Topology Optimization of Structure Using Differential Evolution

Chun-Yin Wu
Department of Mechanical Engineering, Tatung University,
Taipei, 104, Taiwan, R.O.C

and

Ko-Ying Tseng
Department of Mechanical Engineering, Tatung University,
Taipei, 104, Taiwan, R.O.C

ABSTRACT

The population-based evolutionary algorithms have emerged as powerful mechanism for finding optimum solutions of complex optimization problems. A promising new evolutionary algorithm, differential evolution, has garnered significant attention in the engineering optimization research. Differential evolution has the advantage of incorporating a relatively simple and efficient form of mutation and crossover. This paper aims at introducing differential evolution as an alternative approach for topology optimization of truss and continuous structure with stress and displacement constraints. In comparison the results with other studies, it shows that differential evolution algorithms are very effective and efficient in solving topology optimization problem of structure.

Keywords: differential evolution, topology optimization, truss design.

1. INTRODUCTION

Design optimization of structure has been an interesting area of research in the field of engineering optimization and there are still many studies making notable progress for last decade. Optimization of structures can be classified into three categories: sizing, shaping, and topology optimization. In the topology optimization, it is concerned with the structure members and connectivity between members. In general, it is easily represented by discrete variables rather than by those used for continuous optimization problems. Topology optimization is the most difficult and complex among three categories and it is special useful in developing innovative conceptual designs.

In the recent years, optimization has become one of the most important topics of engineering applications. The demanding computational cost for engineering optimization is often very high because the analysis for engineering model takes lots of time in finding required data for calculating objective function of optimization problem. Various mathematical programming methods have been used to solve engineering optimization problems. But those methods need to find the first or second order differentiation that will increase the difficulty in searching optimum solution.

Besides, the mathematical programming methods are easily to fall into local optimum. In order to improving efficiency in global optimization search of engineering problems, many heuristic algorithms have been developed such as genetic algorithms[6-8], ant algorithm[5], evolutionary algorithm[3]. Especially the genetic algorithms have been broadly applied in solving various structural optimization problems[8,14]. The studies of those papers all aimed at developing a robust and efficient algorithm for searching global optimum solution for engineering problems.

Differential evolution (DE) is one of the recent developed population-based technique which was invented by Price and Storn in 1995 [2]. It uses real-value vector for design variables and the number of control parameters is usually three: crossover parameter, scaling factor, and population size. DE has been proven by many researchers that it was powerful and efficient in different global optimization problems[9-12]. But there are few studies of application of DE as a search tool in field of optimization of structure. In this study DE will be integrated with finite element method for topology optimization of truss and continuous structure with stress or displacement constraints.

2. DIFFERENTIAL EVOLUTION

Differential evolution(DE) is a population-based stochastic optimization algorithm for real-valued optimization problems. In DE each design variable is represented in the chromosome by a real number. The DE algorithm is simple and requires only three control parameters: weight factor(F), crossover rates(CR), and population size(NP). The initial population is randomly generated by uniformly distributed random numbers using the upper and lower limitation of each design variable. Then the objective function values of all the individuals of population are calculated to find out the best individual $x_{best,G}$ of current generation, where G is the index of generation. Three main steps of DE, mutation ,crossover, and selection were performed sequentially and were repeated during the optimization cycle.

Mutation
For each individual vector $x_{i,G}$ in the population, mutation operation was used to generate mutated vectors in DE according to the following scheme equation:

$$v_{i,G+1} = x_{best,G} + F(x_{r1,G} - x_{r2,G}) \ , i=1,2,3......NP \quad (1)$$

In the Eq. 1, vector indices r1 and r2 are distinct and different population index and they are randomly selected. The selected two vectors, $x_{r1,G}$ and $x_{r2,G}$ are used as differential variation for mutation. The vector $x_{best,G}$ is the best solution of current generation. and $v_{i,G+1}$ are the best target vector and mutation vector of current generation. Weight factor F is the real value between 0 to 1 and it controls the amplification of the differential variation between the two random vectors. There are different mutation mechanisms available for DE, as shown in Table 1, which may be applied in optimization search process. The individual vectors $x_{r1,G}, x_{r2,G}, x_{r3,G}, x_{r4,G}, x_{r5,G}$, are randomly selected from current genration and these random number are different from each other. So the population size must be greater then the number of randomly selected iion if choosing Rand/2/exp mechanism of DE mutation, the NP should be bigger than 5 to allow mutation.

