Journal of Systemics, Cybernetics and Informatics (2025) 23(7), 183-187
https://doi.org/10.54808/JSCI.23.07.183

They Learned the Course! Why then do They Come to Tutorials? Part I

Russell Jay HENDEL
Department of Mathematics, Towson University
Room 316, 7800 York Road
Towson, Maryland 21252, USA
RHendel@Towson.Edu

ABSTRACT

The paper explores after-course preparation for certification exams. The question addressed is, "What are the broad themes for seeking tutorial help *after* course completion? What *types* of problems are typical during tutorial help? What are their characteristics?" The paper is based on the author's experience in tutorial assistance for Society of Actuary certification exams at Towson University and additionally to help-desk experiences with ACTEX an online educational company that assists in exam preparation. This is part I of what hopefully will become a series of such explorations. In this part I, we explore the Probability course. Four broad areas of seeking help emerge including (i) problems integrating multiple course modules, (ii) many-parameter problems, (iii) modules with multiple vs. single formulas, and (iv) modules where solution is accomplished by an algorithm vs. a formula. The study, besides being of use to those conducting tutorials, is useful to instructors teaching the course since it emphasizes potentially weak spots in the curriculum needing strengthening. This study is consistent with and reflects several important constructs from the psychology of pedagogy.

Keywords: pedagogy, challenging problems, tutorials, integrated problems, multi-parameter problems, graphical aids

1. THE PROBLEM

1.1 Background. The immediate background for the material in this paper is the Society of Actuary (SOA) Preliminary Probability Exam. This examination is based on a standard introductory course in Probability theory. Upon completion of the course, students may attempt to pass the SOA Exam P, corresponding to the course; passing Exam P is a certification and one important step in attaining internships, hirings, and promotions.

In preparing for the exam, many students who have already completed and passed the course may seek additional help. There are several online educational companies specializing in providing assistance for preparing for these exams.

The material in this paper is based on the author's experience assisting and tutoring for exam preparation at both Towson University, and the help desks of the ACTEX corporation, an online educational company specializing in a rich variety of exam preparation materials for a variety of examinations. These experiences provided the author with valuable background facilitating discovery of broad themes and areas of commonality in the help requested by the students.

By way of acknowledgement, Towson University has achieved the prestigious Center of Actuarial Excellence (CAE) and University Earned Credit (UEC) designation by the Society of Actuaries and is one of the approximately three dozen CAE schools in the world. ACTEX is unique among online educational companies in that it is structured as a benefit corporation and relies on a network of academic volunteers to produce manuals, flash cards, practice problems, and a variety of other assistive tools housed under the Actuarial University section of its software. It has also achieved international exposure working on U.N. projects on educating under-developed nations. The author expresses his gratitude for the opportunity to be a part of these two great institutions.

1.2 The Problem. The statement of the problem is remarkably simple. These students are motivated, having taken the Probability Course in order to prepare them for the SOA examination. So, what did they miss? If they passed the course and presumably mastered the material, why are they seeking help with specific problems? What are the broad themes and characteristics of these problems?

In this paper we explore four broad areas of problem difficulty that lead to seeking extra help: (i) problems that integrate multiple modules, (ii) excessively many-parameter problems, (iii) modules with multiple versus single formulas for the same item, and (iv) modules which teach algorithmic vs. formula-driven solutions, for example, problem solution approaches using graphical aids.

These four problems categories, besides having an intuitive ring, suggesting difficulty, are consistent with principles of psychology which we review in the next section. Then, in Sections 3-6 we study each of the four areas of difficulty just mentioned.

This paper, besides being of use to those tutoring and assisting students, facilitates instruction itself. Being aware of what is likely to be forgotten in a course alerts the instructor to the need for increased study time, illustrative examples, and practice exercises.

2. UNDERLYING PSYCHOLOGICAL THEORY: ATTENTION and INTERFERENCE

2.1 Attention. An established adage is that learning cannot take place without prior attention to the material being learned. The following punchy example found in several memory books amusingly illustrates this:

> Example: Prior to the advent of I-phones people ascertained time by wrist watches. Memory books ask, "Without looking at your watch, state whether your watch uses Arabic numerals for time (1,2,3,4) or roman numerals (I, II, III).

