(I_b)$ is a function of $I_b - S_b$ that can be computed as the solution of the following optimization problem

$$Q^*(I_b) = \arg\left\{\max_{0 \leq Q} \{\Pi(I_b, Q, S_b)\}\right\}. \qquad (13)$$

By concavity of $\Pi(I_b, Q, S_b)$ with respect to Q, and for any given S_b value, the optimal solution $Q^*(I_b)$ is given either by

$$Q^*(I_b) = 0 \qquad (14)$$

if $-c + b + p + (s_e - b - p) F(I_b - S_b) \leq 0$, or by

$$Q^*(I_b) = F^{-1}\left(\frac{b + p - c}{b + p - s_e}\right) - I_b + S_b \geq 0 \qquad (15)$$

if $-c + b + p + (s_e - b - p) F(I_b - S_b) \geq 0$.

Optimality Conditions for S_b^*

The partial derivative of $\Pi(I_b, Q, S_b)$ with respect to S_b is given by

$$\frac{\partial \Pi(I_b, Q, S_b)}{\partial S_b} = s_b - b - p + (b + p - s_e) F(I_b + Q - S_b). \qquad (16)$$

For any given Q value satisfying $0 \leq Q$, the optimal ordering quantity $S_b^*(I_b)$ is defined as the solution of the following optimization problem

$$S_b^*(I_b) = \arg\left\{\max_{0 \leq S_b \leq I_b} \{\Pi(I_b, Q, S_b)\}\right\}. \qquad (17)$$

By concavity of $\Pi(I_b, Q, S_b)$ with respect to S_b, and for any given Q value, the optimal solution $S_b^*(I_b)$ is given either by

$$S_b^*(I_b) = 0 \qquad (18)$$

if $s_b - b - p + (b + p - s_e) F(I_b + Q) \leq 0$, or by

$$S_b^*(I_b) = F^{-1}\left(\frac{b + p - s_b}{b + p - s_e}\right) - I_b - Q \geq 0 \qquad (19)$$

if $s_b - b - p + (b + p - s_e) F(I_b + Q) \geq 0$.

Critical Threshold Levels

From the above optimality conditions, two threshold levels appear to be of first importance in the optimal policy characterization,

$$Y_1^* = F^{-1}\left(\frac{b + p - c}{b + p - s_e}\right) \quad \text{and} \quad Y_2^* = F^{-1}\left(\frac{b + p - s_b}{b + p - s_e}\right), \qquad (20)$$

with, from assumption (5), are related by:

$$Y_1^* \leq Y_2^*. \qquad (21)$$

These threshold levels can be interpreted as values such as

$$-c + b + p + (s_e - b - p) F(Y_1^*) = 0, \qquad (22)$$

and

$$s_b - b - p + (b + p - s_e) F(Y_2^*) = 0. \qquad (23)$$

As the function $F(\cdot)$ is monotonously increasing, for any I_b values such that $I_b < Y_1^*$ (resp. $I_b > Y_1^*$), one finds

$$-c+b+p+(s_e-b-p)F(I_b) > 0$$
$$(\text{resp. } -c+b+p+(s_e-b-p)F(I_b) < 0)\,, \quad (24)$$

and for any I_b values such that $I_b > Y_2^*$ (resp. $I_b < Y_2^*$), one finds

$$s_b-b-p+(b+p-s_e)F(I_b) > 0$$
$$(\text{resp. } s_b-b-p+(b+p-s_e)F(I_b) < 0)\,. \quad (25)$$

Critical Threshold Levels and Structure of the Optimal Policy We show below that the structure of the optimal policy is in fact fully characterized by these two threshold levels as depicted in Figure 2.

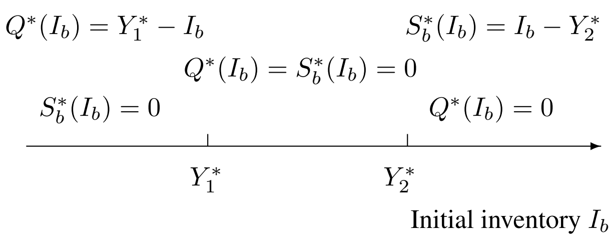

Figure 2: Structure of the optimal policy

Lemma 2 *For $Y_1^* \le I_b \le Y_2^*$, the optimal solution is given by*

$$Q^*(I_b) = S_b^*(I_b) = 0. \quad (26)$$

Proof 3 *For $Y_1^* < I_b < Y_2^*$, one finds*

$$\frac{\partial \Pi(I_b,0,0)}{\partial Q} < 0 \text{ and } \frac{\partial \Pi(I_b,0,0)}{\partial S_b} < 0, \quad (27)$$