Table 1. The mutation mechanism of DE

Mechanism	Mathematical equation
Best/1/exp	$v_{i,G+1} = x_{best,G} + F(x_{r1,G} - x_{r2,G})$
Rand/1/exp	$v_{i,G+1} = x_{r3,G} + F(x_{r1,G} - x_{r2,G})$
Rand-to-Best/1/exp	$v_{i,G+1} = x_{i,G} + F(x_{r1,G} - x_{r2,G})$
Best/2/exp	$v_{i,G+1} = x_{best,G} + F(x_{r1,G} + x_{r2,G} - x_{r3,G} - x_{r4,G})$
Rand/2/exp	$v_{i,G+1} = x_{r5,G} + F(x_{r1,G} + x_{r2,G} - x_{r3,G} - x_{r4,G})$

Crossover

In the crossover operator, the trial vector $u_{i,G+1}$ is generated by choosing some parts of mutation vector, $v_{i,G+1}$ and other parts come from the target vector $x_{i,G}$. The crossover operator of DE is shown in Fig. 1

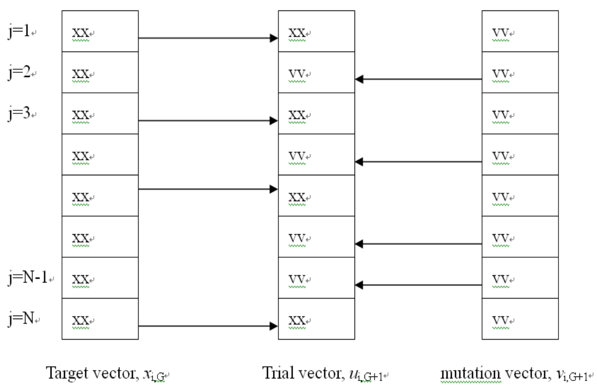


Fig. 1. The Schematic diagram of crossover operation

Where Cr represents the crossover probability and j is the design variable component number. If random number R is larger than Cr value, the component of mutation vector, $v_{i,G+1}$,will be chose to the trial vector. Otherwise, the component of target vector is selected to the trial vectors. The mutation and crossover operators are used to diversify the search area of optimization problems.

Selection operator

After the mutation and crossover operator, all trial vectors $u_{i,G+1}$ have found. The trial vector $u_{i,G+1}$ are compared with the individual vector $x_{i,G}$ for selection into the next generation. The selection operator is listed in the following description:

$$x_{i,G+1} = u_{i,G+1} \ , \text{if } f(u_{i,G+1}) > f(x_{i,G}) \ ,$$
$$x_{i,G+1} = x_{i,G} \ , \text{if } f(u_{i,G+1}) \le f(x_{i,G}) \ , i=1,2....NP \quad (2)$$

If the objective function value of trial vector is better than the value of individual vector, the trial vector will be chosen as the new individual vector $x_{i,G+1}$ of next generation. On the contrary, the original individual vector $x_{i,G}$ will be kept as the individual vector $x_{i,G+1}$ in next generation. The optimization loop of DE run iteratively until the stop criteria are met. There are three stop criteria used in the program. The first criterion is maximum number of optimization generation. The second criterion is maximum number of consecutive generations that no better global optimum is founded in the whole process. If the improvement of objective function between two consecutive generations is less than the threshold set by program, it will be considered as fitting convergence requirement. The last stop criterion is conformed if the accumulated number of generations fitted convergence requirement is greater than maximum counter set by the program. The flowchart of DE is shown in Fig. 2.

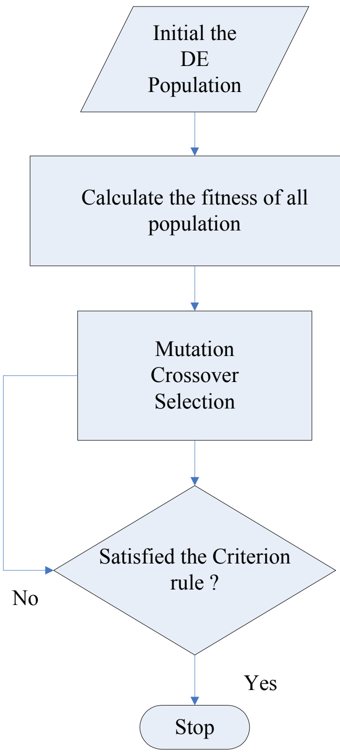


Fig. 2 The flowchart of differential evolution

3. TRUSS OPTIMIZATION

Three truss cases used in literatures of truss optimization problem were illustrated in this study to check the performance of DE in searching optimum truss structure. The objective of those cases of truss optimization is to maximize the utilization of geometry and material for the lightest structure satisfying all the design constraints. The commercial software ANSYS was used for finite element analysis of truss structure and it was integrated with DE optimization program to check constraint violation of displacement and stress. The formulation of the truss-structure optimization problem can be described as following.