Many people don't know the answer to this question because they never paid attention to learn how their watch displays time.

Attention as a prerequisite for learning is well established and involves three stages: *Encoding* the information, *storing* the information, and allowing access and *retrieva*l of the information [1]. Our understanding of how the brain uses attention to learn has greatly expanded with several competing theories explaining the mechanism [2,3]

2.2 Interference. In 1935 Stroop performed an experiment in which the subjects saw words that denoted colors (red, green, blue and yellow) each word printed in either the corresponding color (e.g., the word red was written in red ink) or in a noncorresponding color (e.g., the word blue was written in red ink).[5] The subjects were required by instruction to identify the ink color either orally or by pressing a button.

The response time for each identification was measured. Averages for recognizing congruent items (where the meaning of the word and its ink color were the same) and incongruent items (where the meaning of the word was different than the ink color in which it was presented) were calculated. Stroop discovered a faster average response time for recognizing congruent items than for recognizing incongruent items. Stroop named this psychological phenomenon "interference," and it frequently is called Stroop interference in the literature in recognition of his discovery.

Later studies, besides measuring average response time, also measured omission rates (not responding to a particular trial) and error rates (e.g., saying that the word red in a blue font has a red font).[4] Stroop interference affects all three of these measures: incongruent items elicited slower response rates, higher omission rates, and higher error rates.

3. EXAMPLES: INTEGRATED PROBLEMS

In the remainder of the paper, we explore four frequent themes of tutorial visits. We begin our exploration with integrated problems.

Quite simply an integrated problem is a problem whose solution requires both the techniques of the current course and module one is learning and techniques from either other modules in the same course or possibly other courses. We illustrate with the module of probability distributions in the Probability course.

A major topic in any introductory probability course is probability distributions. Each distribution is characterized by the number of variables (typically one or two), its attributes (continuous or discrete), and the number of parameters it has. Each probability distribution has its own name, typical problems to which it applies, and important formulas for its expected value, variance, and probability densities.

General probabilities can then be calculated using either summation or integral techniques for discrete and continuous distributions, respectively. However, all the textbooks are quick to point out that sometimes alternative computational techniques exist utilizing methods from other courses. In the example below, a geometry approach using such simple formulas as those for the area of a square and triangle are much more efficient than traditional integrals and sums. But students sometimes think they must stick to course methods. They so to speak mentally block out *interference* (actually, in this case, *assistance*) from other courses. This leads to them seeking tutorial help.

Typical Problem: Jack and Jill agree to meet each other for a date by each arriving separately at the Irish Pub sometime between 5 and 6 PM. The deal is whoever comes first will wait 10 minutes and if the other party does not show up, the first party is free to socialize with whomever they chose and perhaps just recently met. Calculate the probability that Jack and Jill will actually date.

In formulating an approach to a solution, it is useful to look at some possible cases. If Jack arrives at 5:25 and Jill arrives at 5:30 then the date is on. We can encode this as (25,30). Contrastively, if Jack arrives at 5 and Jill arrives at 5:30 they do not date; we can encode this as (0,30). It emerges that each possibility of date or no-date is dependent on a pair of numbers which of course can be neatly illustrated in a Cartesian system. Figure 1 shows this graphical representation as well as the date (D) and no-date (N) regions.

