Journal of Systemics, Cybernetics and Informatics (2021) 19(8), 128-138
https://doi.org/10.54808/JSCI.19.08.128

"Historical path in mathematical games": a didactic laboratory activity dedicated to upper secondary school students

*F. S. Tortoriello [1] and I. Veronesi[2]

[1,2]University of Salerno, Italy

[1] fstortoriello@unisa.it, [2]iveronesi@unisa.it

Abstract

In this work we intend to share an educational path on mathematical games developed in the last classes of the high schools that participate to the research-project Mathematical High School Project, a project elaborated by the research group in mathematics education of the Department of Mathematics of the University of Salerno (Italy) and dedicated to scientific high school students. The "Historical path in mathematical games" is a didactic laboratory activity dedicated to students of the last year of scientific high schools. The researchers collaborated with the teachers of the schools and carried out an interdisciplinary path on puzzles and logic games invented or reworked by important mathematicians of the last two centuries. Through these themes that were scientific, historical, artistic, musical, literary ones, connections have been created thank to the presentation of texts, images, paintings, drawings, musical and literary pieces.

Keywords*: Mathematical High School Project, Constructivism, Interdisciplinarity, New Technologies, Logic Puzzles, History of Mathematics*

1. The Mathematical High School Project

The research group in mathematics education of the Department of Mathematics of the University of Salerno (Italy) has been developing for six years a research project called Mathematical High School Project which supports the ordinary curricular path of the scientific high school with laboratory activities that take place in the afternoon in which students are offered themes that reposition mathematics within

[1] The authors acknowledge Prof. Giovanna Bimonte (Department of Statistics, University of Salerno, Italy) as peer-editor who proofread and edited the final version of the paper.

the network of competences shared with the other disciplines and mathematics becomes the bridge between scientific and humanistic culture thanks to interdisciplinary paths (Prigogine, 2004), (Rogora, 2021).

The project has mathematics as a common denominator to read and interpret reality in its continuous cultural, economic and social evolution in an interdisciplinary key and it aimed at a vision of knowledge not fragmented into individual disciplines (Bauman, 2000). The project includes extra-curricular activities to be carried out with laboratory activities.

These activities take place on a weekly basis, meetings lasting 2/3 hours from November to May. Due to the lockdown imposed by covid-19 pandemic, the researchers and the teachers of the schools involved in the project, reworked the activities in order to be carried out with distance learning through e-learning platforms while maintaining interaction with the students.

Previously, students, researchers and teachers carried out the laboratories in the classroom and saw a conspicuous and continuous relationship between the students. The focus of the project is an active didactic so it was decided to use the technological interfaces, such as the Breakout rooms which allowed to maintain active participation and a collaborative approach with group activities and the forms to be filled in online and discussed in real time to analyze the feedback of the works.

2. The context

The most requested skill today as a result of school experience is the ability to know how to create connections between different fields and disciplines. It has been observed that teachers usually develop links between different topics and problems while remaining mainly within related subjects, while the search for the connection between disciplines of different fields is rare.

The Mathematical High School Project calibrates the focus of learning not on individual disciplinary knowledge but on the development of transversal skills that allow students to acquire skills that can be declined into a global knowledge and skills in various specific areas, in fact in a world that is rapidly transforming the indispensable competence for youngers is knowing how to transform their own knowledge by integrating it into the reference context

In particular, the Economist Intelligence Unit (Tabary, 2015), following an analysis of the labor market and the needs of contemporary society, highlights that the world of work is rapidly evolving due to disruptive technological implementation and globalization processes and the objectives that can be assessed for the future are changing. Problem solving, team working, and communication (commonly known as "21st century skills") are the most requested objectives in the world of work and creativity and enterprising curiosity are gaining importance in recent years.

The choice of the Mathematical High School is to incorporate these transversal objectives within didactic modules specially designed and elaborated for an innovative approach to teaching, a change of observation point that approaches knowledge from a perspective that goes beyond the fragmentation of individual disciplines also thanks to the use of the most modern technologies that allow an implemented teaching-learning (Barzel, 2007), (Gartner, 2017), (Vigotsky, 1987), (Wellings, 2009). This way of understanding knowledge follows in the footsteps of the thought of the Nobel laureate Ilya Prigogine who, in an article published after his death (Prigogine, 2004), stated that "In the 19th century fragmentation played an important role in the establishment of separate disciplines for biology, chemistry, physics, mathematics, psychology, sociology, etc. But when we consider the great challenges facing humanity today we see that we need an interdisciplinary approach. Therefore, at this historical moment, I think it is really very important to emphasize the end of fragmentation, or at least the overcoming of fragmentation". It is therefore essential to contribute to a new culture that makes it possible to understand and deal critically, both at a cultural and at a professional level, with an

increasingly complex and ever more rapidly evolving reality.