which induces, by concavity, that the solution $Q^*(I_b) = S_b^*(I_b) = 0$ is the optimum of the profit function for these I_b values. If $Y_1^* = I_b$, one finds

$$\frac{\partial \Pi(I_b,0,0)}{\partial Q} = 0 \text{ and } \frac{\partial \Pi(I_b,0,0)}{\partial S_b} < 0, \quad (28)$$

which leads to the same conclusion. If $Y_2^* = I_b$, one finds

$$\frac{\partial \Pi(I_b,0,0)}{\partial Q} < 0 \text{ and } \frac{\partial \Pi(I_b,0,0)}{\partial S_b} = 0, \quad (29)$$

which leads to the same conclusion.□

Lemma 3 *For $I_b \le Y_1^*$, the optimal solution is given by*

$$Q^*(I_b) = Y_1^* - I_b \text{ and } S_b^*(I_b) = 0. \quad (30)$$

Proof 4 *For $I_b \le Y_1^*$, one finds that*

$$\frac{\partial \Pi(I_b, Y_1^*-I_b, 0)}{\partial Q} = 0 \text{ and}$$
$$\frac{\partial \Pi(I_b, Y_1^*-I_b, 0)}{\partial S_b} < 0 \quad (31)$$

which induces, by concavity, that the solution $Q^*(I_b) = Y_1^* - I_b$ and $S_b^*(I_b) = 0$ is the optimum of the profit function for such I_b values.□

Lemma 4 *For $Y_2^* \le I_b$, the optimal solution is given by*

$$Q^*(I_b) = 0 \text{ and } S_b^*(I_b) = I_b - Y_2^*. \quad (32)$$

Proof 5 *For $Y_2^* \le I_b$, one finds that*

$$\frac{\partial \Pi(I_b, 0, I_b - Y_2^*)}{\partial Q} < 0 \text{ and}$$
$$\frac{\partial \Pi(I_b, 0, I_b - Y_2^*)}{\partial S_b} = 0 \quad (33)$$

which induces, by concavity, that the solution $Q^*(I_b) = 0$ and $S_b^*(I_b) = I_b - Y_2^*$ is the optimum of the profit function for such I_b values.□

As in the classical *Newsvendor* model, it follows from the previous derivations that the optimal policy does not explicitly depend on the pair (p,b) but only on the sum $p+b$. In particular, a model with a unit selling price p and a penalty cost $b > 0$ is equivalent to a model with a penalty cost $b' = 0$ and a unit selling price $p' = p + b$.

4. NUMERICAL EXAMPLES AND INSIGHTS

The fundamental properties of the considered model will be illustrated by some numerical examples. In a first example, we illustrate the structure of the optimal policy as a function of the initial inventory I_b. Then we exhibit, via a second numerical example, the impact of the demand variability on the structure of optimal policy. A third example illustrates the effect of s_b, the salvage value at the beginning of the horizon. In the last example, we compare the considered extended model with the classical *newsvendor* model with initial inventory, and we show the potential benefit associated with the initial salvage process.

For these numerical applications, we assume that the demand has a truncated-normal distribution, corresponding to a normal distributed demand, $D \sim N[\mu,\sigma]$ truncated at the zero value (we exclude negative demand values). Without loss of generality we also assume that the inventory shortage cost is zero, namely $b = 0$.

In the following figures, $Q^*(I_b)$ and $S_b^*(I_b)$ represent the optimal values of the decision variables, and $E[S_e^*(I_b)]$ is the *expected* optimal value of the decision variable $S_e(I_b)$, which is given by

$$E[S_e^*(I_b)] = \int_0^{I_b+Q^*(I_b)-S_b^*(I_b)} \Big(I_b +$$
$$Q^*(I_b) - S_b^*(I_b) - D\Big) f(D)\, dD. \quad (34)$$

This is to account for the fact that the variables Q and S_b are decided before the demand is known while the variable S_e is decided after the demand is realized.

Optimal Policy

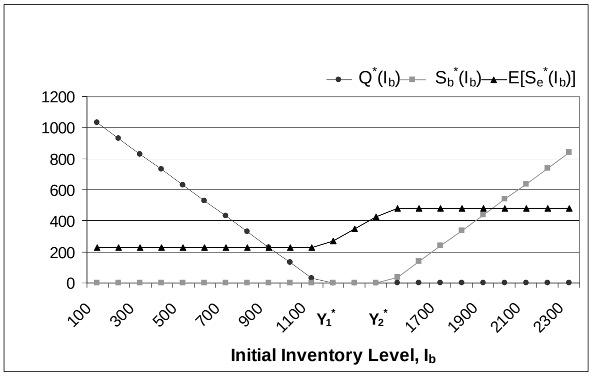

Figure 3: Optimal policy

In this first example, we depict the behaviour of the optimal decision variables as a function of the initial inventory I_b. The numerical values for the parameters are the following: $\mu = 1000$, $\sigma = 400$, $p = 100$, $s_b = 30$, $c = 50$ and $s_e = 20$.