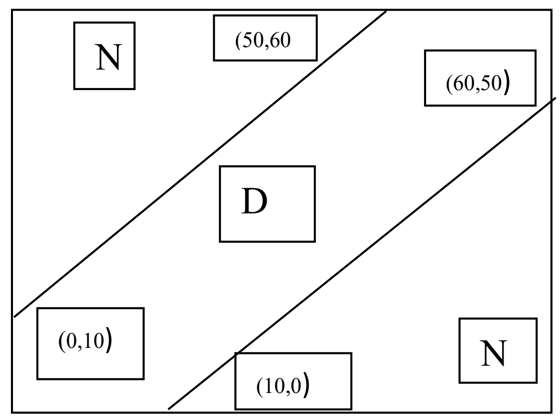

Figure 1: The rectangle represents all meeting times (x,y) within the hour of 5 PM to 6 PM. In the upper left, the slanting line joins x=0 and y=10 (representing arrival times 5:00 PM and 5:10 PM for Jack and Jill), and x=50 and y=60 (arrival times 5:50 and 6:00 for Jack and Jill). For all points in this upper right triangle, for example, x=5, y=20 or x=35, y=55 (5:35 and 5:55 arrival tiems) the date (N) is not on. A similar interpretation applies to the lower right. For all remaining points in the middle region the date is on.

We have implicitly assumed that it is equally likely for each of Jack and Jill to arrive at any time between 5 and 6. In probability theory this assumption is formulated as the uniform distribution. The student could then proceed to calculate probabilities using integrals. (The uniform distribution could also have been stated explicitly in the problem).

But a much more elegant (and quicker) solution can be accomplished by Geometry if one does not let it *interfere* with the probability module and instead actively thinks about it. The following imaginary dialogue illustrates how an instructor-tutor and student might interact.

- Student: I forgot the formula for the area of *D*
- Tutor: *There is no formula for D*

- Student: I am confused. I thought this approach would give us the solution
- Tutor: *It does, but you give up too easily*
- Tutor: *You told me what you forgot; what do you remember?*
- Student: I can compute the areas of the squares and the triangles but that is not what I want. It also doesn't seem to do any good because I want the area of *N* which is oddly shaped.
- Tutor: *Sum of all areas = Area of square!*
- Student: Oh!

To show how easy the solution is, when approached geometrically instead of by the traditional integration techniques, we actually compute the solution:

- Basic formula: Area(D)+Area(N)=Area(Square)
- Area of square = 60*60 =3600
- Area of each triangle = ½*50*50 = 1250
- Plugging into the formula, we obtain
- We obtain 3600 = Area(D) +1250 + 1250 = Area(N) +2500
- So, Area(D) = probability of dating = 1100/3600 = 30.6%

In summary, integrated problems are a typical challenge for students for which they seek help. Integration occurs in many forms and formulating integrated problems is a favorite among exam and item writers.

4. EXAMPLES: MULTI-PARAMETER PROBLEMS

Formulas that solve the problems taught in mathematics' courses have parameters, with many problems typically having 2-3 parameters. The problem is then solved by *plugging in values* for the parameters.

Plugging in is a basic math skill at which good students excel. However, if a formula has many parameters, it may pose a challenge. During class, students cannot give sufficient *attention* to learning the problem resulting in them *taking notes* with the intention *of learning it before the test*. This is why these problems are frequently the subject of tutorial sessions during which sufficient time *is* devoted to mastering the formula.

As mentioned earlier, this paper has implications and benefits for both tutors and instructors. In this case, instructors are advocated to allocate more time in teaching these problems, thus obviating the problem of needing help later on. The following illustrative problem should provide readers with a feel of how overwhelming these problems can be. Particularly note how the multiple parameters compete for *attention* thus creating *interference*.

> Illustrative Problem: At a social party hosted by parents, baby Semantha unexpectedly walks in and sticks her hand in a grape-bowl. She gleefully manages to take out 6 grapes. But 4 of them are rotten grapes. Her parents wish to assure guests about the grapes: "There is probably at most only 1 more rotten grape."
> The bowl initially had 100 grapes. What is the probability that the parents are correct?

Illustrative solution: Re-reading the problem we see three numbers giving rise to six parameters presented in this bulleted list with the # sign indicating number or cardinality. For convenience, we have labeled the 6 numbers *A,R,G,a,r,g* as shown.

- Bowl = Population=> # All grapes =100 = *A*
- Bowl = Population => # Rotten grapes = 5 =*R*
- Bowl =Population => # Good grapes = 95= *G*
- Hand = Sample => # all Grapes = 6= *a*
- Hand = Sample => # rotten grapes = 4=*r*
- Hand = Sample => # good grapes = 2 =*g*.