3. Didactic methodologies

The course activities were developed according to a socio-constructivist conception of learning (Harel, 1991), (Kafai, 1996), (Morin, 1999), (Papert, 1990), the processes of active knowledge processing were relevant both for the pre- established knowledge objectives, but above all for the context and dialogic relationships of different actors, peer to peer education activities and those between trainers and students that distinguish learning experiences (Damon, 1984).

The choice of collaborative and inclusive teaching methods has allowed students to reach higher levels of acquisition of skills starting from the resolution of various problematic situations proposed with different semantic and symbolic codes, based on comparison and collaboration between equals (Vigotsky, 1987) with an instrumental approach in which the artifacts, both technological, such as the software available, and the traditional mathematical tools perform the function of semiotic mediators (Rabardel, 2002) for an embodied learning.

To overcome the difficulties in learning the elements of mathematics decontextualized from the traditional school path, difficulties that originate from the ambiguity in language in the traditional dichotomy between thought and communication, the students worked to refine interpersonal communication, in the direction that Sfard defines "comognition" that is the combination of communication (self-referential and recursive) and cognition (ability to accumulate complexity) (Sfard, 2021). Students worked on languages to acquire mathematical discourse.

4. Didactic laboratory activities

In this paragraph, the path proposed by the students will be summarized with references to some of the works that have been analyzed during the activities. These works are famous and easily available on the Internet, so we have chosen not to include them in the article for copyright reasons.

Students met the mathematicians who dealt with mathematical games, starting from the historical and cultural framework, working to solve the proposed riddles and then facing the theory underlying the logical or mathematical resolution of the questions.

Through group laboratory dynamics, the intent of the activities is to develop the awareness that mathematics acts as cultural glue, as a bridge between humanistic and scientific culture to highlight how much mathematics is also underlying areas not conventionally associated with it.

The choice of proposing well-known texts by significant authors allowed students to face a problem-posing and problem-solving process starting from the analysis of sources in peer-to-peer education mode.

In order to be consistent with the historical period addressed in the curricular programs, researchers have focused the activities on the most charismatic and interesting figures from the didactic and heuristic research point of view. Students met August Ferdinand Mobius and the famous ribbon, they explored geometry by building and cutting the ribbon at various levels observing geometric objects with different topological characteristics, their twists and then they discovered the references in Escher's works through the analysis of some of his works.

Researchers talked about infinity by observing Escher's “Ants”, in an engraving from 1963, ants that walk indefinitely on the belt, covering its entire surface.

Researchers introduced "Relativity" with the lithography made in 1953 which summarizes the concept of "paradox", an impossible image made up of many possible scenes, men going up or down stairs, others looking out onto a balcony, paths they result in incommunicability, in a relative world, a world without a precise point of view, and it is this element that gives the work its title. But the couple walking embraced represents the hope of finding themselves in a common perspective.

The circularity of time has been modeled on the geometric figure of the Moebius strip that Friedrich Nietzsche renders, in philosophy, with his theory of the Eternal Return of the Equal. Nietzsche declaring in Ecce Homo (Nietzsche, Ecce Homo, 1888) that he was struck by this thought, perhaps the most profound and decisive of all his philosophy in 1881 and provides a formulation of it just a year later with Aphorism 341 of The gay science (Nietzsche, The Gay Science, 1882), in which he speaks of his belief that all the events of the universe are destined to repeat themselves indefinitely equal to themselves within a vicious circle in which the cosmos itself is located, and which leads it to originate and dissolve forever.

But the famous strip is also represented in music, in particular students analyzed the enigmatic Canon 1 à 2 from J. S. Bach's Musical Offering (1747) The manuscript depicts a single musical sequence that is to be played front to back and back to front with symmetries and transpositions that recall symmetries and translations in geometry. To integrate the analysis from an interdisciplinary point of view, numerous passages from the book "Gödel, Escher, Bach" have been proposed (Hofstadter, 1984).

Interesting are ideas offered by the students who cited the film "Avengers: Endgame" (2019) where Mobius Strip is used as a representation of the circularity of time. In a scene, the protagonist Tony Stark designs a time travel model on the paradoxical geometric figure of the Mobius Strip, the image par excellence of a vicious circle.