Minimize: $W(A) = \sum \rho_i L_i A_i$

Subject to $G_1 =$ Truss is kinetically stable

$G_2 = \sigma_i \le \sigma_{allow}$

$G_3 = \delta_i \le \delta_{allow}$

$G_4 = A_i^{min} \le A_i \le A_i^{max}$

The design variables A_i is the cross-sectional areas of the structural members. The parameter σ_{allow} and δ_{allow} indicate the allowable strength of the member and the allowable deflection of the node defined by the designer, respectively. The following cases of topology optimization of truss use the same model described above.

Case one

The loadings and geometry of 10-member truss with 6 nodes are shown in Fig. 3. The material properties and design constraints are listed in the following.

Modulus of elasticity $E=1\times10^4$ksi

Density ρ=0.1lb/in^3

Maximum allowable stress σ_a=25ksi

Allowable displacement δ_{allow}=2in.

The loadings are p1=150 ksi and p2=50 ksi. The area of cross section is limited between 0.1in^2 and 30.0in^2. The results of optimum solution found by differential algorithm were compared with the results published in literature [7] as shown in Table 2.

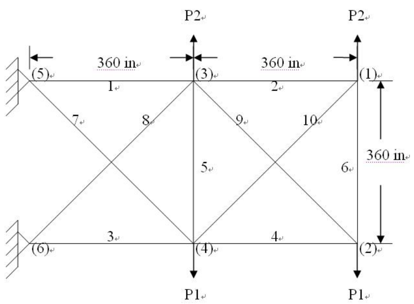


Fig. 3 Truss structure with 10 members and 6 nodes.

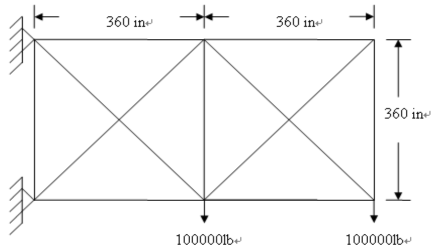
Table 2. Member areas of the optimized truss for case one

Truss Number	Kang Seok Lee[7]	This Paper
1	23.25	23.502
2	0.102	0.100
3	25.73	25.311
4	14.51	14.364
5	0.100	0.100
6	1.977	1.969
7	12.21	12.384
8	12.61	12.830
9	20.36	20.339
10	0.100	0.100
Total Weight	4668.81	4676.92
Max. Displacement	2.0039	2.000
Max. Stress	25041.0	25000.000

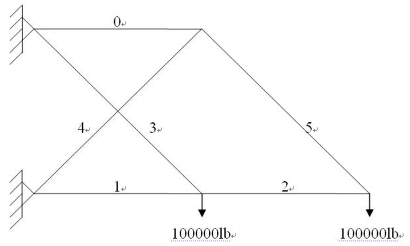
The total weight of optimum solution found by differential evolution is 5060.91lb. Although it is a little heavier than total structure weight published in reference[7], but the maximum displacement and maximum stress were lightly violated in Lee's study by using ANSYS for finite element analysis. On the contrary there is no violation in both maximum displacement and maximum stress constraints in this study.

Case two

The geometry model and loading conditions of 11-members truss with 6 nodes are show in Fig. 4a. The material properties are same as those used in case one. The area of cross section is limited between 0in^2 and 35in^2. The optimized truss with 5 truss members removed is illustrated in Fig. 4b.



(a)



(b)

Fig. 4 (a)11-members, 6 node truss structure, (b) optimized truss structure

The total weight of the best truss structure obtained using DE is better than the results of literature[8]. The maximum displacement is located on the constraint boundary as listed in Table3. The total number of calculation of objective function of DE is 20000 and it is only half of number of objective function calculation used in the reference.