Notice how the adjectives *All, Rotten, Good,* could apply to any similar problem. The idea that the population cardinalities are indicated by capital letters while the sample cardinalities are indicated by lower case letters also facilitates the use of this notation for any problem. The number of parameters as well as mnemonical approaches to their mastery should be mentioned at the beginning of the solution to serve as a navigation of what is to come.

This type of mnemonical notation is one important component of multi-parameter problems.

The solution may now be provided by numerically computing three numbers:

- There are *C(A,a)* ways to pick *a* grapes from *A*
- There are *C(R,r)* ways to pick *r* rotten grapes from *R* rotten grapes.
- There are *C(G,g)* ways to pick *g* good grapes from *G* good grapes.

The function *C* is a known counting function taught in all probability courses. Once these numbers are known the desired probability asked by the question can be obtained from the following formula:

> The probability that there are only
> *R*=5 rotten grapes among the
> *A*=100 grapes from which baby Semanta when grabbing
> *a* = 6 grapes had
> *r*=4 rotten grapes and
> *g*=2 good grapes = *C(R,r)*C(G,g) / C(A,a)*.

Plugging in we obtain a probability of 4 in a million, not very likely! Thus, there are probably more rotten grapes in the bowl.

5. EXAMPLES: NON – FORMULA PROBLEMS

A very frequent theme in student inquiries is problems whose solution is preferably not done with formulas. To appreciate this, consider the alternative. Many mathematics problems have solutions with a focus on *the formula.* The student learns the formula, understands what the formula parameters refers to, and then is able to solve problems by *plugging in.* Plugging in does not only involve numbers; some formulas require plugging in functions for the parameters. A non-formula problem simply refers to a problem whose easiest solution approach does not involve formulas. It may involve an algorithm, use of graphical aids, or an iterated procedure. The student may have even learned the *best* solution. But because students tend to perceive mathematics as a collection of formulas, they feel stuck and seek help.

We present a simple example from probability theory.

> Typical Non-formula problem: A density function of a two variable distribution is given by
>
> $f(x,y) = (x + 2y) / 19$, with (x,y) in {1,2}.
>
> Calculate $\boldsymbol{E}(X \mid y = 1)$.

There is a formula approach to the definition

$$\boldsymbol{E}(X \mid y = 1) = \sum x * f(x,1) / \sum f(x,1)$$

where the sum is over all possible x. A student attempting to use this formula may lapse into difficulties motivating seeking assistance from the help-desk.

However, the following graphical solution is much more readable, easier to memorize and retain, and less subject to error The solution consists of 3 tables. We start with the probability table, presented in Table 1, whose interpretation should be clear without need for further elaboration.

TABLE 1: Complete Probability Distribution

	$y = 1$	$y = 2$
$x = 1$	Probability =f(1,1)= 3 / 19	Probability= f(1,2)= 5 / 19
$x = 2$	Probability=f(2,1) = 4 / 19	Probability = f(2,2)= 6 / 19

Since we are interested in ***Pr***($y \mid x$=1), and since $Pr(y$=1) = 3/19+4/19, we can next calculate the probability of distribution when y is fixed at 1, presented in Table 2, whose interpretation is again clear, without need of further elaboration.

Table 2: Probability distribution of x when $y = 1$.

	$x = 1$	$x = 2$
Probability	3/7 = f(1,1)/ f(1,1)+f(2,1)	4/7 = f(2,1)/ f(2,1)+f(2,1)

Since Table 2 is a finite univariate distribution we can proceed to take the average normally either by formula or by table, as shown in Table 3. The expected value is 3/7+8/7= 11/7.

TABLE 3: Computation of Average Value of x

	$x = 1$	$x = 2$
Probability	3 / 7	4 / 7
Product ($x * Pr$)	1*3/7 = 3/7	2 * 4/7 = 8/7

Even after seeking help many students prefer to try and master formulas. It is important to emphasize both during tutorial sessions as well as during instruction sessions that non-formula approaches to mathematical problems, particularly those involving graphical aids such as tables, are very normal, may be preferred, are easier to memorize, easier to read, and easier to retain.