Starting from the 1940s, we also find many literary tales in which the Moebius strip plays an important role and one of the first and most creative is "No-Sided Professor" written by Martin Gardner (Gardner, The No-Sided Professor, 1946). In history the members of the Moebius Society, an organization of mathematicians who devote themselves to topology, meet and sit around a table set with silver Moebius ribbon-shaped napkin rings and Klein bottles as coffee cups.

From 1956 to 1981, Martin Gardner edited, for the magazine Scientific American, a column of mathematical puzzles and games, which became popular all over the world.

With him students tackled also the puzzle stories he published in the columns of a science fiction newspaper, discovering that thanks to his popularizing efforts, an asteroid now bears his name: 2587gardner.

One of the most fascinating problems disclosed by Martin Gardner, in the course of his long career, can be considered the paradox of the gambler who proposes to the bystanders three hole cards, one of which is an ace: “You choose one, betting that it is the ace”. At this point, the player reveals one of the two remaining cards, showing the bystanders that it is not the ace, and offers them to change their initial choice. What do they decide to do?

Gardner first presented this problem, in October 1959, in a different formulation (instead of the three cards, there were three prisoners, one of whom had been pardoned by the local governor). Then in 1990, Marilyn vos Savant, author of a popular column in Parade magazine, proposed a further version, which is now the best known version in its classical form, the Monty Hall Problem (MHP):
“You are a player on a game show and are shown three identical doors. Behind one is a car, behind the other two are goats. Monty Hall, the host of the show, asks you to choose one of the doors. You do so, but you do not open your chosen door. Monty, who knows where the car is, now opens one of the doors. He chooses his door in

accordance with the following rules:

- Monty always opens a door that conceals a goat.
- Monty never opens the door you initially chose.
- If Monty can open more than one door without violating rules one and two, then he chooses his door randomly. After Monty opens his door, he gives you the choice of sticking with your original choice or switching to the other unopened door. What should you do to maximize your chances of winning the car?"

The problem quickly acquired global popularity, coming to be evaluated as the most beautiful probabilistic paradox of the second millennium (Gillman, 1992).

This very famous paradox was an opportunity to present activities related to the calculation of probability and riddles with counterintuitive solutions.

In the laboratory, a connection has been developed between Gardner and the author who was treated soon after, Lewis Carroll, in fact the classics Alice's Adventures in Wonderland in the version "The Annotated Alice" (Gardner, The Annotated Alice, 1960), include numerous side notes by Martin Gardner on the meanings behind the words and the solutions to the riddles buried in the famous text.

Lewis Carroll's excerpts from "Alice in Wonderland" (Carrol, 1865) have been analyzed, with real logical and verbal games that are always fun. Students told of having rediscovered a story they had read when they were children but without having grasped its subtlety and wit. Furthermore, students met some mathematical puzzles selected from the "Pillow Problems" that Carroll never managed to publish and, in particular, they dedicated times to solve the questions through the Numbers Theory.

Continuing with the path in the activities of the mathematicians of logic games, the students encountered and discovered other connections: Escher, who they had already encountered with the Moebius strip, came back when researchers introduced the tessellations. In Escher's drawings the shape of the pieces (usually real or

fantastic animals) is carefully studied in order to allow a perfect correspondence as for the pieces of a puzzle ...

In 1964 Escher said "I was carried away by the irresistible pleasure I felt in repeating the same figures on a piece of paper". It was therefore natural for students to investigate the mathematics (in particular the theory of groups) that makes it possible to classify the various periodic tessellations from the point of view of their structure. Tessellations with regular polygons and non-periodic ones were constructed. Among the most famous non-periodic tessellations researchers proposed the masterpieces of Roger Penrose. The artistic beauty of the numerous tessellation examples fascinated students and they designed and customized examples of tessellations thanks to the use of dynamic geometry software.

The activities also included a discussion of the works of Roger Penrose who devoted himself to the study of black holes and general relativity, but who also a great expert in math games. He rightly believed that mathematics is the funniest game and in his works he addressed the issues that made him popular, in particular mathematical games and tessellations, including his best-known book, “The Emperor's New Mind” which was published in 1989 and immediately became a best seller (Penrose, 1999). "A wonderful book for intelligent laymen", defined it Martin Gardner to whom, curiously, Penrose entrusted the preface to the book.

We concluded the educational project in a “journey” into logic thanks to Raymond Smullyan, undoubtedly one of the most important contemporary authors of logical and mathematical games, who led the students to travel between worlds inhabited by lying and truthful characters and made it possible to apply the contents of classical logic. In fact, these riddles can be translated into a compact logical form through truth tables and logical connectives and can be solved both through the narrative developed with natural language and through the analysis of synthetic logical structures.