The two thresholds $Y_1^* = 1127$ and $Y_2^* = 1460$ have been represented in Figure 3. For $Y_1^* \leq I_b \leq Y_2^*$, one has $Q^*(I_b) = S_b^*(I_b) = 0$, while $E[S_e^*(I_b)]$ is increasing. For $I_b < Y_1^*$, Q^* decreases linearly as a function of I_b, which corresponds to the order-up-to-level policy defined in Section 3. For $I_b > Y_2^*$, $S_b^*(I_b) > 0$ is a linear increasing function of I_b, which corresponds to the salvage-up-to-level policy defined in Section 3.

Variability Effect

In this extended model, the decision variables Q and S_b are fixed before demand occurrence and are thus, in one way or another, affected by demand variability. On the other hand, the decision variable S_e is fixed once the demand is perfectly known. From an intuitive point of view, the more variable the demand the more profitable is postponement of the decisions.

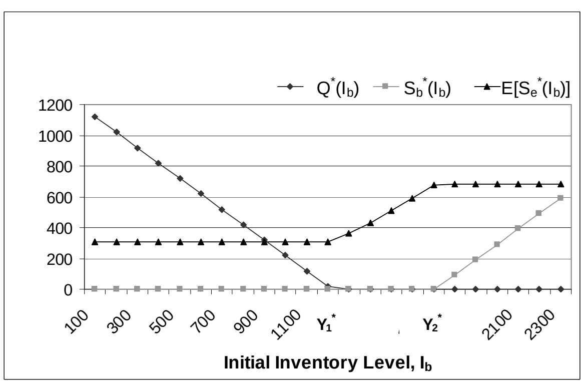

Figure 4: Optimal policy for high demand variability

In order to show the consequence of the variability, we compare the example of Figure 3 with two other examples with higher (Figure 4) and lower (Figure 5) variability. All the numerical parameters are the same as in the first example, except demand variability. The demand standard deviations are respectively $\sigma = 600$ for the second example in Figure 4 and $\sigma = 200$ for the third example in Figure 5.

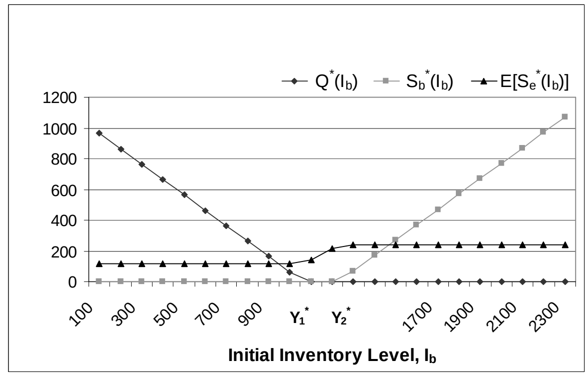

Figure 5: Optimal policy for low demand variability

The thresholds are $Y_1^* = 1191$ and $Y_2^* = 1690$ for Figure 4 and $Y_1^* = 1064$ and $Y_2^* = 1230$ for Figure 5. It can be seen that the values of both thresholds increase with the standard deviation of the demand. The increase of Y_1^* is in accordance with standard *newsvendor* results with normally distributed demands, where for given costs, the order quantity increases linearly with the demand standard deviation. The same justification is valid for the increase of Y_2^*. This increase in the Y_1^* and the Y_2^* values is accompanied by an increase of the optimal Q^* or the decrease of the S_b^* for a given initial inventory value.

The increase of the optimal Q value permits the manager, for a given initial inventory, to stock a bigger quantity to face demand variability. The same is true for the decrease of the optimal S_b value.

The increase in the Y_1^* and Y_2^* values is accompanied by an increase in the difference $Y_2^* - Y_1^*$. In the interval $[Y_1^*, Y_2^*]$, the value of $E[S_e^*(I_b)]$ increases with I_b. The fact that the interval width $Y_2^* - Y_1^*$ increases with demand standard deviation leads to an increase of $E[S_e^*(I_b)]$, which may be interpreted as a postponement of the decision until season's end.

Effect of Initial Salvage Value s_b

In this section, we study the effect of the s_b value on the optimal policy. We compare the nominal example (defined in Section (4), Figure 3), with two other examples with different s_b values. We consider for the first example a high s_b value, i.e. $s_b = 35$, and for the second a low s_b value, i.e. $s_b = 25$. By (20), it is explicit that Y_1^* does not depend on s_b. By (20) also, it is also explicit that Y_2^* is a decreasing function of s_b, as it is the case for the optimal policy behaviour. For $s_b = 35$, we find $Y_1^* = 1127$ and $Y_2^* = 1355$, while for $s_b = 25$, these values become $Y_1^* = 1127$ and $Y_2^* = 1614$.