Table 3 Member areas of the optimized truss for case two

Truss Number	Deb[8]	This paper
0	29.68	30.111
1	22.07	22.121
2	15.30	14.998
3	6.09	6.081
4	21.44	21.254
5	21.29	21.337
Weight	4899.15	4898.41
Maximum Displacement	1.9999 in	2.000 in
Maximum Stress	23222.0 lb	23253.2 lb

Case three

The material properties are same as those used in previous cases. The maximum allowable stress is 40ksi and the maximum allowable deflections are both 0.35in at node one and node two. The design variables were also divided into eight groups. The loading conditions are described in Table 4, and optimal design of structure must satisfy boundary constrain under both two loading condition. The geometric model of 25-member space truss is shown in Fig. 5. The final results were compared with the results of other literatures[8,15] as illustrated in Table 5. The total weight of optimized truss structure of this study is the best of three without violating the stress and displacement constraints. The optimum solutions obtained by Deb and Arora were similar in all eight groups. But the optimum solutions searched by differential evolution in this study are quite different in compared with those two solution sets. The problem should be a multi-modal optimization problem and there may have some other better solutions. It is worth further study.

Table 4 The loading setting in case three

Loading condition	Node	Force Fx(lb)	Force Fy(lb)	Force Fz(lb)
(1)	1	1000	10000	-5000
	2	0	10000	-5000
	3	500	0	0
	6	500	0	0
(2)	5	0	20000	-5000
	6	0	-2000	-5000

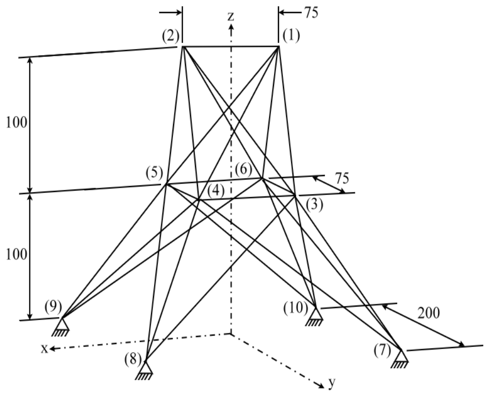


Fig.5 25-members, 10-node space truss structure

Table 5 Member areas of the optimized truss for case three

Member	Length	Deb[8]	Arora[15]	This Paper
1-2	75.0	0.006	0.010	0.0208
1-4,2-3,1-5,2-6	130.504	2.092	2.048	1.1043
2-5,2-4,1-3,1-6	106.80	2.884	2.997	2.9999
3-6,4-5	75.0	0.001	0.01	0.2699
3-4,5-6	75.0	0.001	0.01	0.2699
3-10,6-7,4-9,5-8	181.142	0.690	0.685	0.9268
3-8,4-7,6-9,5-10	181.142	1.640	1.622	1.4072
3-7,4-8,5-9,6-10	133.464	2.691	2.671	2.9999
Weight (lb)		544.984	545.050	523.32
Node 1 Max. Displacement		0.3500	0.3500	0.3485
Node 2 Max. Displacement		0.3500	0.3500	0.3500

4. TOPOLOGY OPTIMIZATION OF CONTINUOUS STRUCTURE

Topology optimization[16,17] is useful for concept design during development of new innovative product. When using finite element method to analyze structure, every element without constraints can be eliminated or kept in design domain. This freedom of change may create whole new design out of experiences and professional knowledge of designers. The long computational time needed in complex design will cause the difficulty for applying topology optimization in many real-world applications.

The differential evolution is a real-value optimization algorithm and has good performance in continuous optimization problem. In topology optimization for a practical and feasible structure, a binary code should better be used in search optimum structure. The element can be grouped into solid and void only to representing corresponding 1 and 0 in binary code. A simple filtering mechanism is used to convert a real-value density to binary code in this study. The rule used in filtering conversion is as followings.

$$\rho_i = 0, \text{ if } x_{i,j} \le 0.5$$
$$\rho_i = 1, \text{ if } x_{i,j} > 0.5 \quad (3)$$

After binary conversion, the binary string was decoded as a continuous structure.

The two-dimensional continuous structure is subdivided into elements for finite element model. A binary string is used to store the shape information for every design variables. The code "1" in the string means that corresponding solid element of the structure in finite element model. On the contrary, the code "0" in the string represents the corresponding void element of the structure. The string can be mapped into a two-dimensional design domain as shown in Fig. 6.