And like the Moebius strip that after each path brings us back to the starting point,

even when the students treated Smullyan, they met again an author they had already dealt with: Carroll. Smullyan in fact set a series of riddles in the world of "Alice in Puzzle-land", a world in which the characters had the peculiar characteristic of being always wise (and sincere) or crazy (and liars), (Smullyan, 1986)

5. Conclusions

In the final brainstorming debate and in the forms fulfilled by the students, it emerged their high involvement; they enjoyed challenging each other in teams in the resolution of the proposed questions. A significant acquisition of the contents addressed and a high appreciation for the didactic choice of linking mathematics with literature, history, art, music and science emerged in the answers of the questionnaires.

Students were also very impressed by the choice of the methodology for developing the issues encountered. In fact, in the planning of the didactic activity it was decided to meet the characters, their stories, their cultural context, the mathematical stimuli not with a chronological narrative but looking for the connections that link ideas, people, thematic.

If we imagine the themes developed in the project as a ball of wool, in the activity we have chosen to cross it like a sock needle that passes from one side to the other by touching the various threads, rather than unravelling it starting from one end and unrolling it a little at a time.

6. Acknowledgements

The authors acknowledge Prof. Antonio Di Nola (Department of Mathematics, University of Salerno, Italy) as beta-reader of the paper.
The authors acknowledge Prof. Benedetto Di Paola (Department of Mathematics, University of Palermo, Italy) and Prof. Silvia Benvenuti (Department of

Mathematics, University of Bologna, Italy) as non-anonymous reviewers of the paper.

7. References

Barzel, B. (2007). New Technology? New ways of teaching- Noo time left for that! *International Journal of Technologies in Mathematics Education, 14*(2), 77-86.

Bauman, Z. (2000). *Liquid modernity.* Cambridge: Polity Press.

Carrol, L. (1865). *Alice in Wonderland.* London: Macmillan & Co.

Damon, W. (1984). Peer Education: The Untapped Potential. *Journal of Applied Developmental Psycology, 5*, 331-343.

Gardner, M. (1946). *The No-Sided Professor.* Prometheus Books.

Gardner, M. (1960). *The Annotated Alice.* WW Norton & Co Inc.

Gartner, M. (2017). *Gartner Insights on How to Lead in a Connected World.* Retrieved from www.gartner.com: http://www.gartner.com/imagesrv/books/iot/iotEbook_digital.pdf

Gillman, L. (1992). The Car and the Goats. *The American Mathematical*, 3-7.

Harel, I. P. (1991). *Constructionoism.* Ablex Publishing.

Hofstadter, D. (1984). *Godel, Escher, Bach.* Basic Books.

Kafai, Y. R. (1996). *Constructionism in practice: Designing, thinking and learning in a digital world.* Routledge.

Morin, E. (1999). *Seven complex lessons in education for the future.* UNESCO.

Nietzsche, F. (1882). *The Gay Science.* Barnes & Noble.

Nietzsche, F. (1888). *Ecce Homo.* Adelphi.

Papert, S. (1990). *Mindstorms; Children, computers and powerful ideas.* Basic Books Inc.

Penrose, R. (1999). *The Emperor's New Mind.* OUP Oxford.

Prigogine, I. (2004). Beyond, Being and Becoming. *New Perspectives Quarterly, 21*(4), 5-12.

Rabardel, P. (2002). *People and Technology - A Cognitive Approach to Contemporary Instruments.* Retrieved from http://ergoserv.psy.univ.paris8.fr

Rogora, E. T. (2021). Interdisciplinarity for learning/teaching mathematics. *Bolema: Boletim de Educacao Matematica*.

Sfard, A. (2021). Development of Mathematical discourse: Some insights from communicational research. *International Journal of Education Research, 51-52*, 1-9.

Smullyan, R. (1986). *Alice in Puzzle-Land.* William Morrow & Co.

Tabary, Z. (2015). *The skills agenda: preparing students for the future.* Retrieved from The Economist Intelligence Unit: https://eiuperspectives.economist/talent-education/driving-skills-agenda/blog/skills-agenda-preparing-students-future

Vigotsky, L. (1987). *Mind in society: The development of higher psychological processes.* Harvard University Press.

Wellings, J. L. (2009). *The digital promise: transforming learning with innovative uses of technology.* Sesame Workshop.