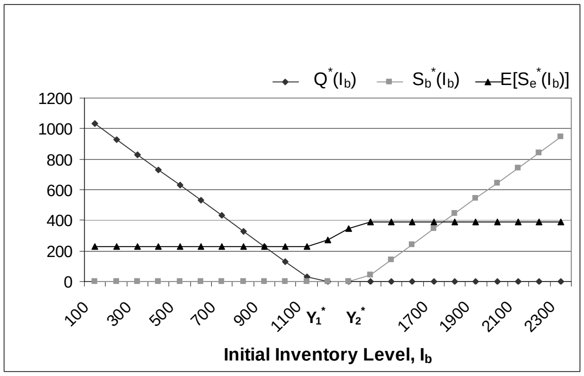

Figure 6: Optimal policy for high s_b value

An increase of s_b automatically induces a decrease of Y_2^*, which means that for a given value of the initial inventory I_b, the salvaged quantity $S_b^*(I_b)$ will increase. This increase will be accompanied by a decrease of the expected value of $S_e^*(I_b)$. This can be summarized as follows: the higher the salvage value of the parallel market, the higher the salvaged quantity $S_b^*(I_b)$ and the lower the expected salvaged quantity at the end of the season $E[S_e^*(I_b)]$.

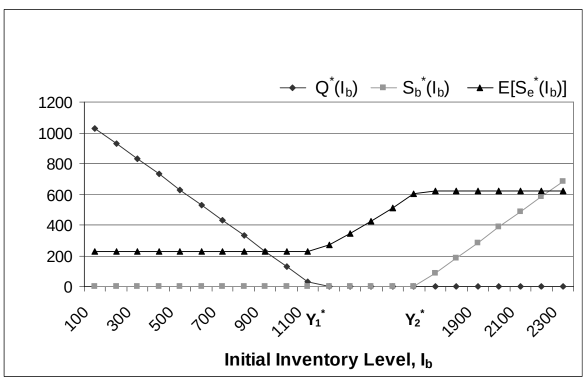

Figure 7: Optimal policy for low s_b value

Numerical Comparison with the Classical *Newsvendor* Model with Initial Inventory

Our extended model introduces the additional variable S_b, which appears to be useful in presence of high initial inventory level. In order to illustrate the magnitude of the benefits potentially associated with S_b, we compare our model with the classical initial inventory *newsvendor* model, where $s_b = 0$. For the same numerical parameters values, we have measured the relative difference between the expected objective (profit) functions of the two models. We have considered three values of the salvage value s_b for our model: the nominal value, $s_b = 30$; a high value, $s_b = 35$; a low value, $s_b = 25$. The comparison is shown in Figure 8.

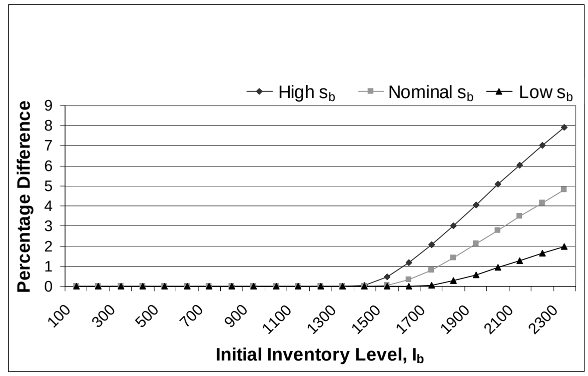

Figure 8: Comparison with the classical *newsvendor* model

Figure 8 shows that the benefits associated with the S_b variable can be non-negligible for high values of I_b. Clearly, it is equal to zero for the I_b values that are less than Y_2^*, where $S_b^* = 0$. Via Figure 8, one may conclude that:

- the difference, between the two expected optimal objective functions, is greater for high s_b values. This increase corresponds logically to the fact that the s_b term only appears in the objective function of the extended model and not in the *newsvendor* model.
- the threshold Y_2^* decreases with s_b. For high s_b value, the difference becomes positive.

This can be summarized as follows: the extended model is profitable for high s_b values and/or high I_b values.

5. SUMMARY AND CONCLUSIONS

This paper presents a new extension to the initial inventory *newsvendor* model in which a part of the initial inventory can be salvaged to a parallel market before demand occurrence. We have shown that in the case of a high initial inventory level, or a high initial salvage value s_b, this feature can be useful. The structure of the optimal policy is characterized by two threshold levels. Via numerical applications, we have illustrated the theoretical properties and given some managerial insight.

The extension of this model to a multi-periodic framework or to a model with pricing decisions is an ongoing research avenue.

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