6. MULTIPLE FORMULAS

6.1 Overview. As noted in the introduction, *interference* from similar but different items is a leading source of confusion for students and a key contributor to tutorial visits. We illustrate with a simple example from the SOA Probability course, where the problem solution has two possible formula approaches.

> Illustrative example: Jack and Jill are playing a game of guessing the last digit in a phone number. Jill picks a phone number which she does not reveal to Jack. Jack pays a penalty \$p for every wrong guess but receives \$1**the number of steps needed to guess correctly.* At each step Jill uses a different phone number.
>
> For example, if Jill picks the phone number 123 456 7890 and Jack guesses 1, he must pay \$*p*. If Jill next picks 123 729 3544 and Jack guesses 5, he must pay \$*p*. If on the 3rd try Jill picks 878 234 5642 and Jack guesses 2, Jack receives \$1*3rd try = \$3.
>
> Determine the value of p that would make Jack's expected payment (or rewards) \$0.

6.2 **Discussion of Solution.** The problem is not that hard. Suppose, for example, Jack guesses correctly on the i-th try. Then he receives \$$i$ but pays \$(i-1)**p;* the probability of this happening is p_i . Then the total Jack receives at guess i, is \$$i$ – $(i\text{-}1)\ p$ and the expected value is **Exp**(i) – **Exp**($(i\text{-}1)p$) where i is geometrically distributed with probability 0.1. It is then straightforward to plug into known formulas and calculate p.

The problem arises because there are two formulas for the geometric distributed variable; one focusing on the waiting time to success and the other focusing on the number of failures prior to the first success. Both approaches naturally occur in this problem. A student may try to memorize both formulas thereby creating *interference.*

6.3 A Simple Correct Approach. One better approach in these problems is to stick to one formula from which the others are derived as needed in each problem. While this takes more time, it creates less *interference*, and hence retention is maximized.

Suppose a student chooses the approach of the waiting time to the first success. Then we must solve the equation

$$0= \mathbf{Exp}(i) - \mathbf{Exp}((i\text{-}1)p) = \mathbf{Exp}(i) - p\ \mathbf{Exp}\ (i) - p\ \mathbf{Exp}(1).$$

Since **Exp**(i) = 1/**Pr**(*Success*), we must solve

$$0 = 10 - 10p + p \Rightarrow p = 10/9 = \$1.11.$$

7. CONCLUSION

This paper has reviewed some typical patterns resulting in learning not taking place in class but requiring tutorial visits. We believe the cases reviewed will benefit both tutors, who are given tools to identify the source of difficulty, and course instructors, who are urged to spend more time on course problems where difficulty is likely. Discussions with others involved in tutorial help showed agreement that these four categories are important weak spots to emphasize.

8. REFERENCES

[1] A. W. Melton, "Implications of short-term memory for a general theory of memory", **Journal of Verbal Learning and Verbal Behavior**, Vol. 2, 1963, pp. 1–21.

[2] Isabel A Muzzio, Clifford Kentros, Eric Kandel, "What is remembered? Role of attention on the encoding and retrieval of hippocampal representations", **J. Physiol.,** Vol. 587, No. 12, 2009, pp. 2837–2854.

[3] K. Oberauer, "Working Memory and Attention – A Conceptual Analysis and Review", **Journal of Cognition**, Vol. 2, No. 1, 2019, Article 36, pp. 1–23.

[4] M. Rezaei and F. Azizian, "Neuropsychological Decomposing Stroop Interference into Different Cognitive Monitoring: An Exploratory Factor Analysis", **Basic and Clinical Neuroscience,** Vol. 10, No. 5, 2019, pp. 475–484,

[5] J. R. Stroop, "Studies of Interference in Serial Verbal Reactions", **Journal of Experimental Psychology**, Vol. 18, No. 6, 1935, pp. 643–662.

.