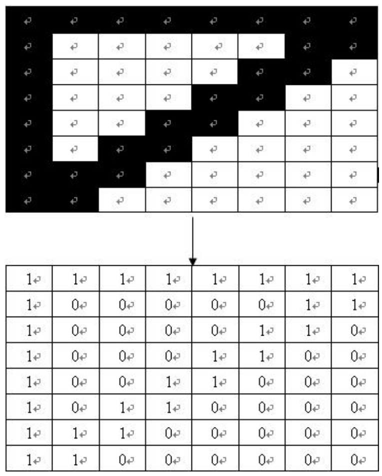
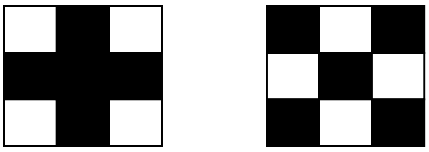


Fig.6 The translation of binary code to topology structure

The discontinuity of structure may be happened when the topologic representation of structure is changed by mutation or crossover operation in search process. But only continuous structure can be analyzed by the finite element method, it is necessary to develop an algorithm to modify the discontinuous structure into continuous structure. The criterion of continuity condition is defined as that any element of structure must share at least one common edge with any other element in finite element model as shown in Fig. 7.



(a) (b)

Fig.7 (a) The continuous structure (b) discontinuous structure

For checking the continuity of structure before finite element analysis, a special algorithm is developed and stated as following:

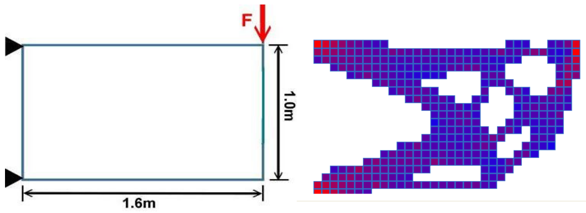
1) First, we must select a starting element that must be kept in the structure. The node with support or the node with loading should be selected as the starting element, because this element must be existed in structure for maintaining the consistence of physical model. The starting element is assigned with code "2".
2) The structure configuration is decoded from binary code of string and only the element with code "1" will be checked for the continuity. The checking procedure begins at the starting element. If there is an element with code "1" sharing a common edge with the element with code "2", it is also set to code "2". The procedure will be repeated until no element is changed from code "1" to code "2" any more.
3) If the elements with code "1" are all changed to code "2", it is said to be a continuous structure. Otherwise, if there is any element still with code "1", it is said to be a discontinuous structure.

In order to make the value of objective function independent from magnitude of loadings, the data of stress, displacement and volume of full structure without void will be used as base for normalization of objective function. The objective function is defined as in Eq.(4).

$$F_{objective} = V_{normal} * D_{normal} / Penalty \quad (4)$$

Where V_{normal} is normalized volume of full structure divided by the volume of current structure. D_{normal} is normalized maximum displacement of full structure divided by maximum displacement of current structure. Penalty is multiplication of stress penalty and displacement penalty. Allowable stress and allowable maximum displacement will be used as constraints. When the maximum stress is over the allowable stress or the maximum displacement is over the allowable displacement, two penalty factors will be multiplied as total penalty for constraint violation.

The finite element program ANSYS is also used in structural analysis for this case. Geometric dimensions of cantilever structure are 1.6m in width, 1.0m in height and 0.01m in thickness. It is subjected to a load F=50kN. The nodes at top and bottom on left hand side are set as fixed nodal displacement as shown in Fig. 8a. The optimum solution is shown in Fig. 8b. The performance of differential evolution applied in topology optimization of structure is good enough in comparison with other algorithms such as genetic algorithms.



(a) (b)

Fig. 8 Cantilever structure and result for topological optimization

5 CONCLUSIONS

There are some conclusions may be drawn from this study.

1) Differential evolution algorithm is very effective in solving size and topology optimization problems of truss structure. This study illustrates the potential of using differential evolution as alternate optimization tool in structural optimization.
2) The results obtained in this study are better than results of previous literatures without violating stress and displacement constraints. The total weight of structure in case one is little heavier than result of previous study [7]. But there were constraint violations in Lee's study checked by using finite element program ANSYS.
3) Some advanced mechanisms such as multi-population evolution, paralleled processing, and global-local search, should be studies further to understand the performance of differential evolution in complex real life engineering optimization problems.

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